Multidisciplinary Design Optimization of a Hybrid Rocket

Gustavo Hideki Yamada

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Supervisors:  Prof. Afzal Suleman
              Dr. Frederico José Prata Rente Reis Afonso

Examination Committee
Chairperson:  Prof. Fernando José Parracho Lau
Supervisor:   Prof. Afzal Suleman
Member of the Committee: Dr. Simão Santos Rodrigues

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This master thesis is dedicated to my family and friends from Brazil and Portugal, that supported me through this arduous route.
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Resumo

Esta tese de mestrado propõe um modelo teórico conceitual para o desenvolvimento de um foguete de propulsão híbrida movido a óxido nitroso e parafina. Na área da propulsão, o tanque do oxidante, o tanque do combustível, câmara de combustão e bocal são modelados separadamente. Adicionalmente, é feito um modelo para a transmissão de calor no tanque. Para o estudo do dimensionamento, dados experimentais são usados para desenvolver modelos empíricos para estimar a massa e comprimento de componentes do foguete que combinados a modelos teóricos permitem estimar as dimensões necessárias do veículo. Um estudo da trajetória e estabilidade do foguete são incluídos na disciplina de aerodinâmica. O modelo é validado com dados experimentais de foguetes híbridos.

Uma otimização multidisciplinar é conduzida com o objetivo de minimizar a massa total do veículo e de maximizar seu impulso específico. As variáveis de desenho selecionadas são o volume do tanque, área efetiva de injeção, comprimento do grão de combustível, diâmetro inicial do orifício do combustível, diâmetro da garganta e razão de áreas do bocal, comprimento da câmara de combustão, raio do tanque de oxidante, dimensões das aletas e massas adicionais para estabilização. Uma série de constrangimentos é imposta para assegurar a coerência física do modelo e impor a altitude pretendida.

A otimização é ainda multiobjetivo de forma a combinar os benefícios da diminuição da massa e aumento do impulso específico. O estudo dos resultados é feito utilizando a fronteira de Pareto que facilita a visualização gráfica e seleção dos resultados.

Palavras-chave: Foguete Híbrido, Propulsão Híbrida, Otimização Multidisciplinar, Otimização Multiobjetivo
Abstract

This master thesis proposes a theoretical model for the conceptual design of a nitrous oxide-paraffin hybrid rocket. On the propulsion subject, the oxidizer tank, injector, combustion chamber, and nozzle are modeled separately. Additionally, a heat transfer model is developed for the oxidizer tank. For the mass and sizing study, experimental data are used to develop an empirical model for mass and length estimation, which is combined with theoretical models to estimate the dimensions and masses of the components. A trajectory and stability analysis are performed in the aerodynamics discipline. The model is validated with experimental data from similar hybrid rockets.

Multidisciplinary design optimization is performed with the minimization of the lift-off mass and maximization of the specific impulse as objectives. The selected design variables are the oxidizer tank volume and radius, effective injector area, fuel grain length, fuel grain initial port diameter, nozzle throat diameter and area ratio, combustion chamber length, fins dimensions and additional masses for stabilization. Several constraints are imposed to ensure the physical coherence of the model and precisely select the target altitude.

Multi-objective optimization is then performed to combine the benefits of a higher specific impulse and lower mass. A Pareto frontier is used to analyze and select the results graphically.

Keywords: Hybrid Rocket, Hybrid Propulsion, Multidisciplinary Design Optimization, Multi-objective Optimization
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Nomenclature

Greek symbols

\( \alpha \)  Thermal Diffusivity
\( \beta \)  Expansion Coefficient
\( \Delta \)  Level change of unit input
\( \kappa \)  Dyer Parameter
\( \nu \)  Kinematic Viscosity
\( \rho \)  Density.
\( \sigma \)  Normal Stress
\( \zeta \)  Correction Factor

Roman symbols

\( A \)  Area
\( a \)  Regression Rate Coefficient
\( \text{accel} \)  Acceleration
\( \text{alt} \)  Altitude
\( B \)  Root Chord
\( b \)  Tip Chord
\( c \)  Constraints
\( C_d \)  Injector Discharge Coefficient
\( C_N \)  Normal Force Coefficient
\( c_p \)  Heat capacity
\( \text{CG} \)  Center of Gravity distance to nose cone tip
\( \text{CP} \)  Center of Pressure distance to nose cone tip
D  Diameter

d  Distance

e  Error

F  Force

G  Oxidizer Mass flow per unit of Area

g  Gravitational Acceleration

h  Specific Enthalpy

hc  Convection Coefficient

I  Impulse

I_{sp}  Specific Impulse

K  Thermal Conductivity

k  Heat capacity ratio

L  Length

\dot{m}  Mass Flow Rate

M  Mach Number

m  Mass

N  Nozzle Cover Factor

n  Regression Rate Exponent

\text{Nu}  Nusselt Number

p  Pressure

Pr  Prandtl Number

\dot{Q}  Heat Transfer Rate

\bar{R}  Ideal Gas Constant

\dot{r}  Regression Rate

R  Thermal Resistance

r  Radius

Ra  Rayleigh Number

Re  Reynolds Number
S  Span
s  Specific Entropy
SM Static Margin
T  Temperature
t  Thickness
U  Internal Energy
u  Specific Internal Energy
V  Volume
v  Velocity
\{x\} Design Variables Vector
x  Quality
\{y\} State Variables Vector
\{y^t\} Target State Variables Vector

Subscripts
0  Stagnation
add Additional
addcone Additional at nose cone
air Air property
amb Ambient property
atm Atmospheric property
av Avionic System property
body Rocket External Structure property
c  Combustion
CC Combustion Chamber property
cond Conduction
cone Nose cone property
conv Convection
cylinder Cylinder property
ray  Rayleigh
rec  Recovery System property
rocket  Rocket property
sat  Saturation property
sound  Sound
SPI  Single-Phased-Incompressible
structural  Structural
t  Thrust
tank  Tank content property
th  Throat property
tot  Total
trans  Cross section
vap  Vapour property

Superscripts
-  Average
-  Temporal derivative
## Acronyms and abbreviations

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<td>AAO</td>
<td>All-at-once.</td>
</tr>
<tr>
<td>CEA</td>
<td>Chemical Equilibrium with Applications.</td>
</tr>
<tr>
<td>CFD</td>
<td>Computational Fluid Dynamics.</td>
</tr>
<tr>
<td>COPV</td>
<td>Composite Overwrapped Pressure Vessel.</td>
</tr>
<tr>
<td>COTS</td>
<td>Commercial-Off-the-Shelf.</td>
</tr>
<tr>
<td>DRMOGA</td>
<td>Divided Range Multi-Objective Genetic Algorithm.</td>
</tr>
<tr>
<td>GUI</td>
<td>Guide User Interface.</td>
</tr>
<tr>
<td>HDPE</td>
<td>High Density Polyethylene.</td>
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<tr>
<td>HEM</td>
<td>Homogeneous Equilibrium Model.</td>
</tr>
<tr>
<td>HTPB</td>
<td>Hydroxyl-terminated-Polybutadiene ( (C_4H_6)_n ).</td>
</tr>
<tr>
<td>IDF</td>
<td>Individual Discipline Feasible.</td>
</tr>
<tr>
<td>IREC</td>
<td>Intercollegiate Rocket Engineering Competition.</td>
</tr>
<tr>
<td>IST</td>
<td>Instituto Superior Técnico.</td>
</tr>
<tr>
<td>MDF</td>
<td>Multidisciplinary Feasible.</td>
</tr>
<tr>
<td>MDO</td>
<td>Multidisciplinary Design Optimization.</td>
</tr>
<tr>
<td>NASA</td>
<td>National Aeronautics and Space Administration.</td>
</tr>
<tr>
<td>NIST</td>
<td>National Institute for Standards and Technology.</td>
</tr>
<tr>
<td>NaN</td>
<td>Not a number.</td>
</tr>
<tr>
<td>O/F</td>
<td>Oxidizer-Fuel ratio.</td>
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<tr>
<td>PMMA</td>
<td>Polymethyl Methacrylate.</td>
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<tr>
<td>SAND</td>
<td>Simultaneous analysis and design.</td>
</tr>
<tr>
<td>SPI</td>
<td>Single-Phased-Incompressible.</td>
</tr>
<tr>
<td>SQP</td>
<td>Sequential Quadratic Programming.</td>
</tr>
<tr>
<td>SRAD</td>
<td>Student Researched and Developed.</td>
</tr>
<tr>
<td>USA</td>
<td>United States of America.</td>
</tr>
<tr>
<td>USP</td>
<td>University of São Paulo.</td>
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XDSM  Extended Design Structure Matrix.
Chapter 1

Introduction

This chapter introduces the topics presented in this master thesis, explaining the motivation, objectives and thesis outline.

1.1 Motivation

The study of engineering is one of the most relevant knowledge for technological enhancement. However, most of the universities around the world prioritize the theoretical study, leaving a gap for the practical part. For this reason, activities such as the Formula Student or the Solar Boat from Instituto Superior Técnico (IST), Projeto Jupiter from the University of São Paulo (USP) or UVic Rocketry University of Victoria (UVic) are essential to engage the engineering students.

Projeto Jupiter is a rocket design team from the Escola Politécnica of the University of São Paulo, composed mostly of engineering students, which has as its main purpose the design and building of a sounding rocket, to be launched at national and international competitions. The project has over 50 members divided into four technical areas: aerodynamics, propulsion, recovery, and avionics. With a similar ideal, the UVic rocketry is the rocket design team from the University of Victoria. The team's logos are shown in Figure 1.1. Both teams’ last projects were a hybrid rocket designed to reach a 10000 feet apogee, propelled by paraffin wax as the fuel and nitrous oxide (N₂O) as the oxidizer, to be launched at the Spaceport America Cup. A scheme for the rocket mission is shown in Figure 1.2.

