Optimizing maintenance decision in rails – a Markov Decision Process approach

Luís Pedro Correia Beatriz Sancho
luis.sancho@tecnico.ulisboa.pt

Instituto Superior Técnico, Universidade de Lisboa – Portugal
April 2020

Abstract. Nowadays, railway transport is one of the most sustainable means of transportation, guaranteeing a quick movement of passengers and freights over short and medium distances with reduced usage of fossil fuels. The growing demand for this transport mode represents a major challenge to railway infrastructure managers in an attempt to guarantee cost-effective solutions without compromising the safety and reliability of railway infrastructures. The aim of the present dissertation is to provide an optimal decision map to support maintenance decisions in rail component. A Markov Decision Process (MDP) approach is followed to derive an optimal policy that minimizes the total costs over an infinite horizon depending on the different condition states of the rail. A practical example is explored with the estimation of the Markov Transition Matrices (MTMs) and the corresponding cost/reward vectors. The MTMs states are defined in terms of rail width, height, accumulated Million Gross Tons and damage occurrence. The optimal policy represents a condition-based maintenance plan with the aim of supporting railway infrastructure managers to take the best maintenance decision among a set of three possible actions depending on the state of the rail. The results obtained indicate that UIC60 rail profile requires that preventive maintenance actions are performed earlier than UIC54 rail profile.

Keywords: Railway maintenance, condition-based maintenance**,** optimizing maintenance, Markov Decision Process (MDP), wear, damage.

1. Introduction

With the increase of worldwide population, railway transport is becoming an even more relevant mean of transportation as an alternative to road and air vehicles. Climate changes and traffic demand are also increasing and it is very important to reduce the usage of fossil fuels and guarantee a quick transportation of passengers and freights at the same time. To fulfil these growing needs it is important to improve reliability and reduce the life-cycle cost of railway infrastructures, namely construction and maintenance costs. Railway track geometry and design as well as construction, inspection and maintenance procedures must comply with a set of technical standards. Nowadays, rail transport is characterized by the safety, comfort, low-cost usage and quick transportation of passengers, freights or other goods over short and medium distances.

A railway track is responsible for supporting and guiding the rail vehicle along the track. The track is subjected to dynamic forces resultant from the contact between rails and wheelsets, which leads to the degradation of rail profiles and deviations in track geometric parameters and track layout, commonly known as track irregularities.

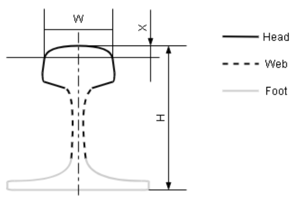

Figure 1: Rail profile.

As Figure 1 shows, rail profiles are characterised by several dimensions, being two of the most important:

- Height (H) – the linear distance between the two intersections of rail symmetry line with rail profile;

- Width (W) – the linear distance between the two intersections with rail head of a line located X mm below the top intersection of rail symmetry line with rail profile. In the present dissertation, data from two rail profiles is used: UIC54 (54E1) and UIC60 (60E1). For the UIC54 rail profile, $X = 14.1$ mm, $W = 70$ mm and $H = 159$ mm and for the UIC60 rail profile, $X = 14.3$ mm, $W = 72$ mm and $H = 172$ mm. UIC54 and UIC60 rail profiles are manufactured and tested according to the European standard EN 13674-1 [1].

The main objective of this research work is to create an optimal decision map using the MDP approach in order to support maintenance decisions for the rail component in Portuguese railways using a condition-based maintenance policy which implies that maintenance actions are triggered by the actual condition of the rail.

2. Degradation, inspection and maintenance in the railway track

2.1 Degradation

Rail profiles are continuously changing due to the loads and the high speeds that the rail is subjected during the passage of the vehicles, besides environmental conditions. High normal and lateral forces in the contact zone between wheelsets and rail tracks resultant from the traction and braking of rail vehicles may lead to yielding and fatigue of rail material. Consequently, vehicle dynamics are affected as track irregularities and worn rail profiles result in an increase in rail vehicle dynamic loads, vibrations and noise and may bring dangerous consequences such as derailments.

On a straight track, wheel treads are in contact with the top of the rail head and rolling interactions are more significant, typically producing lower wear rates than sliding interactions. In a curve or transition curve, the wheel flange might be in contact with the gauge corner of the rail head and sliding interactions can become predominant. Under these circumstances, the load is applied in a smaller area, which results in higher contact stresses, predominantly above the elastic limit, and consequent plastic deformation of the rail head leading to higher wear rates [2].

Besides wear, another important degradation/failure mechanism in rail profiles and the whole track is damage occurrence. According to EN 13231-5 [3], rolling contact fatigue (RCF), which results from the stresses' characteristics of the contact zone between wheels and rails, is the most common rail degradation/failure mechanism on European tracks.

2.2 Inspection

Due to railway track degradation, regular measurements of the track are required in order to detect either functional or safety failures. Track geometric parameters and rail profiles should be measured on a regular basis according to technical procedures defined in the European Standards.

Rail profile wear must be within specified tolerance ranges defined in the Portuguese standard IT.VIA.021 [4]. Non-destructive testing (NDT) is also a common practice within rail inspection procedures for the detection of either internal or external rail defects.

2.3 Maintenance

Railway maintenance is a set of operations carried out by railway infrastructure managers in order to keep railway infrastructures and equipment in a good, reliable and safe operational condition according to several quality and safety standards at minimum cost**.**

Rail reprofiling consists in removing surface defects such as corrugations or track irregularities resultant from rail vehicle operation or small manufacturing defects and is useful to maintain wheel-rail contact conditions at an acceptable level. According to EN 13231-3 [5], three different tolerance classes are specified for rail reprofiling.

A track renewal should occur when rail dimensions or track geometric parameters are out of the standardized tolerances. Several aspects are also taken into account such as the quality of the sleepers, fasteners, rail pads as well as the actual and future associated maintenance costs.

3. Survival Analysis and Markov Decision Process

3.1 Survival Analysis

Reliability is the probability that a component can perform its service functions longer than a specific period of time (called "survival time") under service conditions. Denoting the survival time of a component by T and the specific time value for that variable by t, the reliability function $R(t)$ represents the probability that the variable T exceeds any given value of time *t* as written in the equation below:

$$R(t) = \mathrm{P}(T > t) \tag{1}$$

$F(t)$ represents the probability of failure of a component until or at the instant of time *t* and is given by:

$$F(t) = 1 - R(t) \tag{2}$$

The time to failure distribution $f(t)$ represents the probability of failure of the component at the instant of time *t* and is obtained by derivation of expression (2).

$$f(t) = F'(t) = -R'(t) \tag{3}$$

The Hazard Rate $h(t)$ represents the instantaneous potential per unit of time that the failure of the component occurs at instant time *t*, given that no failure occurred until time *t:*

$$h(t) = \frac{f(t)}{R(t)} = -\frac{R'(t)}{R(t)} \tag{4}$$

The Cumulative Hazard Rate $H(t)$ is obtained by integrating $h(t)$ over time:

$$H(t) = \int_0^t h(t)dt = -\ln(R(t)) \tag{5}$$

3.2 Markov Decision Process

An MDP is a sequential decision-making process. It involves taking the best decision or action from a finite set of actions $a \in \{1,2, \dots A\}$, for each state the chain is with $s \in \{0,1,2,3, \dots, N\}$. The main objective is to create a set of decisions for each state, called policy, in order to maximize the sum of all rewards.

Markov chain model consists on a random (or stochastic) sequence of states equally spaced in points of time called epochs (n). The time between each epoch is called period or step. At each epoch n, the chain is described by a random vector of states X_n which represents the probability of the chain be in each one of the states (s) with s ∈ $\{0,1,2,3,\ldots,N\}$. The sum of the entries of vector X_n is 1 since the chain is in any of the states at each epoch [6].

$$X_n = [P(s=1)\ \ P(s=2)\ldots P(s=N)] \quad (6)$$

The transition probabilities for a Markov chain represent a one-step or one-period transition probability and are organized in a $N \times N$ square matrix where N represents the number of possible states. This square matrix is called Markov Transition Matrix (MTM). An MTM is composed by a set of conditional probabilities that state the probability p_{ij} that the chain is in state j at epoch $n+1$, given that it is in state i at epoch n and action k is taken. Each MTM has non-negative entries and no entries greater than 1.

$$P = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1N} \\ p_{21} & p_{22} & \cdots & p_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ p_{N1} & p_{N2} & \ldots & p_{NN} \end{bmatrix} \quad (7)$$

Let the chain be described by a vector of states X_n at epoch n. Then, using the MTM, denoted by P, the vector of states X_{n+1} is obtained for epoch $n+1$ using the following expression:

$$X_{n+1} = X_n.P \quad (8)$$

Given a chain described by an initial vector of states X_0 and an MTM denoted by P, the vector of states X_n after n epochs is obtained applying the following expression:

$$X_n = X_0.P^n \quad (9)$$

The reward vector q_i is a vector whose entries represent the immediate rewards earned at the end of each epoch by visiting state i and taking a specific action. In the majority of the cases, immediate rewards are assumed to be stationary over time. This means they do not depend on the epoch they are earned, but only on the state the chain is and the action taken.

$$q_i = [q_1\ \ q_2\ \ q_3 \ldots q_N]^T \quad (10)$$

The vector of expected rewards R earned after n steps is given by:

$$R = P^n.q \quad (11)$$

The optimal policy obtained is given by a decision vector $d(n)$ that defines the best action to take at each epoch n given that the chain is in state i at epoch n.

$$d(n) = [d_1(n)\ \ d_2(n) \ldots d_N(n)]^T \quad (12)$$

For an infinite horizon, decision vector d obtained is stationary over time which implies that it will always specify the same action depending on the state the chain is, regardless of the epoch.

An optimal maintenance decision plan for Iranian railways was obtained by Shafahi and Hakhamaneneshi [7] considering a planning horizon of 10 years where a Markov chain is used to predict track deterioration and the ability of the track to perform its function, which are described by the Track Quality Index (TQI). Sharma et al. [8] provided an MDP optimal maintenance policy based on data collected from a Class I railroad in North America in terms of the TQI considering three possible actions ("Major Maintenance", "Minor Maintenance" and "No Maintenance").

However, none of these previous studies has considered wear in terms of rail profile height and width dimensions, neither have they included the accumulated Million Gross Tons (MGT) of traffic that have passed over the rail as a variable in Markov chain states. Moreover, the "Grinding" action is not considered in any of the previous studies regarding optimal maintenance policies for railways using MDP.

4. Application

In this fourth chapter, the main steps to estimate the Markov Transition Matrices' (MTMs) probabilities are provided, as well as an MDP application for this practical case. The problem is divided into three possible actions that can be performed after rail inspection: "Do Nothing", "Renewal" and "Grinding". Since two rail profiles (UIC54 and UIC60) are considered in this practical case, two optimal maintenance strategies will be obtained.