![Projeto Jupiter Logo](image1.jpg)

![UVic Rocketry Logo](image2.jpg)

Figure 1.1: Rocket Design Teams logo [1][2]
Callisto is a hybrid rocket designed by Projeto Jupiter with a lift-off mass of 21.5 kg, 3.39 m of length, a diameter of 0.128 m and a 2884.34 m apogee at ground level. Using a Pro75-6118M3100-P (White Thunder) by Cesaroni Technology as its motor, the vehicle carries a 2.9 kg of propellant, with a 6131.3 N s total impulse and 2 seconds burn time. The calculated error for the theoretical apogee was 4%, showing that the simulation results are reliable for comparison [4]. A half-section rocket view of Callisto is shown in Figure 1.3.

The Spaceport America Cup, previously known as IREC (Intercollegiate Rocket Engineering Competition), is a student competition set between the cities of Las Cruces and Truth or Consequences, New Mexico, USA. The participants must design and craft a rocket caring a scientific related payload, to be launched at a safety area during the competition. The IREC has participants from different countries, as shown in Figure 1.4, with over 120 teams, mostly from the USA and Canada, but also four teams from Brazil [5].

The competition is divided into categories based on the type of rocket fuel (solid, liquid or hybrid), the target altitude (10000 or 30000 feet) and according to the motor fabrication (COTS - commercial-off-the-shelf or SRAD - student researched and developed). The main goal is to reach the specified altitude with maximum precision, with a penalty for a higher or lower distance. The Spaceport America Cup is the main rocket design competition around the world and allows engineering students to apply their theoretical knowledge, helping to develop an interest in the subject. In Figure 1.5 the rocket Future Heavy designed by the United Launch Alliance and the teams from the 2018 Spaceport America Cup are shown.
1.2 Topic Overview

Since the advance of the Space Race in the 20th-century, spacial research is gaining importance. Spacial structures, satellites and especially rockets are drawing attention on the aerospace engineering teams to develop environmentally friendly and low-cost technology [6, 7]. The study of chemical rockets has therefore been shown to be key to avoid the risks and high costs related to this launch vehicle, allowing scientists to research the outer space and low gravity environments more safely and reliably [8].

Chemical rockets work by reacting fuel and an oxidizer, generating a high pressure and temperature gaseous mixture in the combustion chamber that is expelled through the nozzle at high speeds, creating thrust for the vehicle. The most common types of chemical rockets are the solid rockets, which uses a solid fuel and a solid oxidizer, or liquid rockets, which uses liquid fuel and liquid oxidizer. Recently an interest to develop a model that combines the advantages of the solid and liquid models has surged [9].
Hybrid rockets employ a fuel and an oxidizer in different physical states, being the most common applications a solid fuel and liquid oxidizer [6]. This model has shown to have a higher performance than solid rockets, due to more energetic chemicals, while having lower costs than the liquids, due to the mechanical simplicity of the system, requiring feed and storage for only one component [10]. Additionally, by presenting a fuel and an oxidizer in different physical states, the risk related to the unwanted reaction of the components and consequent explosion is significantly lower.

Virgin Galactic, a commercial spaceline company, has implemented hybrid motors with hydroxyl-terminated polybutadiene (HTPB) and N$_2$O as propellants, on the suborbital spaceplanes SpaceShipOne and SpaceShipTwo, as shown in Figure 1.6.

![Figure 1.6: Virgin Galactic - SpaceShipTwo [11]](image)

Design optimization is an important engineering tool for the development of the most effective design. A mathematical statement of an optimization problem is shown in Equation 1.1, where $f(x)$ is the variable to be minimized, known as objective function, $x$ are the inputs of the system, named as design variables, and $g(x)$ and $h(x)$ are the problem’s inequality and equality constraints, respectively.

$$\text{Minimize} \quad f(x)$$
$$\text{w.r.t.} \quad x$$
$$\text{subject to} \quad g(x) \leq 0$$
$$h(x) = 0$$

1.3 Objectives

The main objective of this work is to develop an engineering tool to generate an optimal multidisciplinary conceptual design of a N$_2$O-paraffin wax hybrid rocket. This design must attend to all the mission requirements and constraints, according to the requested inputs, even under the expected model and physical limitations and variations.

A separated theoretical model for each discipline will be created and then assembled into the main optimization function. The study of four disciplines is conducted: mass and sizing, aerodynamics, propulsion, and stability. Each model should correspond to a theoretical model and will be validated with
experimental data, when possible, to ensure accuracy.

Multidisciplinary design optimization algorithms are then applied to the combination of the separated model, creating an objective function focused on maximizing the specific impulse and minimizing the total rocket weight. The design variables and constraints are appropriately defined to ensure physical coherence.

Multi-Objective optimization is then applied to find a solution that combines the combustion efficiency of a higher specific impulse with the lower costs of a lower lift-off mass rocket.

1.4 Thesis Outline

This document is divided into eight chapters, including the introduction. In chapter 2 a theoretical overview of each of the rocket's technical areas, as well as the state-of-the-art in hybrid rockets and optimization is conducted. Chapter 3 describes the theoretical models used to describe each one of the disciplines considered: propulsion, aerodynamics, mass and sizing, stability. The model's limitations are further discussed in the chapter.

In chapter 4 the multidisciplinary design optimization theory is presented. A description of the design and state variables, as well as the system constraints, are stated. Chapter 5 brings a software overview, describing the methodology implemented in the Matlab code.

Chapter 6 presents a model validation using the experimental data from three previously tested hybrid rockets. Chapter 7 presents the results of the simulations to maximize the impulse and minimize the rocket's weight, as well as the multi-objective solution. Chapter 8 brings the conclusions and recommendations for future works.
Chapter 2

Background

The literature overview is a fundamental component of any scientific work. It shows the previous works done in the subject, allowing the definition of what has already been done and the possible improvements.

This chapter illustrates a brief overview of the theory and previous works related to hybrid propulsion, hybrid rockets modeling and optimization.

2.1 Theoretical Overview

2.1.1 Propulsion System

As stated in Section 1.2, chemical rockets can be classified by the state of the used propellants. The four types of chemical rockets are gaseous, solid, liquid and hybrid [6].

In the gaseous engines, a high-pressure propellant, in the gaseous state, is injected in the feed system, being ejected through a nozzle in high velocities. Although this system is simpler and has cleaner propellants, such as hydrogen, the low performance, and heavy propellant tank makes these systems not suitable to be used as a primary source of thrust.

In the solid engines, the solid propellant, stored in the combustion chamber, contains all the chemicals to perform a complete burn when ignited, being highly reactive and potentially explosive [12]. These vehicles are easy to operate and are relatively smaller than the other chemical rockets, but are harder to manufacture and handle, for safety reasons, and has a smaller performance than the liquid engines.

The liquid engines contain the propellant stored in a high-pressure tank at the liquid phase, that reacts when transported to the combustion chamber, creating hot gases that are spelled through a nozzle and generates thrust. The high-pressure levels that the propellant is maintained require a complex feed system of valves, pumps, and turbines that are related to the high costs of this type of engine. This engines usually have the highest performance, using propellants like liquid oxygen.

In a hybrid engine, in most cases, the fuel is stored as a solid grain in the combustion chamber, while the liquid oxidizer is stored in the oxidizer tank. The connection between the combustion chamber and the oxidizer tank is done by an injector valve, responsible for controlling the oxidizer mass flow, and the
combustion gases leave the system through a nozzle, as shown in Figure 2.1. This engine combines the advantages of solid and liquid engines, being, therefore, an important object of study.

![Figure 2.1: Hybrid Propulsion Scheme](image)

Figure 2.1: Hybrid Propulsion Scheme [13]

The main advantages of a hybrid rocket propulsion according to Sutton and Biblar [6], Humble et al. [14] and Jansen and Kletzkine [15] are:

- Safer than a solid engine since the fuel grain is inert and therefore insensitive to overpressure due to fissures
- Higher specific impulse than a solid engine
- Less needs for pumping than a liquid engine, resulting in a simpler and low-cost design.
- Start-Stop-Restart system which contributes to the system safety
- Environmentally safe exhaust gases, since most of the fuels used are hydrocarbons
- Possibility to introduce additives to the fuel, as aluminum powder or stearic acid, to increase the fuel mass flow and specific impulse and to reduce the oxidizer-fuel ratio, reducing the amount of liquid oxidizer needed.

However, this model also presents disadvantages such as:

- Mixture ratio and specific impulse variation during the steady-state operation
- Oxidizer-fuel ratio varies as the fuel grain port increases during burning, reducing the engine performance
- Lack of experimental data for some propellants combinations

As for propellants, the most commonly used fuels are hydroxyl-terminated polybutadiene (HTPB) \((\text{C}_4\text{H}_6)\) and paraffin-wax \((\text{C}_{20}\text{H}_{42})\) and as oxidizer the most common are nitrous oxide \((\text{N}_2\text{O})\) and oxygen \((\text{O}_2)\) [6, 12]. The combination of HTPB and nitrous oxide has been used by Virgin Galactic Space Ship One on a successful manned orbital craft and is also applied in sounding rockets. As shown by de Oliveira Alcaria Guerreiro [12], the specific impulse generated by nitrous oxide or oxygen combination does not vary significantly, but nitrous oxide can be considered an easier to handle and safer solution, since oxygen requires storage with cryogenic temperatures. Also, paraffin is shown to have a slightly better performance than HTPB, being easy to cast into fuel grains and to mix with additives. Considering
these advantages and that the previous projects from Projeto Jupiter and UVic Rocketry used these propellants, paraffin-wax and nitrous oxide were selected to be approached in this work.