4.1 Definition of MDP state space

The MDP state space is defined as a combination of four variables: rail width (W), height (H), accumulated MGT and damage occurrence. Rail width (W) and height (H) variables are grouped in 1 mm intervals (I), assuming a maximum (M) and minimum (m) value. The minimum values considered for rail profile width (W) and height (H) variables for UIC54 and UIC60 rail profiles are defined according to the alert limit wear values for track speeds less or equal than 80 km/h specified in IT.VIA.021 [4]. The maximum values are defined according to the initial width (W) and height (H) dimensions for UIC54 and UIC60 rail profiles. The states in which width (W) and/or height (H) reach their minimum interval are called scrap states. Thus, the rail is considered to be in a scrap state when the minimum interval in width (W), in height (H) or in both width (W) and height (H) states is reached. Moreover, the damage variable only assumes two nominal values: "with damage" or "without damage"; whereas the accumulated MGT variable is discretized in steps of 8 MGT, from 0 MGT to 352 MGT.

The summary of the MDP state space for UIC54 and UIC60 rail profiles is presented in Table 1.

Table 1 – Summary of the MDP state space for UIC54 and UIC60 rail profiles.

		UIC54 profile			UIC60 profile		
Var.	**I**	**m**	**M**	**Nr of states**	**m**	**M**	**Nr of states**
W	1 mm	57 mm	70 mm	13	56 mm	72 mm	16
H	1 mm	145 mm	159 mm	14	156 mm	172 mm	16
MGT	8	0	352	45	0	352	45
Dam.	-	-	-	182	-	-	256
Total nr of states	-	8 372			11 776		

4.2 MTM for the "Do Nothing" action ($a = 1$)

The "Do Nothing" action ($a = 1$) consists in assuming that the rail is in an acceptable condition and able to continue in service.

4.2.1 Wear analysis

This analysis is based on data provided by the Portuguese railway infrastructure company from inspections that were made in "Linha de Cintura". The values of the change in the rail width due to wear (ΔW) and change in the rail height due to wear (ΔH) as a function of the accumulated MGT for UIC54 rail profile are represented, respectively, in Figures 2 and 3. The values of the change in the rail width due to wear (ΔW) and change in the rail height due to wear (ΔH) as a function of the accumulated MGT for UIC60 rail profile are represented, respectively, in Figures 4 and 5.

The probabilities that, at any given state, rail profiles width (W) or height (H) states decrease one interval can be derived by assuming only neighbouring state transitions are possible. For instance, an UIC54 rail profile can only decrease 1 mm in width (W) every 8 MGT with a certain probability p_{W54} or remain with the same width (W) with probability $1 - p_{W54}$. Then, the average width (W) wear can be expressed as $p_{W54} \cdot (1) + (1 - p_{W54}) \cdot (0)$, and by making it equal to the mean wear estimated in Figure 2, the expression (13) can be estimated. The same is applicable for UIC54 rail profile height (H) and UIC60 rail profile width (W) and height (H). The probabilities that rail width (W) or height (H) states of UIC54 and UIC60 rail profiles decrease one interval from epoch *n* to epoch *n+1* depend only on the rail accumulated MGT at epoch *n.*

- **UIC54 rail profile**

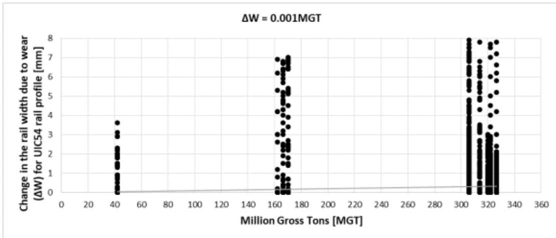

Figure 2 – Change in the rail width due to wear (ΔW) for UIC54 rail profile.

$$p_{W54}(MGT) = \frac{\Delta W(MGT+8) - \Delta W(MGT)}{1} = 0.008, \quad MGT = 0, 8, 16, 24, \ldots, 344 \tag{13}$$

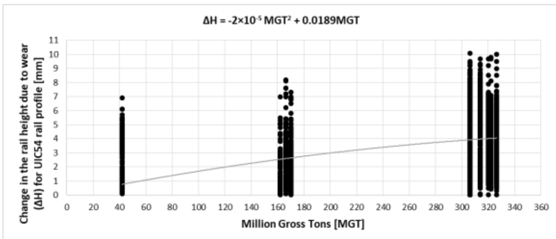

Figure 3 – Change in the rail heigth due to wear (ΔH) for UIC54 rail profile.

$$p_{H54}(MGT) = \frac{\Delta H(MGT+8) - \Delta H(MGT)}{1}, \quad MGT = 0, 8, 16, 24, \ldots, 344 \tag{14}$$

- **UIC60 rail profile**

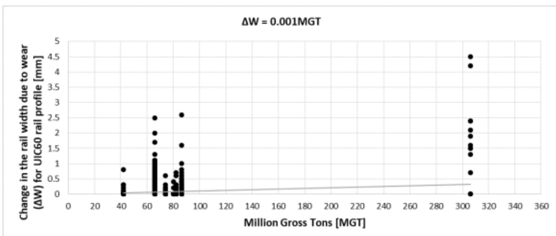

Figure 4 – Change in the rail width due to wear (ΔW) for UIC60 rail profile.

$$p_{W60}(MGT) = \frac{\Delta W(MGT+8) - \Delta W(MGT)}{1} = 0.008, \quad MGT = 0, 8, 16, 24, \ldots, 344 \tag{15}$$

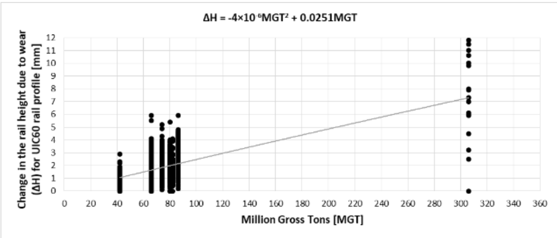

Figure 5 – Change in the rail heigth due to wear (ΔH) for UIC60 rail profile.

$$p_{H60}(MGT) = \frac{\Delta H(MGT+8) - \Delta H(MGT)}{1}, \quad MGT = 0, 8, 16, 24, \ldots, 344 \tag{16}$$

4.2.2 Damage analysis

The first stage of analysis is based on data from "Linha do Norte". These points are plotted in Figure 6 and represent the defects detected per kilometer per year for each year of rail lifetime. The rail maximum lifetime is here assumed to be 45 years.

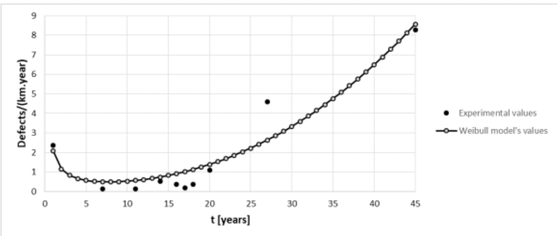

Figure 6 – Comparison between the experimental values and the values obtained from the additive Weibull model.

The points represented in the graph above can be modelled as an additive Weibull model's Bathtub-Shaped Curve $B(t)$ as a function of t, in years, which consists on the combination of two Weibull distributions and is given by:

$$B(t) = ab(at)^{b-1} + cd(ct)^{d-1}, t \geq 0 \tag{17}$$

The parameters a, b, c and d are estimated by minimizing the nonlinear least-square errors (with the MATLAB function *slqcurvefit*). Given initial values $a = 0.05$, $b = 2$, $c = 10$ and $d = 0.2$, the solution obtained is $a = 0.0895$, $b = 3.3911$, $c = 21\,499\,000$ and $d = 0.1539$. Having estimated $B(t)$ for our rail damage occurrence data, the rate of defects per kilometre per year can be predicted for each of the 45 years of rail lifetime. The number of accumulated defects per kilometre $N(t)$ in defects/km for each of the 45 years of rail lifetime t are obtained from the following equation:

$$N(t) = \sum_{j=0}^{t} B(j), t = 0, 1, \ldots, 45 \tag{18}$$

It is now possible to do a survival analysis considering a group of 128.85 individuals in order to obtain the reliability values for each of the 45 years of rail lifetime. An annual average traffic value of 8 MGT is assumed for "Linha do Norte". The reliability curve obtained for this case is shown in Figure 7.

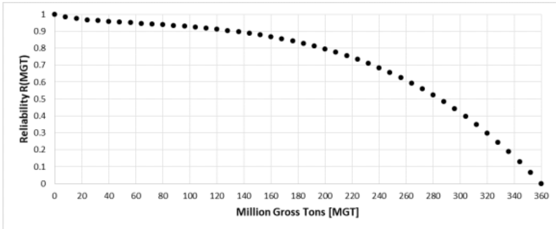

Figure 7 – Reliability values for each of the 45 years of rail lifetime (a maximum of 360 MGT).

The cumulative hazard rate $H(MGT)$ is obtained by replacing in expression (19) the reliability $R(MGT)$ values obtained previously.

$$H\,(MGT) = -\ln\big(R(MGT)\big), \quad MGT \in \{0, 8, 16, 24, \ldots, 352\} \tag{19}$$

The hazard rate associated with damage can then be estimated, which is assumed to be equal to the probability of transiting to a damaged state. Therefore, it represents the probability that a rail at a certain state at epoch *n* transits to a state with damage at epoch *n+1*. These probabilities depend on the rail accumulated MGT at epoch n and are estimated using following expression:

$$h_{Damage}(MGT) = p_{Damage}(MGT) = H(MGT+8) - H(MGT), \quad MGT \in \{0, 8, 16, 24, \ldots, 344\} \tag{20}$$

4.2.3 Markov Transition Matrices

Considering transitions in wear and in damage as independent events, the probability of their joint transitions can be computed as:

$$p(Trans.\,in\,Wear \wedge Trans.\,to\,Damage) = p(Trans.\,in\,Wear).p(Trans.\,to\,Damage) \tag{21}$$

Regarding the "Do Nothing" action ($a = 1$) MTM probabilities for both UIC54 and UIC60 rail profiles, several assumptions are made:

- The probability of an increase in rail width (W) or height (H) states is assumed to be zero;

- The transitions to the next states are limited, which means that a two or more intervals decrease of rail width (W) or height (H) states is considered impossible, i.e. with null probability;

- The probability of a state transition only depends on the rail accumulated MGT, regardless of the rail width (W) or height (H) states;

- The transition step between each epoch is 8 MGT. Therefore, when the rail transits to a state without damage, it transits to a state with 8 more MGT, unless it is in a state with damage or in a state with 352 accumulated MGT;

- The state transition to a damaged state assumes that the rail maintains the same width (W) and height (H) states.

4.3 MTM for the "Renewal" action ($a = 2$)

The "Renewal" action ($a = 2$) consists in assuming that the rail is unable to continue in service and replace it, regardless of the state it is. Therefore, for every state the rail is at epoch *n*, when a "Renewal" action occurs, it is certain that it transits to the initial state s_1 at epoch *n+1*.

- **UIC54 rail profile**

$$P_2^{54} = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 1 & 0 & 0 & \cdots & 0 \\ 1 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 0 & 0 & \cdots & 0 \end{bmatrix} (8\,372 \times 8\,372) \qquad (22)$$

- **UIC60 rail profile**

$$P_2^{60} = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 1 & 0 & 0 & \cdots & 0 \\ 1 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 0 & 0 & \cdots & 0 \end{bmatrix} (11\,776 \times 11\,776) \qquad (23)$$

4.4 MTM for the "Grinding" action ($a = 3$)

The "Grinding" action ($a = 3$) consists in removing small or more severe surface defects on the rail head. For this particular practical case, only preventive grinding (for the undamaged states) and corrective grinding (for the damaged states) are considered. Regarding the "Grinding" action ($a = 3$) MTM probabilities for both UIC54 and UIC60 rail profiles, four assumptions are made:

- Only rail height (H) is affected, which means that the rail remains in the same width (W) state and can only decrease intervals in height (H) state in each transition;

- The rail transits to a state with 0 accumulated MGT since it is considered to be completely repaired, and a new damage cycle is reinitiated;

- Unlike in the "Do Nothing" action case, the transition probabilities for preventive and corrective grinding are independent of the rail accumulated MGT;

- As it is not possible to grind a rail beyond the minimum interval in height (H) state, when the rail reaches this condition and the probabilities obtained indicate height losses that go beyond that minimum interval in height (H) state, then the probabilities of the remaining transitions are summed up, becoming the probability value to stay at the state with minimum interval in height (H).