For some types of propellants, a tank containing an inert gas, usually helium, is necessary to pressurize the oxidizer tank and force the injection into the combustion chamber. However, if a volatile fluid is being used as the oxidizer, the gas by itself creates enough pressure to perform the injection, creating a self-pressurized tank [16]. The N\textsubscript{2}O used in this work as the oxidizer is considered a volatile fluid.

### 2.1.2 Aerodynamics System

The aerodynamic analysis is a key component to optimize the rocket, since it is related to the stability and dynamic forces acting on the rocket during the mission. The use of Computational Fluid Dynamics (CFD) tools, like Ansys Fluent, Star-CCM+ or OpenFOAM, to simulate the body pressure distribution and aerodynamic forces is a common approach. The nose cone, tail cone and fins are the main components of the rocket aerodynamics system. CFD simulations for the pressure distribution on a nose cone and a fin cross-section selection are shown in Figure 2.2.

![Pressure distribution over nose cone](image1.png) ![Fin cross section selection](image2.png)

(a) Pressure distribution over nose cone [17]  
(b) Fin cross section selection [17]

Figure 2.2: Computational analysis for aerodynamics structures[17]

The nose cone is directly related to the vehicle drag forces. At low subsonic speeds the skin friction drag is predominant, meaning that nose cone shapes with smaller surface area are prioritized. As the rocket velocity range increases to the high transonic speeds, wave drag becomes the predominant source of drag, leading to shapes with a smoother transition to prevent the premature formation of shockwaves [17]. The nose cone designs as a function of the Mach number are shown in Figure 2.3. The rankings in this figure are 1 - superior, 2 - good, 3 - fair and 4 - inferior.

The tail cone is a component that is not commonly studied in-depth as the other aerodynamics components. However, a paper published by the Military Technical College in Cairo, Egypt, found that the drag coefficient could be minimized by nearly 50\% with the use of the tail cone in subsonic and transonic regimes [19].

The fins are the most important component for the rocket stability. The study of the fins cross-section is performed in the previously mentioned CFD software to minimize the structure drag. The selection of the airfoil, as in the nose cone, depends on the rocket’s Mach number.
These aerodynamic structures are normally made of composite materials, such as carbon fiber-epoxy and glass fiber-epoxy, to minimize the structural weight. However, it is important to notice that the manufacturing of these structures can have a higher cost than the available budget by a university team, taking into account the system’s dimensional tolerances. It is important to consider this fact during the selection of the aerodynamic components since the theoretical optimal solution might not be a feasible one.

2.1.3 Recovery System

The recovery system of the rocket is responsible for slowing down its descent and is a key feature for preventing damages to the rocket structure and payload, being also mandatory for safety reasons. The main components of the recovery system are the deployment subsystem, the drogue and main parachutes.

The deployment subsystem is responsible to eject the parachutes, after being activated by the avionics system. The main methods to deploy the parachutes are the use of black powder or compressed CO$_2$ to release the nosecone with sufficient velocity to guarantee the parachutes opening. A common ejection system used is the 3-ring release system, which is used to decrease the required forces to release the parachute. A main parachute and a 3-ring system are shown in Figure 2.4.

The sounding rockets usually present two different parachutes: the drogue parachute, with smaller dimensions, and the main parachute, with larger dimensions. The usage of two parachutes is recommended, since the main parachute opening can generate high stresses due to aerodynamic forces sufficient to cause structural failure and the drogue parachute is not able to slow the vehicle down enough to prevent a high impact when the structure lands, having a terminal velocity of 80-100 ft/s, while the main parachutes leads to a terminal velocity of 15-25 ft/s. Using the data from rockets launched in the 2018 Spaceport America Cup, the drogue diameter varies between 20 and 30 inches (50.8 to 76.2 cm), the main diameter has a larger variation, varying between 90 and 300 inches (228.6 to 762 cm) [20–24]. The drogue launch occurs near the rocket's apogee, to minimize the aerodynamic forces related to its
opening, leading to the vehicle descent until the main parachute is launched, slowing the structure until it reaches the terminal velocity.

The parachutes present a large variety of shapes (round, cruciform, annular and pull-down apex, ribbon and ring, ram air, square and toroidal geometries [20, 25]) and materials (silk, nylon, Kevlar, terylene [26]) that need to resist the aerodynamic forces and present a proper porosity to ensure the parachute shape during the operation. It is common for the main parachute to have a central annular hole, named air vent, being responsible for the parachute shape stability. The parachute proper folding is crucial to ensure that it opens after its release, being commonly storage inside a deployment bag to prevent damages from the ejection process.

2.1.4 Avionics System

The avionics system is responsible to collect the rocket flying data and to activate the recovery system in the correct moment. The components have sensors as altimeter, accelerometer, barometer and gyroscopic sensor. The most common avionic component between the participant teams from Spaceport America Cup is the Stratologger CF, responsible for the main and drogue parachutes, using the barometric pressure to determine the vehicle apogee. A Stratologger CF is shown in Figure 2.5.
2.2 Hybrid Rocket

As mentioned before in Section 1.2, the researches about spacecraft have been constantly increasing in the last decades, especially the ones related to hybrid propulsion, for the reasons presented in Subsection 2.1.1. An accurate hybrid rocket model has been proven difficult to develop, since the simulation of complex physical phenomenons, as the oxidizer tank emptying process, requires the usage of high computational power [12].

Rocket propulsion textbooks as Sutton and Biblar [6] describes low fidelity steady-state models used for preliminary studies. Sutton divides the jet propulsion into two classes: rocket propulsion, where the thrust comes from the storage expelled gases, and duct jet propulsion, which uses the surrounding fluid as opposed to storage gases. Rocket propulsion is considered a pure reaction system since it only depends on the usage of the internal medium to generate the propulsive force. The classification of rocket propulsion is based on the energy source and the conversion process: the internal energy source as the chemical and nuclear sources, and the ones that use external energy sources, like the Sun.

As mentioned earlier in Subsection 2.1.1, the chemical rockets can be separated into gaseous, solid, liquid and hybrid engines [6, 14]. Another classification proposed by Zandbergen [28] divides the rocket systems according to its goals: weapons of war, as missiles, and peaceful space exploration, as space launchers.

At the Spaceport America Cup, teams from all over the world present a large variety of rocket designs. On this annual competition several hybrid rockets engines are presented and launched [3, 17, 20, 21, 29–35]. Other entities also developed researches related to hybrid rocket [36–38]. These projects discuss the development of each subarea. In the aerodynamic subsection, it is described the CFD simulations for the design of the aerodynamic components, such as the nose cone and fins, the manufacturing process and material selection of these components and the theoretical models adopted. In general, carbon fiber-epoxy is used for the rocket body and fins, adopting the usage of glass fiber-epoxy for the nose cone, since these composite materials present a high strength to weight ratio, being ideal to minimize the vehicle gross weight. As the main manufacture procedure for these components, the majority of the presented works present the usage of molds. To select the proper shapes for the aerodynamic components, the study of the rocket velocity and manufacturing feasibility is essential, as previously discussed in Subsection 2.1.2.

For the propulsion subsystem, are presented the design and manufacture of the oxidizer tank, injector, combustion chamber, and nozzle, as well as the theoretical models adopted, results of previously performed tests and propellants selection. The main material used for the combustion chamber is the Aluminum 6061-T6 for its capability to support the imposed pressure and temperature, around 500 psi and 3000 K, respectively, with the addition of a thermal protection system [20]. In some cases, a composite overwrapped pressure vessel (COPV) such as the 6061-T6 liner used by the University of Calgary team [21] is adopted as an alternative solution. The most common propellants in these projects are the nitrous oxide N₂O as the oxidizer and the paraffin wax or the hydroxyl-terminated polybutadiene (HTPB) as the fuel. The finite element method is computed by some of the teams to analyze the stress on the
combustion chamber, oxidizer tank and bulkheads.

The recovery system shows the specifications of the parachutes and ejections system, such as main and drogue manufacturing, shape and material selection, theoretical models used and possible tests performed. The avionics subsystem describes each component utilized and its function. The payload of each project is mostly related to scientific researches. The variety of payload carried is very large, from the measurement of the capabilities of certain material to shield from the effects of space radiation [21] to the acquisition of the flight data for further analysis.

The regression rate is the rate of which the radius of the port in the fuel grain increases. This variable is directly related to the oxidizer mass flow and varies according to the pair of fuel-oxidizer and fuel grain properties that are being used. For this reason, many tests using different combinations of propellants were performed to estimate the regression rate constants [39–41]. The research performed in 1987 by Korting et al. [42] concluded that the regression rate is a function of the mass flux, geometry, presence of oscillations, propellant composition, burn time and pressure. The tests performed in Ziliiac and Karabeyoglu [43] and Doran et al. [44] shows the regression rate constants using O₂ and N₂O as the oxidizer and Hydroxyl Terminated Polybutadiene (HTPB), Polymethyl Methacrylate (PMMA), High Density Polyethylene (HDPE) and Paraffin wax as the tested propellants. The issues related to modeling the regression rate axial variations are shown in Karabeyoglu [45].

In the hybrid rocket design, one of the hardest physical phenomenons to model is the oxidizer tank emptying. Two models developed by Newlands [46] and Fernandez [47] uses thermodynamics, heat transfer and compressible fluid dynamics equations to model the tank emptying. In Fernandez [47] a comparison between the ideal and non-ideal gas is made, concluding that the non-ideal shows better results than the ideal model, as expected.

The theoretical model developed by Guerreiro [12] uses a different design, adding a heat transfer system to cool down the nozzle and heat the oxidizer tank, as shown in Figure 2.6. The theoretical model is divided by scale parameters, where parameters as burn time and exhaust mass flow rate are settled, burning parameters, where regression rate, oxidizer to fuel ratio O/F, oxidizer and fuel mass flow are defined, injection and exhaust parameters, defining the injection area and the pressure constraints, chamber and nozzle geometry, choosing appropriate values for the combustion chamber length, and heat transfer system. The experimental values are used to validate the theoretical model.