- **Preventive grinding**

For the preventive grinding the transitions to the next states are limited, which means that a two or more intervals decrease of rail height (H) state is considered impossible. Denoting p_{pg} as the probability of a one interval decrease in rail height (H) state, the possible transitions are represented below for a rail height (H) state above the minimum interval.

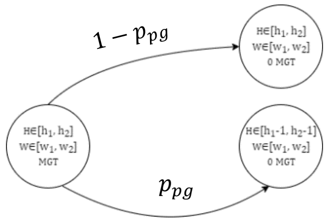

Figure 8 – Possible transitions for a state with height (H) above the minimum interval for the preventive grinding.

The average rail wear μ_{pg} associated to a preventive grinding is around 0.3 mm. Considering a wear of 0 mm for the case in which the rail remains in the same height (H) state, and 1 mm of wear for the one interval decrease in height (H) state, the transition probability p_{pg} can then be obtained as follows:

$$\mu_{pg} = 0.3 \Leftrightarrow \qquad (24)$$

$$p_{pg} \cdot (1) + (1 - p_{pg}) \cdot (0) = 0.3 \Leftrightarrow p_{pg} = 0.3$$

- Corrective grinding

For the corrective grinding, the number of occurrences of four different crack lengths observed in a railway track is obtained. The corrective grinding depth is inferred from this sample of crack lengths (see Table 2).

Table 2 – Number of occurrences of four different crack lengths.

Occurrence number (i)	**Crack length (x_i)**	**Number of occurrences (F_i)**	**Relative frequency (f_i)**
1	4 mm	7	7/24
2	5 mm	8	8/24
3	6 mm	8	8/24
4	9 mm	1	1/24
		$\sum F_i = 24$	$\sum f_i = 1$

A normal distribution is used to model the change in the rail height (H) as a result of a corrective grinding operation and is denoted by $X \sim N(\mu, \sigma^2)$. The parameters μ and σ are estimated below:

$$\hat{\mu} = 5.208333 \text{ mm} \qquad (25)$$

$$\hat{\sigma} = 1.141287 \text{ mm} \qquad (26)$$

Bearing in mind the two estimated parameters of the normal distribution, the values of the change in the rail height (H) as a result of a corrective grinding operation are assumed to be between 0 and 10 mm.

4.5 Rewards/cost function

The MATLAB toolbox function chosen to solve this problem uses a reward maximization to derive an optimal policy which maximizes the total rewards earned over an infinite horizon.

4.5.1 "Do Nothing" action ($a = 1$)

The "Do Nothing" action ($a = 1$) does not hold any operational cost. However, it is important to guarantee that when the rail reaches scrap states, states with 352 accumulated MGT or damaged states, other option different from "Do Nothing" action must be chosen. A penalty of -200 thousand monetary units/km is assigned to these critical states.

- UIC54 rail profile

The reward vector for the UIC54 rail profile, $q^{1\,54}(8\,372 \times 1)$ (see equation (27)) is divided into two sub-vectors, $q_{\alpha}^{1\,54}(182 \times 1)$ and $q_{\beta}^{1\,54}(182 \times 1)$ (see equations (28) and (29), respectively).

$$q^{1\,54} = \begin{bmatrix} q_{\alpha}^{1\,54} \\ q_{\alpha}^{1\,54} \\ \vdots \\ q_{\alpha}^{1\,54} \\ q_{\beta}^{1\,54} \\ q_{\beta}^{1\,54} \end{bmatrix} \begin{matrix} \leftarrow & 0\ MGT \\ \leftarrow & 8\ MGT \\ \leftarrow & \vdots \\ \leftarrow & 344\ MGT \\ \leftarrow & 352\ MGT \\ \leftarrow & Damage\ states \end{matrix} \qquad (27)$$

$$q_{\alpha\,i}^{1\,54} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ q_{\alpha\,13}^{1\,54} \\ 0 \\ \vdots \\ 0 \\ q_{\alpha\,26}^{1\,54} \\ 0 \\ \vdots \\ 0 \\ q_{\alpha\,169}^{1\,54} \\ q_{\alpha\,170}^{1\,54} \\ \vdots \\ q_{\alpha\,182}^{1\,54} \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ -200\,000 \\ 0 \\ \vdots \\ 0 \\ -200\,000 \\ 0 \\ \vdots \\ 0 \\ -200\,000 \\ -200\,000 \\ \vdots \\ -200\,000 \end{bmatrix} \qquad (28)$$

$$q_{\beta\,i}^{1\,54} = \begin{bmatrix} q_{\beta\,1}^{1\,54} \\ \vdots \\ q_{\beta\,182}^{1\,54} \end{bmatrix} = \begin{bmatrix} -200\,000 \\ \vdots \\ -200\,000 \end{bmatrix} \qquad (29)$$

- UIC60 rail profile

The reward vector for the UIC60 rail profile, $q^{1\,60}(11\,776 \times 1)$ (see equation (30)) is divided into two sub-vectors, $q_{\alpha}^{1\,60}(256 \times 1)$ and $q_{\beta}^{1\,60}(256 \times 1)$ (see equations (31) and (32), respectively).

$$q^{1\,60} = \begin{bmatrix} q_{\alpha}^{1\,60} \\ q_{\alpha}^{1\,60} \\ \vdots \\ q_{\alpha}^{1\,60} \\ q_{\beta}^{1\,60} \\ q_{\beta}^{1\,60} \end{bmatrix} \begin{matrix} \leftarrow & 0\ MGT \\ \leftarrow & 8\ MGT \\ \leftarrow & \vdots \\ \leftarrow & 344\ MGT \\ \leftarrow & 352\ MGT \\ \leftarrow & Damage\ states \end{matrix} \qquad (30)$$

$$q_{\alpha\,i}^{1\,60} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ q_{\alpha\,16}^{1\,60} \\ 0 \\ \vdots \\ 0 \\ q_{\alpha\,32}^{1\,60} \\ 0 \\ \vdots \\ 0 \\ q_{\alpha\,240}^{1\,60} \\ q_{\alpha\,241}^{1\,60} \\ \vdots \\ q_{\alpha\,256}^{1\,54} \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ -200\,000 \\ 0 \\ \vdots \\ 0 \\ -200\,000 \\ 0 \\ \vdots \\ 0 \\ -200\,000 \\ -200\,000 \\ \vdots \\ -200\,000 \end{bmatrix} \qquad (31)$$

$$q_{\beta\ i}^{1\ 60} = \begin{bmatrix} q_{\beta\ 1}^{1\ 60} \\ \vdots \\ q_{\beta\ 256}^{1\ 60} \end{bmatrix} = \begin{bmatrix} -200\ 000 \\ \vdots \\ -200\ 000 \end{bmatrix} \quad (32)$$

4.5.2 "Renewal" action ($\boldsymbol{a} = \mathbf{2}$)

It was considered for the "Renewal" action ($a = 2$) a cost value of -67.554 thousand monetary units/km. The reward vectors for the UIC54 and UIC60 rail profiles, $q^{2\ 54}(8\ 372 \times 1)$ and $q^{2\ 60}(11\ 776 \times 1)$ respectively, are represented in equations (33) and (34).

- **UIC54 rail profile**

$$q^{2\ 54} = \begin{bmatrix} q_1^{2\ 54} \\ \vdots \\ q_{8\ 372}^{2\ 54} \end{bmatrix} = \begin{bmatrix} -67\ 554 \\ \vdots \\ -67\ 554 \end{bmatrix} \quad (33)$$

- **UIC60 rail profile**

$$q^{2\ 60} = \begin{bmatrix} q_1^{2\ 60} \\ \vdots \\ q_{11\ 776}^{2\ 60} \end{bmatrix} = \begin{bmatrix} -67\ 554 \\ \vdots \\ -67\ 554 \end{bmatrix} \quad (34)$$

4.5.3 "Grinding" action ($\boldsymbol{a} = \mathbf{3}$)

It was chosen a value of -22.630 thousand monetary units/km for the "Grinding" action ($a = 3$). However, when the rail reaches a scrap state, a "Renewal" action is needed rather than a "Grinding" action. Thus, a penalty of -200 thousand monetary units/km is assigned to this critical state.

- **UIC54 rail profile**

The reward vector for the UIC54 rail profile, $q^{3\ 54}(8\ 372 \times 1)$ (see equation (35)) is divided into a sub-vector, $q_\alpha^{3\ 54}(182 \times 1)$ (see equation (36)).

$$q^{3\ 54} = \begin{bmatrix} q_\alpha^{3\ 54} \\ q_\alpha^{3\ 54} \\ \vdots \\ q_\alpha^{3\ 54} \\ q_\alpha^{3\ 54} \\ q_\alpha^{3\ 54} \end{bmatrix} \begin{matrix} \leftarrow & 0\ MGT \\ \leftarrow & 8\ MGT \\ \leftarrow & \vdots \\ \leftarrow & 344\ MGT \\ \leftarrow & 352\ MGT \\ \leftarrow & Damage\ states \end{matrix} \quad (35)$$

$$q_{\alpha\ i}^{3\ 54} = \begin{bmatrix} -22\ 630 \\ \vdots \\ -22\ 630 \\ q_{\alpha\ 13}^{3\ 54} \\ -22\ 630 \\ \vdots \\ -22\ 630 \\ q_{\alpha\ 26}^{3\ 54} \\ -22\ 630 \\ \vdots \\ -22\ 630 \\ q_{\alpha\ 169}^{3\ 54} \\ q_{\alpha\ 170}^{3\ 54} \\ \vdots \\ q_{\alpha\ 182}^{3\ 54} \end{bmatrix} = \begin{bmatrix} -22\ 630 \\ \vdots \\ -22\ 630 \\ -200\ 000 \\ -22\ 630 \\ \vdots \\ -22\ 630 \\ -200\ 000 \\ -22\ 630 \\ \vdots \\ -22\ 630 \\ -200\ 000 \\ -200\ 000 \\ \vdots \\ -200\ 000 \end{bmatrix} \quad (36)$$

- **UIC60 rail profile**

The reward vector for the UIC60 rail profile, $q^{3\ 60}(11\ 776 \times 1)$ (see equation (37)) is divided into a sub-vector, $q_\alpha^{3\ 60}(256 \times 1)$ (see equation (38)).