The project brings a possible system design, highlighting the need for further analysis of the engine stability, since the heat transfer system promotes an increase on the tank pressure, causing the oxidizer mass flow rate and O/F ratio to increase and creates instability.

The theoretical model designed by Klammer [13] was used as the basis of the propulsion subsystem. The work analyzes each component of the propulsion subsystem, namely the oxidizer tank, injector, combustion chamber and nozzle, separately using thermodynamics, compressible fluid dynamics and numerical equations. The oxidizer tank, combustion chamber and nozzle are considered adiabatic, axial and radial variation on the combustion chamber, as well as the transient effects, are neglected. A mass estimation and trajectory system are also developed, using data from previously launched hybrid rockets. The optimization used the maximization of the specific impulse as the objective function, finding
a design with 34.8 kg liftoff mass and 201.8 s specific impulse. From the global sensitivity analysis, the characteristic velocity and thrust correction factor were the parameters that influenced the most the altitude and specific impulse. The tank fill level, volume and pressure also influenced the altitude.

2.3 Design Optimization

With the evolution of scientific discoveries, the rising quest for the best performance design of an engineering project has become even more relevant to achieve a cost-effective solution. For this reason, the study of design optimization has gained increasing relevance. In a high-cost project, such as the design of a hybrid rocket, the study of the optimal design is a key component to make the correct investments.

Multidisciplinary Design Optimization (MDO) theory is shown by Martins [48] on his course notes. MDO can be divided into monolithic architectures and distributed architectures. Monolithic structures solve the MDO as a single optimization problem being divided into four problem statements, according to the strategies used to achieve the multidisciplinary feasibility: all-at-once (AAO), simultaneous analysis and design (SAND), individual discipline feasible (IDF) and the multidisciplinary feasible (MDF). These architectures will be further explored in Section 4.5. The distributed architectures divide the optimization problem into sets of optimization subproblems with the same solution when assembled.

The study of MDO of hybrid rockets is also explored in the literature. Hao et al. [49] creates a model to estimate the rocket mass, associating the structural mass to the rocket diameter, and its trajectory. A sensitivity analysis is performed on the rocket parameters, with the fuel regression rate precision coefficient having the highest impact on both the rocket liftoff mass and length to diameter ratio since the propellants represent the main mass and dimensions of the rocket. The oxidizer density and yield limit of the oxidizer tank shell material are shown to have the highest impact on the maximum altitude and rocket maximum acceleration.

A multidisciplinary design exploration methodology paper was developed by Kosugi et al. [50]. A multi-objective problem is created, using the maximization of the flight altitude and minimization of lift-off
weight as the objective functions. A theoretical model for the thrust and weight estimation are used to estimate the flight altitude of the vehicle. A payload of 40 kg is considered, using polypropylene as the fuel and a swirling oxidizer type of hybrid rocket engine. A divided range multi-objective genetic algorithm (DRMOGA) is used as the optimization algorithm, found that the model maximum altitude was 180 km and that after the 150 km maximum altitude, the rocket weight increase as maximum altitude becomes even higher than for vehicles with an apogee less than 150 km. A Pareto curve is plotted and a model with 334 kg of gross weight and 108.2 km altitude is selected. The oxidizer mass flow was found to be the most influential parameter for the rocket gross weight and maximum altitude.

In the paper developed by DaLin et al. [51], the comparison of the wheel and star grain shapes, finding that for a higher payload mass of 10 to 13 kg, a shorter length to diameter ratio and better thermal protection is found with the wheel grain. It is concluded that the main factors that alter the motor performance and size of the hybrid rocket are the fuel grain geometry, chamber pressure and O/F ratio.
Chapter 3

Hybrid Rocket Modelling

With the increasingly scientific and industrial advances, theoretical models that simulate and predict the behavior of the physical world become even more necessary than before. This has encouraged universities and companies around the world to invest in engineering researches in Finite element's structural analysis, computational fluid dynamics (CFD) simulations and many other tools that analyze problems efficiently. For this reason, a theoretical model with reliable and realistic results is necessary for any project.

To model the hybrid rocket, a subdivision of the different disciplines and corresponding physical equations was created. The system was subdivided into four disciplines: Propulsion, Aerodynamics, Mass and Sizing, Stability.

3.1 Propulsion

The propulsion sub-area is responsible to simulate the interactions between the oxidizer and fuel, using thermodynamics, heat transfer and compressible fluid dynamics theoretical models to calculate the fuel burn process.

This area was subdivided into four components: the Self-Pressuring Oxidizer Tank, Injector, Combustion Chamber and Nozzle. A representation of hybrid motor control volumes is represented in Figure 3.1.

The propulsion system was based in Klammer [13], with the added heat transfer system on the oxidizer tank.

3.1.1 Self-Pressurizing Oxidizer Tank

The oxidizer tank is the component responsible to store the nitrous oxide (N\textsubscript{2}O) both in liquid and gaseous states. Since N\textsubscript{2}O is a volatile fluid, having a high vapor pressure at the operating temperatures, the oxidizer pressurizes the storage tank, making it a self-pressurizing oxidizer tank. The self-pressurizing mechanism allows a high enough tank pressure to sustain the flow rate. A schematic is presented in Figure 3.2.
The emptying of the oxidizer tank assumes that both liquid and gaseous states are in thermal equilibrium, on the saturated state, as stated in J. E. Zimmerman and Zilliac [16]. This state is important since the thermodynamic properties of the saturated N₂O were registered by the National Institute for Standards and Technology (NIST) [52] that was used to define the thermodynamic state of the oxidizer.

The system was modeled based on two differential equations related to fundamental thermodynamic relations of mass and energy balance. The variation of the oxidizer tank content mass $m_{tank}$ is equal to the oxidizer mass flow rate $\dot{m}_{ox}$, as shown in Equation 3.1. According to the first law of thermodynamics, the variation of the tank’s content internal energy $U_{tank}$ is related to the heat transfer rate $\dot{Q}_{in}$, oxidizer mass flow and its specific enthalpy $h_{ox, out}$, as shown in Equation 3.2.

$$\frac{dm_{tank}}{dt} = -\dot{m}_{ox}$$  \hspace{1cm} (3.1)
\[
\frac{dU_{\text{tank}}}{dt} = -\dot{m}_{\text{ox}} h_{\text{ox,out}} + \dot{Q}_{\text{in}}
\] (3.2)

The emptying process is calculated in two different stages: the first with both liquid and gas N\(_2\)O and the second when the tank runs out of liquid N\(_2\)O.

Using the volume constraint and the table obtained by NIST for the thermodynamic properties of the saturated nitrous oxide\([52]\), it is possible to estimate the tank temperature. The tank quality \(x_{\text{tank}}\) is related to its internal energy and mass, as well as the saturated oxidizer specific internal energy in the liquid \(u_{\text{liq}}\) and vapor \(u_{\text{vap}}\) states, as shown in Equation 3.3. The volume of the tank \(V_{\text{OT}}\) is calculated using the density of the saturated N\(_2\)O in both liquid \(\rho_{\text{liq}}\) and vapor \(\rho_{\text{vap}}\) states, as shown in Equation 3.4. The software searches for the temperature, according to the saturation properties, which results in a tank volume equal to the imposed.

\[
x_{\text{tank}} = \frac{U_{\text{tank}}}{m_{\text{tank}}} = u_{\text{liq}} - u_{\text{vap}}
\] (3.3)

\[
V_{\text{OT}} = m_{\text{tank}} \left( \frac{1 - x_{\text{tank}}}{\rho_{\text{liq}}} + x_{\text{tank}} \frac{1}{\rho_{\text{vap}}} \right)
\] (3.4)

In the stage where only vapor N\(_2\)O remains, the density \(\rho_{\text{tank}}\) is calculated directly from the tank volume and mass, as shown in Equation 3.5. The specific internal energy \(u_{\text{tank}}\) is calculated by using the internal energy and mass, as shown in Equation 3.6. Since the assumption of an ideal gas for N\(_2\)O does not apply to a near saturation state, the Span-Wagner equation of state is used to simulate a real gas behavior \([53]\) \([54]\). The non-ideal model is used to define the tank temperature \(T_{\text{tank}}\) as a function of the \(u_{\text{tank}}\) previously calculated.

\[
\rho_{\text{tank}} = \frac{m_{\text{tank}}}{V_{\text{OT}}}
\] (3.5)

\[
u_{\text{tank}} = \frac{U_{\text{tank}}}{m_{\text{tank}}} = u(\rho_{\text{tank}}, T_{\text{tank}})
\] (3.6)

### 3.1.2 Injector

The injector is the component responsible to interconnect the oxidizer tank and combustion chamber by controlling the amount of N\(_2\)O that reacts with the fuel. One of the critical aspects of the propulsion system modeling is an accurate estimation of the oxidizer flow, since it is responsible for controlling most combustion chamber parameters, making the injector a key component for the model. Since saturated N\(_2\)O flows through the injector, cavitation and two-phased flow will occur with any decrease in pressure \([13]\), as presented in Figure 3.3.

The biphasic flow can be modeled according to the cavitation effect: the single-phased-incompressible (SPI) flow and the homogeneous equilibrium model (HEM).

The SPI model uses the traditional equations and has accuracy when the residence time is much shorter than the bubble growth time. In this model the oxidizer mass flow \(\dot{m}_{\text{SPI}}\) is calculated as a function of the discharge coefficient \(C_d\), the injection area \(A_{\text{inj}}\), the inflow density \(\rho_{\text{in}}\) and pressure
\( p_{in} \) and outflow pressure \( p_{out} \), as shown in Equation 3.7. In the case where the residence time is much longer than the bubble growth time, the HEM model is more accurate. This model assumes an isentropic expansion at saturation and calculates the oxidizer mass flow \( \dot{m}_{HEM} \) as a function of the \( C_d, A_{inj}, \) outflow density \( \rho_{out} \) and specific enthalpy \( h_{out} \) and inflow specific enthalpy \( h_{in} \), as shown in Equation 3.8 [53].

\[
\dot{m}_{SPI} = C_d A_{inj} \sqrt{2 \rho_{in} (p_{in} - p_{out})} \tag{3.7}
\]

\[
\dot{m}_{HEM} = C_d A_{inj} \rho_{out} \sqrt{2(h_{in} - h_{out})} \tag{3.8}
\]

To combine both regimes a non-equilibrium parameter \( \kappa \), that estimate the ratio between residence time to bubble growth time proposed by Dyer et al. [53], presented on Equation 3.9 is used. This parameter is calculated as a function of the pressures of the inflow \( p_{in} \), outflow \( p_{out} \) and saturation state of the inflow \( p_{in, sat} \). By applying the parameter \( \kappa \) the model presented by Dyer is given in Equation 3.10.

\[
\kappa = \frac{p_{in} - p_{out}}{p_{in, sat} - p_{out}} \tag{3.9}
\]

\[
\dot{m}_{DYER} = \frac{\kappa \dot{m}_{SPI} + \dot{m}_{HEM}}{1 + \kappa} \tag{3.10}
\]

To evaluate the downstream properties, related to the combustion chamber, the Equation 3.11 is used at equilibrium saturation values at pressure \( p_{out} \), relating the quality \( x_{HEM} \) to the specific entropy of the inflow \( s_{in} \) and saturated liquid at the outflow \( s_{out, liq} \) and the specific internal energy of the saturated liquid \( u_{out, liq} \) and saturated vapor \( u_{out, vap} \) at the outflow. The downside of this model is the difficulty to account for changes in flow properties between the tank and the injector.

\[
x_{HEM} = \frac{s_{in} - s_{out, liq}}{u_{out, vap} - u_{out, liq}} \tag{3.11}
\]
Between the different models, the SPI shows a better agreement with the experimental data used in the work by Klammer [13], and as such was the selected method, despite the model accuracy restrictions. Since the gas outflow is much smaller than the liquid, these restrictions do not affect the performance of the model. The oxidizer mass flow \( \dot{m}_{ox} \) is presented in Equation 3.12, taking into account a feed system pressure drop term \( p_{feed} \), the density of the discharged fluid \( \rho_{discharge} \), the pressure at the tank \( p_{tank} \) and combustion chamber \( p_{CC} \), the discharge coefficient \( C_d \) and injection area \( A_{inj} \).

\[
\dot{m}_{ox} = C_d A_{inj} \sqrt{2 \rho_{discharge} (p_{tank} - p_{feed} - p_{CC})}
\]  

(3.12)

### 3.1.3 Combustion Chamber

After the oxidizer passes through the injector, it generates a chemical reaction with the solid fuel, initiating the combustion. The fuel grain has a cylindrical shape, fitting narrowly inside the combustion chamber, with the fuel grain external diameter coincident to the combustion chamber internal diameter. The combustion occurs in the cylindrical port, located at the center of the fuel grain. As the fuel is burned, the port radius increases, as shown in Figure 3.4.

![Figure 3.4: Hybrid Rocket Fuel Regression [13]](image)

In this hybrid rocket model, the transient process is not the main focus, since the combustion is much faster than the variation on the oxidizer flow. Therefore, the combustion is modeled as a steady-state process, occurring instantaneously and consuming completely the fuel and oxidizer. The fluid is considered an ideal gas with the same properties in the whole chamber. Therefore, the mass balance equation and the first law of thermodynamics are applied in the system, as shown in Equations 3.13 and 3.14. The mass conservation implies that mass flow that enters the chamber \( \dot{m}_{in} \) is equal to the mass flow that leaves it \( \dot{m}_{out} \). The conservation of energy implies that the energy that enters the control volume as the heat transfer rate \( \dot{Q} \) and inflow specific enthalpy \( h_{in} \) and velocity \( v_{in} \) is equal to the energy that leaves it as the outflow inflow specific enthalpy \( h_{out} \) and velocity \( v_{out} \). Since the variation in height is negligible and the system does not work, these terms were neglect.

\[
\sum \dot{m}_{in} - \sum \dot{m}_{out} = 0
\]  

(3.13)
\[
\sum \dot{m}_{in}(h_{in} + \frac{1}{2}v_{in}^2) - \sum \dot{m}_{out}(h_{out} + \frac{1}{2}v_{out}^2) + \dot{Q} = 0 \quad (3.14)
\]

An important aspect of the process is to measure the burn speed rate, relating it to the mass flux that passes through the port. For this purpose, a variable, named regression rate \( \dot{r} \), was created. The regression rate is related to the oxidizer mass flux \( G \) and the fuel grain geometry and composition, expressed by the constants \( a \) and \( n \), and is one of the most important performance indicators of a hybrid rocket [13][40]. The formula to calculate the regression rate is expressed at the Equation 3.15.

\[
\dot{r} = aG^n \quad (3.15)
\]

This equation considers that the regression rate and oxidizer mass flux remain constant in the axial direction and although a slight variation might occur, the values float less than 10\%, being, this way, negligible [45].

After calculating the regression rate, the fuel mass flow \( \dot{m}_f \) is calculated as a function of the regression rate, fuel grain length \( L_f \), density \( \rho_f \) and port radius \( r_{port} \), as shown in Equation 3.16. To calculate the mass flow through the port an iterative process is needed since the constant used in the regression rate expression (Equation 3.15) were obtained for a local regression rate and therefore requires the total mass flow through the port, instead of just the oxidizer mass flow, to ensure accuracy. The initial step, shown in Equation 3.17, calculates \( G_O \) using only the oxidizer mass flow \( \dot{m}_{ox} \) and fuel grain port, and is used to calculate the fuel mass flow and total mass flow \( \dot{m}_{CC} \) (Equations 3.16 and 3.18).

\[
\dot{m}_f = 2\pi r_{port} \dot{r} L_f \rho_f \quad (3.16)
\]

\[
G_O = \frac{\dot{m}_{ox}}{\pi r_{port}^2} \quad (3.17)
\]

\[
\dot{m}_{CC} = \dot{m}_f + \dot{m}_{ox} \quad (3.18)
\]

After the initial values are calculated, the average mass flux through the port is used to recalculate \( G_{i+1} \), as shown in Equation 3.19, using the new values to reiterate the regression rate \( \dot{r} \) and fuel mass flow \( \dot{m}_f \), recalculating the average mass flux of the oxidizer \( \dot{m}_{ox,i} \) and total \( \dot{m}_{CC,i} \). The iterative process continues until the average mass flux reaches convergence to a constant value.

\[
G_{i+1} = \frac{\dot{m}_{ox,i} + \dot{m}_{CC,i}}{2\pi r_{port}^2} \quad (3.19)
\]

To calculate the pressure in the combustion chamber \( p_{CC} \), the relations for the upstream of a convergent-divergent nozzle are used. Assuming that the nozzle, connected to the end of the combustion chamber, is ideal (isentropic, no shock waves, steady axial flow of a ideal gas and homogeneous fluid) and that the mass flow through the supersonic nozzle is choked, the stagnation pressure \( p_0 \) can be calculated as a function of the mass flow \( \dot{m}_{CC} \), nozzle throat area \( A_{th} \), stagnation temperature \( T_0 \),
fluid heat capacity ratio $k$ and ideal gas constant $R$, as shown in Equation 3.20. To take into account the divergences of the mass flow, due to cooling, changes in properties or incomplete combustion in the real nozzle, a discharge correction factor $\zeta_d$ is added to the equation. Assuming an adiabatic expansion with a constant specific heat ratio, the pressure of the combustion chamber $p_{CC}$ can be related to its stagnation pressure, temperature $T_{CC}$ and stagnation temperature $T_0$, as stated in Equation 3.21.

$$p_0 = \frac{\dot{m}_{CC}}{\zeta_d A_R} \sqrt{\frac{T_0 R}{k} \left( \frac{k + 1}{2} \right)^{\frac{k-1}{k}}}$$  \hspace{1cm} (3.20)$$

$$p_{CC} = p_0 \left( \frac{T_{CC}}{T_0} \right)^{\frac{k}{k-1}}$$  \hspace{1cm} (3.21)$$

To calculate the remaining fluid properties, a computer program written by NASA that calculates the thermodynamic state at the equilibrium of complex mixtures, Chemical Equilibrium with Applications (CEA), is used [55]. The inputs necessary for CEA are the combustion chamber pressure $p_{CC}$, previously calculated, and the oxidizer to fuel ratio (O/F), calculated as in Equation 3.22, and returns the corresponding mixture properties such as stagnation temperature $T_0$, specific heats $c_{p,CC}$ and $k_{CC}$ and density $\rho_{CC}$, as shown in Equation 3.23.

$$O/F = \frac{\dot{m}_{ox}}{\dot{m}_f}$$  \hspace{1cm} (3.22)$$

$$[T_0, \rho_{CC}, c_{p,CC}, k_{CC}] = CEA(p_{CC}, O/F)$$  \hspace{1cm} (3.23)$$

With the estimated properties, it is possible to calculate the velocity of the mixture at the outflow of the chamber $v_{CC}$ as a function of the mass flow $\dot{m}_{CC}$, density $\rho_{CC}$ and area $A_{CC}$, shown in Equation 3.24. Using the relation between the temperature $T_{CC}$ and stagnation temperature $T_0$, as well as the heat capacity $c_p$ and velocity previously calculated, it is possible to determine the combustion chamber temperature, as shown in Equation 3.25. To take into account the imperfections on the real model, such as friction or incomplete combustion, an efficiency correction factor $\zeta_c$ can be applied to modify the theoretical stagnation temperature $T_0$ according to Equation 3.26.