$$q^{3\ 60} = \begin{bmatrix} q_\alpha^{3\ 60} \\ q_\alpha^{3\ 60} \\ \vdots \\ q_\alpha^{3\ 60} \\ q_\alpha^{3\ 60} \\ q_\alpha^{3\ 60} \end{bmatrix} \begin{matrix} \leftarrow & 0\ MGT \\ \leftarrow & 8\ MGT \\ \leftarrow & \vdots \\ \leftarrow & 344\ MGT \\ \leftarrow & 352\ MGT \\ \leftarrow & Damage\ states \end{matrix} \quad (37)$$

$$q_{\alpha\ i}^{3\ 60} = \begin{bmatrix} -22\ 630 \\ \vdots \\ -22\ 630 \\ q_{\alpha\ 16}^{3\ 60} \\ -22\ 630 \\ \vdots \\ -22\ 630 \\ q_{\alpha\ 32}^{3\ 60} \\ -22\ 630 \\ \vdots \\ -22\ 630 \\ q_{\alpha\ 240}^{3\ 60} \\ q_{\alpha\ 241}^{3\ 60} \\ \vdots \\ q_{\alpha\ 256}^{3\ 60} \end{bmatrix} = \begin{bmatrix} -22\ 630 \\ \vdots \\ -22\ 630 \\ -200\ 000 \\ -22\ 630 \\ \vdots \\ -22\ 630 \\ -200\ 000 \\ -22\ 630 \\ \vdots \\ -22\ 630 \\ -200\ 000 \\ -200\ 000 \\ \vdots \\ -200\ 000 \end{bmatrix} \quad (38)$$

4.6 Optimal policy

A graphical representation of the decision map is obtained for UIC54 and UIC60 rail profiles for rail width (W) and height (H) states with the evolution of the accumulated MGT. This representation is provided for the states with damage and without damage and can serve as a guideline for a condition-based maintenance carried out by railway infrastructure managers. The decision maps for UIC54 and UIC60 rail profiles are represented, respectively, in Figures 9 and 10.

- **UIC54 rail profile**

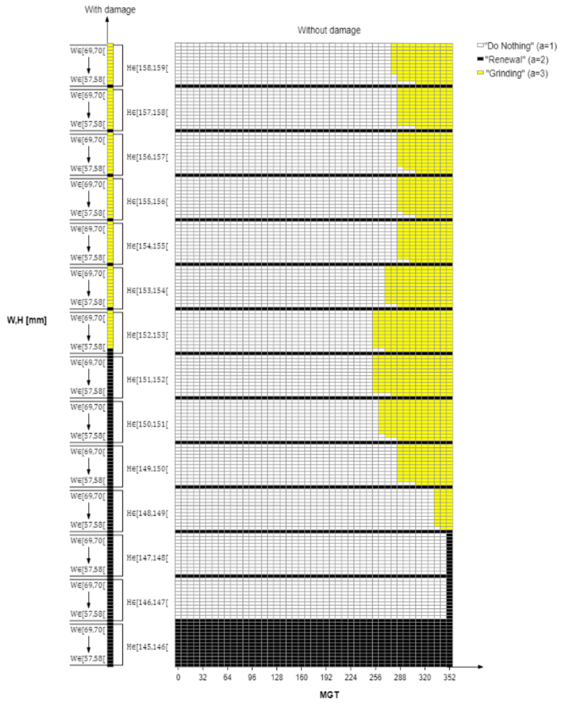

Figure 9 – Decision map for UIC54 rail profile.

- **UIC60 rail profile**

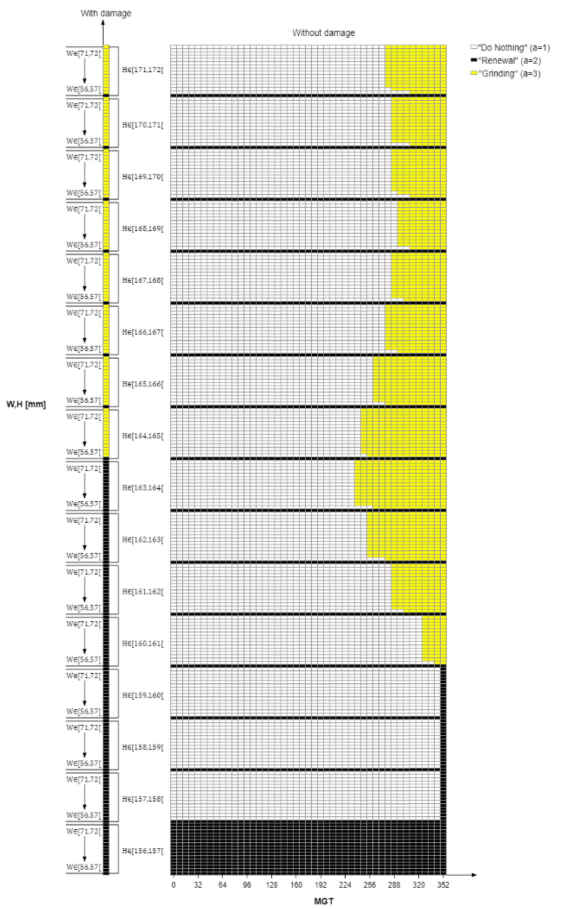

Figure 10 – Decision map for UIC60 rail profile.

5. Conclusions and Further Research

This final chapter presents the main conclusions of the present research work, identifies some limitations and suggests future paths for further research.

5.1 Conclusions

An MDP approach was applied to a practical case of Portuguese railway lines to model rail degradation in terms of rail height (H), width (W), accumulated Million Gross Tons (MGT) and the occurrence of damage. According to these four indicators, an optimal maintenance plan is obtained for UIC54 and UIC60 rail profiles. The state space was divided into 8 372 and 11 776 states, respectively for UIC54 and UIC60 rail profiles. A set of three possible actions were defined: i) “Do Nothing”, ii) “Renewal” and iii) “Grinding”. An optimal decision policy was derived using linear programming and a decision map was provided with the aim of supporting the decision-maker to take the best maintenance decision for each possible rail condition state.