$$v_{CC} = \frac{\dot{m}_{CC}}{\rho_{CC} A_{CC}}$$  \hspace{1cm} (3.24)$$

$$T_{CC} = T_0 - \frac{v_{CC}^2}{2c_p}$$  \hspace{1cm} (3.25)$$

$$T_0 = T_{0,CEA} (\zeta_c)^2$$  \hspace{1cm} (3.26)$$
3.1.4 Nozzle

After leaving the combustion chamber, the high temperature and pressure combustion products are expelled in high speeds through the nozzle, generating thrust for the rocket. The thrust $F_{\text{prop}}$ is calculated using the atmospheric pressure $p_{\text{atm}}$, the nozzle exit area $A_e$, the exhaust mass flow, velocity $v_e$ and pressure $p_e$, calculated as a function of the nozzle geometry and upstream stagnation properties, as shown in Equation 3.27. To take into account the losses that are not modeled in an ideal nozzle, the correction coefficient $\zeta_t$ is added to the equation.

$$F_{\text{prop}} = \zeta_t ( \dot{m}_{\text{CC}} v_e + (p_e - p_{\text{atm}}) A_e )$$ (3.27)

To calculate the downstream properties an ideal isentropic nozzle model is adopted. In this model, all the exit properties can be calculated as a function of the Mach number and stagnation state properties. Initially, the Mach number $M_e$ is calculated iteratively by the area ratio constraint, relating the nozzle throat area $A_{th}$ to the nozzle exit area as shown in Equation 3.28. With this result it is possible to calculate the pressure $p_e$, temperature $T_e$ and velocity $v_e$ at the exit, as shown at Equations 3.29, 3.30 and 3.31. The thermodynamic diagram of the nozzle is shown in Figure 3.5.

$$\left( \frac{A_e}{A_{th}} \right)^2 = \frac{1}{M_e^2} \left[ \frac{2}{k+1} \left( 1 + \frac{k-1}{2} M_e^2 \right) \right]^{\frac{k+1}{k-1}}$$ (3.28)

$$T_e = T_0 \left( 1 + \frac{k-1}{2} M_e^2 \right)^{-1}$$ (3.29)

$$p_e = p_0 \left( 1 + \frac{k-1}{2} M_e^2 \right)^{-\frac{k}{k-1}}$$ (3.30)

$$v_e = M_e \sqrt{kR T_e}$$ (3.31)

Figure 3.5: Inlet and Outlet Properties of a Nozzle [13]

This set of equations can be applied on the convergent-divergent nozzle is flowing full and no shockwaves occur in the nozzle. A large difference in the exit and ambient pressure ($p_e < 0.1 - 0.4 p_{\text{atm}}$) can cause the flow to separate and a shockwave can occur in the nozzle [6]. However, since these variations only occur for short periods when the motor is not at full thrust, a simplified model can be applied. As the oxidizer tank runs out of liquid oxidizer, a drop in the combustion chamber pressure can cause the exit pressure to decrease, making the exit flow to separate [13]. In this case, the nozzle throat properties
shall be considered, as shown in Equations 3.32, 3.33 and 3.34.

\[ A_e = A_{th} \]  
(3.32)

\[ p_e = p_0 \left( \frac{2}{k+1} \right)^{\frac{1}{k-1}} \]  
(3.33)

\[ v_e = \sqrt{\frac{2kRT_0}{k+1}} \]  
(3.34)

### 3.1.5 Heat Transfer

The heat transfer occurs in these stances: in the oxidizer tank, as the oxidizer flows, in the combustion chamber, as the fuel is burned, and in the nozzle. However, due to the limitations on the information of the outflow properties, the combustion chamber and nozzle had to be considered adiabatic. In CEA, the lookup table containing the properties of the paraffin-N\(_2\)O mixture does not behave as an injective function, meaning that a combination of a pressure and \(O/F\) values results in a unique set of stagnation temperature, density and specific heats, but a combination of a stagnation temperature and \(O/F\) values results in multiple sets of properties, making the heat transfer system for the combustion chamber and nozzle unfeasible. Since the combustion of a hybrid rocket occurs almost instantly and the amount of heat transferred can be made relatively small compared to the energy in the exhaust flow [12], this consideration does not affect the model results, as it will be shown in Chapter 6.

To model the heat transfer phenomenon on the oxidizer tank, some assumptions were made: the process occurs on a one dimensional steady state, agreeing with the propulsion assumptions, the tank and oxidizer have the same temperature, since the tank’s wall are thin enough for the internal and external diameters to be approximately the same. The model was divided into three different stances: the convection between the oxidizer tank and the air inside the structure, the conduction through the external structure and the external convection occurring with the air, as represented in Figure 3.6. Since there is no internal energy generation and the properties are constant, the thermal resistance analogy approach was selected, as shown in Figure 3.7.

The convection inside the cavity between the tank and external structure was modeled as a free convection flow in an annular space between long concentric cylinders [56], since the external diameter of the oxidizer tank does not coincide with the internal diameter of the structure, as shown in Figure 3.8. The equivalent thermal resistance for cylindrical structures is related to both cylinders’ diameters and lengths, as well as the thermal conductivity of the fluid, in this case, air.

Since the rocket is on a vertical position, the approximation of a vertical rectangular cavity was initially considered. However, since the distance between the tank and external structure is much shorter than the oxidizer tank length, the range of validity of this model is exceeded, opting in the end by a horizontal concentric cylinder approximation instead. The air temperature was set to 301 K, as it will be further discussed in Section 4.1, calculating the necessary properties corresponding to that temperature. The
first step necessary is to calculate the Rayleigh number $Ra_c$. The length scale $L_{ray}$ used to calculated $Ra_c$ is related to the radius of the oxidizer tank $r_{OT}$ and combustion chamber $r_{CC}$, as shown in Equation 3.35. Using the temperature difference between the oxidizer tank $T_{OT}$ and the inside air $T_{air}$, the gravitational acceleration $g$, the air kinematic viscosity $\nu$, expansion coefficient $\beta$ and thermal diffusivity $\alpha$ it is possible to calculate the Rayleigh number, as shown in Equation 3.36.

$$L_{ray} = \frac{2ln\left(\frac{CC}{OT}\right)^{1.333}}{(r_{OT}^{0.6} + r_{CC}^{0.6})^{1.667}}$$  \hspace{1cm} (3.35)

$$Ra_c = \frac{g\beta(T_{OT} - T_{air})}{\nu\alpha L_{ray}^3}$$  \hspace{1cm} (3.36)

To take into account the non-stationary working fluid, a correction factor is applied to its thermal
conductivity $K_f$. The effective thermal conductivity $K_{eff}$ is calculated using the Prandtl number $Pr$, as shown in Equation 3.37. If $K_{eff} < K_f$ the nominal value is used instead. With the thermal conductivity calculated, the internal convection resistance $R_{conv,in}$ is calculated as a function of the oxidizer tank length $L_{OT}$, as shown in Equation 3.38

$$\frac{K_{eff}}{K_f} = 0.386\left(\frac{Pr}{0.861 + Pr}\right)^{0.25}Ra^{0.25}$$  \hspace{1cm} (3.37)

$$R_{conv,in} = \frac{\ln\left(\frac{CC}{OT}\right)}{2\pi L_{OT}K_{eff}}$$ \hspace{1cm} (3.38)

The conduction through the external structure walls $R_{cond}$ is modeled as a one-dimensional cylindrical wall and is a function of the rocket’s external radius $r_{ex}$ and thermal conductivity of the carbon fiber-epoxy $K_{ex}$, resulting in the thermal resistance as written in Equation 3.39.

$$R_{cond} = \frac{\ln\left(\frac{r_{CC}}{r_{ex}}\right)}{2\pi L_{OT}K_{ex}}$$ \hspace{1cm} (3.39)

To model the external convection, the simplified model of a flat plate parallel to the flow with a length of $2\pi r_{ex}$. Taking into account the distance between the rocket nose cone and the oxidizer tank, turbulent flow over an isothermal plate was selected. The initial step is to calculate the local Nusselt number $Nu$ on different sections of the external structure. The Reynolds number $Re$ is calculated as a function of the rocket's velocity $v$, air kinematic viscosity $\nu$ and the position of the analyzed section $d_i$ as written in Equation 3.40. The local Nusselt number is calculated as shown in Equations 3.41.

$$Re = \frac{vd_i}{\nu}$$ \hspace{1cm} (3.40)

$$Nu = 0.0296Re^{0.8}Pr^{0.333}$$  \hspace{1cm} (3.41)

Using the local Nusselt number and the thermal conductivity of air $K_{air}$, it is possible to calculate the local convection coefficient $hc$ as shown in Equation 3.42. By integrating the local convection coefficient over the whole tank length, the average convection coefficient $\bar{hc}$ is calculated as shown in Equation 3.43.

$$hc = \frac{NuK_{air}}{d_i}$$ \hspace{1cm} (3.42)

$$\bar{hc} = \frac{\int_0^{L_{OT}} hc \, dx}{L_{OT}}$$ \hspace{1cm} (3.43)

The convection thermal resistance is then calculated as a function of the tank length, external radius and average convection coefficient, as in Equation 3.44

$$R_{conv,ex} = \frac{1}{2\pi L_{OT}r_{ex}\bar{hc}}$$ \hspace{1cm} (3.44)
Having calculated all the thermal resistances, the heat transferred $Q$ can be calculated using the difference between the oxidizer tank $T_{OT}$ and ambient temperature $T_{amb}$, as well as the sum of the resistances, as stated in Equation 3.45.

$$Q = \frac{T_{OT} - T_{amb}}{R_{\text{conv,in}} + R_{\text{cond}} + R_{\text{conv,ex}}}$$  \hspace{1cm} (3.45)

### 3.2 Aerodynamics

The main mission parameter for the rocket is the apogee since this is one of the most important criteria in the Spaceport America CUP. Knowing this, the study of the rocket's aerodynamics is a key factor for model optimization.

The model developed by Pearson [57] used to predict the rocket's trajectory considers a one-degree-of-freedom movement, taking the rocket altitude $alt$ to calculate the ambient properties, such as the pressure $p_{amb}$ and density $\rho_{amb}$, by the International Standard Atmosphere lookup table, to estimate the rockets Mach number $M$, by dividing its velocity $v$ by the speed of sound $v_{\text{sound,air}}$, and average drag coefficient $C_{\text{drag}}$ using the data of 27 solid rocket motors, as shown in Equation 3.46 [58].

$$alt \rightarrow \text{atmoisa}(alt) \rightarrow v_{\text{sound,air}}, p_{amb}, \rho_{amb} \rightarrow M = \frac{v}{v_{\text{sound,air}}} \rightarrow C_{\text{drag}}$$  \hspace{1cm} (3.46)

Knowing the drag coefficient, the drag force $F_{\text{drag}}$ applied to the rocket is calculated as a function of the rocket's cross section $A_{\text{trans}}$ and air density $\rho_{\text{air}}$, as shown in Equation 3.47. The thrust $F_{t,\text{prop}}$, calculated at the propulsion subsystem Section 3.1, needs to be adjusted to take into account the contribution from the pressure variation at the sea level $p_{\text{atm}}$ and ambient $p_{\text{amb}}$. The effective thrust force $F_t$ is calculated as shown in Equation 3.48.

$$F_{\text{drag}} = 0.5C_{\text{drag}}A_{\text{trans}}\rho_{\text{air}}v^2$$  \hspace{1cm} (3.47)

$$F_t = F_{t,\text{prop}} + (p_{\text{atm}} - p_{\text{amb}})A_e$$  \hspace{1cm} (3.48)

Using Newton's Second law of motion, it is possible to estimate the rocket acceleration $\text{accel}$ as a function of the external forces and the rocket's mass $m_{\text{rocket}}$, as shown at Equation 3.49. The acceleration is numerically integrated to determine the velocity $v$ and altitude $alt$, as in Equations 3.50 and 3.51.

$$\text{accel}_i = \frac{F_t - F_{\text{drag}}}{m_{\text{rocket}}} - g$$  \hspace{1cm} (3.49)

$$\text{accel} = \frac{dv}{dt} \rightarrow v_{i+1} = \text{accel}_i \cdot dt + v_i$$  \hspace{1cm} (3.50)

$$\text{accel} \frac{d^2alt}{dt^2} \rightarrow alt_{i+1} = v_i \cdot dt + \text{accel}_i \cdot \frac{dt^2}{2} + alt_i$$  \hspace{1cm} (3.51)
3.3 Mass and Sizing

A key factor for any flight vehicle is the vehicle’s total mass, for its direct relation to the propulsion subsystem. Create a lighter structure and optimizing the mass distribution in the rocket are key factors to achieve the optimal design and minimize the costs to build the vehicle. The rocket size is a factor as important as the mass since the rocket stability is directly related to its length.

In this section, the models used to calculate and estimate both mass and sizing of the rocket are presented.

3.3.1 Mass Estimation

In a sounding rocket the mass can vary according to the material of the components and the respective dimensions. But since the mass of each compartment is necessary for the model, a multivariate linear regression was done by Klammer [13]. At this linear regression, twelve rockets mass data [3, 20, 21, 29–36, 59] was used to create a linear fit to estimate the structural mass $m_{\text{structural}}$ in function of the rocket external diameter $D_{\text{ex}}$ and propellant mass $m_{\text{prop}}$, calculated as the sum of the fuel mass $m_{\text{fuel}}$ and oxidizer mass $m_{\text{ox}}$, as shown in Equation 3.52. The linear fit is shown in Figure 3.9 and the multivariate linear regression is shown in Equation 3.53.

![Figure 3.9: Linear Regression for Mass Estimation][13]

$$m_{\text{prop}} = m_{\text{fuel}} + m_{\text{ox}} \quad (3.52)$$

$$m_{\text{structural}} = 0.4349m_{\text{prop}} + 259.015D_{\text{ex}} - 13.262 \quad (3.53)$$

The structural mass calculated represents the sum of the masses of all the rockets components. However, since the masses of the rocket external structure $m_{\text{body}}$, the nose cone $m_{\text{cone}}$, the fins $m_{\text{fins}}$, combustion chamber $m_{\text{CC}}$ and oxidizer tank $m_{\text{OT}}$ are calculated, the function can be modified to estimate only the missing values, which are the recovery system mass $m_{\text{rec}}$, avionics system mass $m_{\text{av}}$ and nozzle mass $m_{\text{nozzle}}$. The expression for the estimated mass $m_{\text{estimated}}$ is shown in Equation 3.54.
To distribute the estimated mass to the three remaining components the average value of the masses of each of them were taken from the reference models resulting in a distribution of 60% designated for the recovery bay, since this compartment is composed of two parachutes, the main and drogue, and the parachute launch system, meaning that between the three its mass and volume vary the most, 25% designated for the avionics system, since the electronic components usually are lighter than the recovery components, and 15% for the nozzle mass, since the design and material selection of this part is usually focused to minimize the mass for stability purposes.

\[ m_{\text{estimated}} = m_{\text{rec}} + m_{\text{av}} + m_{\text{nozzle}} = m_{\text{structural}} - m_{\text{body}} - m_{\text{cone}} - m_{\text{fins}} - m_{\text{CC}} - m_{\text{OT}} \] (3.54)

Another mass that is necessary to estimate is the payload mass. At the Spaceport America Cup rules [60] it is stated that the payload mass must be equal or higher to 8.8 lb, or approximately 4 kg. To take this rule into account, the payload mass was seated as 4 kg.

### 3.3.2 Mass Calculation

Having determined a model to estimate the masses of the nozzle, recovery and avionic system, it is also necessary to calculate the masses of the remaining components. These components can be separated into three groups: the cylindrical components, the nose cone and fins.

To calculate the mass of the cylindrical components, such as the combustion chamber, the oxidizer tank and the rocket body, a model of a thin wall cylinder is adopted. The masses of these components are calculated as a function of its density \( \rho_{\text{cylinder}} \) and volume \( V_{\text{cylinder}} \), that can be calculated using the cylinder’s length \( L_{\text{cylinder}} \), internal \( r_{\text{cylinder,int}} \) and external radius \( r_{\text{cylinder,ex}} \), as in Equation 3.55. To calculate the internal radius of the cylinder, a value for the wall thickness is necessary. Using the data from six different rockets launched at the 2018 Spaceport America Cup [3, 17, 20, 21, 29, 30], an average value for the wall thickness was created. The thickness selected for the combustion chamber was \( t_{\text{CC}} = 3 \text{mm} \), for the oxidizer tank \( t_{\text{OT}} = 6 \text{mm} \) and for the rocket body \( t_{\text{body}} = 3 \text{mm} \). Since the external radius is a design variable and therefore is considered a known value, the internal radius can be calculated by subtracting the cylinder’s thickness \( t_{\text{cylinder}} \) as in Equation 3.56.

\[ m_{\text{cylinder}} = \rho_{\text{cylinder}} V_{\text{cylinder}} = \rho_{\text{cylinder}} L_{\text{cylinder}} \pi (r_{\text{cylinder,ex}}^2 - r_{\text{cylinder,int}}^2) \] (3.55)

\[ r_{\text{cylinder,int}} = r_{\text{cylinder,ex}} - t_{\text{cylinder}} \] (3.56)

To define the density of each component it is necessary to define the most commonly used materials. For a sounding rocket, it is a common practice to use carbon fiber-epoxy as the rocket body material, since it presents a lower density in comparison to other lightweight materials (such as aluminum), a high tensile strength, toughness, corrosion and impact resistance, having a high strength to weight ratio [61]. Knowing this, the properties of a carbon fiber-epoxy composite were considered for the rocket body. As
for the oxidizer tank and combustion chamber, using the data from the previously mentioned six rockets, the most commonly used material is the Aluminum 6061-T6 for its low price, for being easily machinable and to attend the resistances requisites during operation[3]. Therefore it was selected as the material for the combustion chamber and oxidizer tank.