From a detailed analysis of the two decision maps obtained, one can conclude that for damaged rails, only “Renewal” or “Grinding” actions must be performed. “Renewal” actions are mandatory for scrap states since a “Grinding” action would wear the rail beyond acceptable values. For UIC54 rail profile, the “Renewal” action is assigned to rail heights (H) until 7 mm above the minimum interval (145 mm), for all the 13 width (W) states. On the other hand, for UIC60 rail profile, a “Renewal” action is mandatory for rail heights (H) until 8 mm above the minimum interval (156 mm), for all the 16 width (W) states. For undamaged rails, “Renewal” actions are also mandatory for scrap states as one might expect. Comparing the two rail profiles for undamaged states with 352 MGT, for the case of UIC54 rail profile, a “Renewal” action must be carried out for height (H) states until 3 mm above the minimum interval in height (H) state, for all the 13 width (W) states, whereas for the case of UIC60 rail profile, a “Renewal” action must be carried out for height (H) states until 4 mm above the minimum interval in height (H) state, for all the 16 width (W) states. In a nutshell, UIC60 rail profile requires that a “Renewal” action must be performed for height (H) values more above the minimum interval than UIC54 rail profile, for both damaged states and undamaged rails with 352 MGT.

Comparing the two rail profiles for the “Grinding” action performed for undamaged rails, it can be derived that for UIC54 rail profile it is required for rails with accumulated MGT between 256 and 352 and height (H) states between 148 mm and 159 mm, except in rail scrap states. For the case of UIC60 rail profile, a “Grinding” action would be advisable for rails with accumulated MGT between 240 and 352 and height (H) states between 160 mm and 172 mm, except in rail scrap states. The lowest value of the accumulated MGT variable for which a preventive grinding is recommended is 256 MGT for UIC54 rail profile and 240 MGT for UIC60 rail profile. In general, for UIC54 rail profile, preventive grinding actions should be performed later (i.e. for higher values of the rail accumulated MGT). As height (H) states decrease below 151 mm for UIC54 rail profile or below 163 mm for UIC60 rail profile, the preventive grinding would be advisable later. The decreasing slope formed in the maps resulting from the transitions between the "Do Nothing" and "Grinding" actions is approximately the same for the two rail profiles.

Overall, these optimal policies require railway infrastructure companies to have a tight control over their assets, in particular railway lines, in order to constantly monitor the actual condition of the rails and perform the adequate maintenance actions according to these optimal policies.

5.2 Limitations

In the research work, several limitations can be identified. First of all, the MDP approach is a random (or stochastic) process that requires defining transition probabilities, which are calibrated based on data from past inspections/samples. Random factors such as the month of inspection or meteorological conditions under which these inspections were carried out may influence the values obtained.

Relatively to the Markov Transition Matrices (MTMs) for the ''Grinding'' action, the probabilities of the corrective grinding were obtained based on the number of occurrences of four different crack lengths observed in the rail component. It is inferred that when a defect occurs, the corrective grinding depth is equal to the crack length, which is a rough assumption. Although the state space was sufficiently large to describe the different width, height, accumulated MGT and damage occurrence states, it did not control the evolution of track geometric parameters such as gauge, cross level, twist, alignment and longitudinal level, which would be desirable to model railway track as a whole system.

Finally, only accumulated MGT and rail profile (P) are considered to influence rail degradation in the estimation of the MTMs probabilities. Several explanatory variables play also a key role on rail degradation process, though they have not been considered in the estimation of the MTMs probabilities. This is mainly due to the need to make the MDP formulation simpler.

5.3 Further Research

Bearing in mind all the limitations pointed out previously, several steps for further research are suggested in order to overcome these obstacles. For the present research work, only information about the number of occurrences of different crack lengths was obtained. However, for further research, data from rail grinding operations carried out in the track section analysed should be collected in order to improve the accuracy in the MTMs estimation for the ''Grinding'' action.

Also, some more explanatory variables should be integrated in the next MDP approach. For instance, rail data from different track curvatures should be analysed separately and a decision map must be provided for each of those track sections in order to improve the feasibility of the results obtained. It would also be recommended to include track geometric parameters in the form of a Track Quality Indicator as a variable to define the state space. Bearing in mind all these variables, an improved optimal maintenance plan would be obtained.

6. References

[1] BS (British Standard), 2017, "Railway Applications — Track — Rail — Part 1: Vignole Railway Rails 46 Kg/m and above: EN 13674-1:2011+A1:2017," BSI.

[2] Iwnicki, S., 2006, *Handbook of Railway Vehicle Dynamics*, CRC Press.

[3] BS (British Standard), 2018, "Railway Applications — Track — Acceptance of Works — Part 5: Procedures for Rail Reprofiling in Plain Line, Switches, Crossings and Expansion Devices: EN 13231-5:2018," BSI.

[4] REFER, 2009, "IT.VIA.021 — Tolerâncias de Desgaste Do Perfil Transversal Do Carril."

[5] BS (British Standard), 2012, "BSI Standards Publication Railway Applications — Track — Acceptance of Works — Part 3: Acceptance of Reprofiling Rails in Track: EN 13231-3:2012," BSI.

[6] Sheskin, T. J., 2011, *Markov Chains and Decision Processes for Engineers and Managers*, CRC Press.

[7] Shafahi, Y., and Hakhamaneshi, R., 2009, "Application of a Maintenance Management Model for Iranian Railways Based on the Markov Chain and Probabilistic Dynamic Programming," Sci. Iran., **16**(1 A), pp. 87–97.

[8] Sharma, S., Cui, Y., He, Q., Mohammadi, R., and Li, Z., 2018, "Data-Driven Optimization of Railway Maintenance for Track Geometry," Transp. Res. Part C Emerg. Technol., **90**, pp. 34–58.