The nose cone mass calculation has a similar procedure to the cylindrical parts calculations. The simplification of a thin-walled nose cone is applied to the model. Since the nose cone shape can vary according to each team’s project, a conical shape was adopted as a reference, with a correction factor \( \zeta_{\text{cone}} \). The nose cone mass can be calculated based on its material density \( \rho_{\text{cone}} \), superficial area \( A_{\text{cone}} \), thickness \( t_{\text{cone}} \) and length \( L_{\text{cone}} \), as shown in Equation 3.57. The material selection for the nose cone is very similar to the one done for the rocket body. However, carbon fiber can generate radio frequency shielding [62] and since the nose cone is commonly used to accommodate electronic sensors, the carbon fiber is not commonly used as the nose cone material. With similar mechanical properties as the carbon fiber, fiberglass is an alternative material that does not generate radio frequency shielding. Therefore, a fiberglass-epoxy composite was selected as the material for the nose cone.

\[
m_{\text{cone}} = \rho_{\text{cone}} \zeta_{\text{cone}} A_{\text{cone}} t_{\text{cone}} = \rho_{\text{cone}} \zeta_{\text{cone}} t_{\text{cone}} \left( \pi r_{ex} \sqrt{L_{\text{cone}}^2 + r_{ex}^2} \right)
\]  

The fins of a rocket are responsible to stabilize the rocket during flight, maintaining its orientation and trajectory [63]. To reduce the structural mass and guarantee higher stability, the fins are usually made of a composite material, like carbon fiber-epoxy. In this model, a trapezoidal carbon fiber-epoxy fin is considered, resulting in a mass expression as in the Equation 3.58. In this equation the mass of the fins \( m_{\text{fin}} \) is calculated as a function of its density \( \rho_{\text{fin}} \), span \( S_{\text{fin}} \), root chord \( B_{\text{fin}} \), tip chord \( b_{\text{fin}} \) and thickness \( t_{\text{fin}} \).

\[
m_{\text{fin}} = \rho_{\text{fin}} t_{\text{fin}} (B_{\text{fin}} + b_{\text{fin}}) S_{\text{fin}} \quad \text{(3.58)}
\]

### 3.3.3 Size Estimation

In the project of a rocket, some variables such as the dimensions of the drogue and main parachutes, the avionic components and the fins may vary according to each team decision making, resources and priorities making it difficult to predict or calculate. However, since this model requires the length of each of the rockets compartments, an approximation needed to be done.

Using the data from the same rockets used on mass calculation (Subsection 3.3.2)[3, 17, 20, 21, 29, 30], a linear regression was done relating the length of the recovery system bay to the length of the rocket and the avionics bay to the recovery system bay. The linear regression comparison with the actual rocket recovery and avionics length is shown in Figure 3.10.

Using this fits, the length of recovery bay \( L_{\text{rec}} \) is calculated as a function of the rocket’s length \( L_{\text{rocket}} \) and the avionics bay \( L_{\text{av}} \) is calculated as a function of \( L_{\text{rec}} \) as expressed in Equations 3.59 and 3.60.

\[
L_{\text{rec}} = 0.2404 L_{\text{rocket}} - 0.4193
\]  

\[
L_{\text{av}} = 0.7121 L_{\text{rec}} - 0.4193
\]  

31
The payloads are separated into two categories: a "boiler-plate" payload, being a non-functional payload, and a scientific experiment or technology demonstration payload. For the boiler-plate payload, a minimum requirement for its length is to fit at least a 3U Cubesat, with 30cmx10cmx10cm. However, the scientific experiment payload may be constructed in any form factor. To attend the non-functional payload rule, a length of $L_{\text{pay}} = 0.4$ m was considered for the payload, taking a tolerance into account. The length of the nose cone and the length of the nozzle are two independent project variables that can vary according to each group’s manufacturing costs and team decisions, therefore are hard to associate with other known parameters. To stipulate these two dimensions, an average value between the six rockets was used leading to a length of $L_{\text{cone}} = 0.6$ m for the nose cone and a length of $L_{\text{nozzle}} = 0.135$ m for the nozzle. The assumed lengths are shown in Table 3.1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length Nose cone</td>
<td>0.6 m</td>
</tr>
<tr>
<td>Length Payload</td>
<td>0.4 m</td>
</tr>
<tr>
<td>Length Nozzle</td>
<td>0.135 m</td>
</tr>
</tbody>
</table>

Table 3.1: Assumed Lengths

To determine the rocket external diameter, a geometrical constraint was considered. The combustion chamber was considered as a straight fit inside the fuselage, meaning that the external diameter of the combustion chamber is equal to the internal diameter of the rocket external tube and that the oxidizer tank diameter is lower than the combustion chambers. Normally the teams apply a thermal protection coating inside or outside the combustion chamber to minimize the heat transfer from the combustion to the chamber and external structure.
3.3.4 Size Calculation

Since the size of the rocket is directly related to its mass and stability, the calculation of the length of each compartment in a precise way is crucial to the model. The components whose lengths can be calculated are the oxidizer tank and the rocket external body.

To calculate the oxidizer tank’s length \( L_{OT} \), a cylindrical shape is considered. Combining two design variables, the tank volume \( V_{OT} \) and tank radius \( r_{OT} \), the length of this component is expressed in Equation 3.61.

\[
L_{OT} = \frac{V_{OT}}{\pi r_{OT}^2}
\]

The rocket external body is a carbon fiber-epoxy tube, which needs to contain the recovery, payload, avionics, oxidizer tank, combustion chamber and part of the nozzle. To take into account the nozzle part that is allocated inside the composite tube, a factor \( N_{nozzle} \) is created. Since the combustion chamber length \( L_{CC} \) is a design variable, the rocket external body \( L_{body} \) and total length \( L_{rocket} \) can be calculated as shown in Equations 3.62 and 3.63.

\[
L_{body} = L_{rec} + L_{pay} + L_{av} + L_{OT} + L_{CC} + L_{nozzle}N_{nozzle}
\]

\[
L_{rocket} = L_{cone} + L_{body} + L_{nozzle}(1 - N_{nozzle})
\]

The initial estimation of the rocket’s length \( L_{rocket,0} \) is performed by substituting the recovery and avionics lengths expressions in Equation 3.62, rearranging it as a function of the known lengths, as shown in Equation 3.64.

\[
L_{rocket,0} = \frac{L_{cone} + L_{pay} + L_{OT} + L_{CC} + L_{nozzle}}{0.5939} = 1.324
\]

3.4 Stability

To study the stability of the vehicle it is necessary to find its center of gravity and center of pressure. In this section concepts about rocket stability are discussed, as well as the calculation of its center of gravity and center of pressure used to calculate the static margin.

3.4.1 Rocket Stability

During the rocket flight disturbances, such as a small wind gust or thrust instabilities, can change the rocket trajectory and, in more extreme cases, cause the vehicle to crash. The position of the center of pressure and center of gravity are key factors for the vehicle stability. With the rocket on a vertical position and the nose cone up, a stable rocket will have its center of gravity above the center of pressure, meaning that the center of gravity is closer to the nose cone. In Figure 3.11, an example of a stable
configuration is shown [64].

On the rocket's rise, there can be 3 states: powered, when the hybrid motor still providing thrust; stable, when the rocket is vertical; and coasting, when the motor has run out of fuel. In the example of Figure 3.11, at the powered state, both lift and drag produce counter-clockwise moments about the center of gravity, moving the rocket to a stable condition. At the coasting state, both lift and drag produce clockwise moments about the center of gravity, moving the rocket to a stable condition as well. These forces are called restoring forces since they allow the rocket to return to its stable state. For the opposite case, where the center of pressure is above the center of gravity, both forces are destabilizing forces, meaning that if a disturbance occurs, both lift and drag will generate moments that will destabilize the rocket even more[64].

### 3.4.2 Center of Gravity

The center of gravity is a geometric property, related to the average location of the weight of an object. This property is directly related to the body dynamics since the center of rotation of a body is its center of gravity, being therefore crucial for the rocket stability.

To calculate the rocket center of gravity, it is necessary to locate each of the component’s center of gravity, relative to the nose cone tip, as well as the weights, as shown in Figure 3.12.

The position of each component’s center of gravity is shown in Table 3.2.

<table>
<thead>
<tr>
<th>Center of Gravity Distance to Nose Cone Tip</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(d_{\text{cone}})</td>
<td>(0.67l_{\text{cone}})</td>
</tr>
<tr>
<td>(d_{\text{rec}})</td>
<td>(l_{\text{cone}} + 0.5l_{\text{rec}})</td>
</tr>
<tr>
<td>(d_{\text{payload}})</td>
<td>(d_{\text{rec}} + 0.5l_{\text{rec}} + 0.5l_{\text{pay}})</td>
</tr>
<tr>
<td>(d_{\text{electronics}})</td>
<td>(d_{\text{pay}} + 0.5l_{\text{pay}} + 0.5l_{\text{ew}})</td>
</tr>
<tr>
<td>(d_{\text{fins}})</td>
<td>(d_{\text{fins}} + 0.5l_{\text{fins}} + 0.5l_{\text{fins}})</td>
</tr>
<tr>
<td>(d_{\text{nozzle}})</td>
<td>(d_{\text{nozzle}} + 0.5l_{\text{cc}} + 0.5l_{\text{nozz}})</td>
</tr>
<tr>
<td>(d_{\text{finsalt}})</td>
<td>(l_{\text{cone}*l_{\text{body}} - b_{\text{fins}} - 0.67(b_{\text{fins}} - b_{\text{fins}})})</td>
</tr>
<tr>
<td>(d_{\text{finsroot}})</td>
<td>(l_{\text{cone}*l_{\text{body}} - 0.5b_{\text{fins}}})</td>
</tr>
</tbody>
</table>

Table 3.2: Components center of gravity
Figure 3.12: Rocket’s components center of gravity position

The position of the rocket center of gravity CG is related to the weighted sum of each component center of gravity $d_i$, weighted according to its mass $m_i$, as shown at Equation 3.65.

$$ CG = \frac{\sum d_i m_i}{\sum m_i} \quad (3.65) $$

### 3.4.3 Center of Pressure

Such as the center of gravity, the center of pressure is another key factor for the rocket stability. The center of pressure of the structure is the average position where the sum of all the pressure fields on the rocket would generate the corresponding aerodynamic forces, drag and lift.

To calculate the center of pressure, the Barrowman equations were applied. This set of equations was developed by James Barrowman from NASA’s Sounding Rocket Branch for subsonic rockets [65]. The Figure 3.13 shows a scheme of the necessary dimensions for the center of pressure calculation.

The Barrowman Equations use the following assumptions: the angle of attack is low ($\alpha < 10^\circ$); the rocket velocities are subsonic; the airflow over the rocket is smooth; the rocket length is much larger than its diameter; the rocket is an axisymmetric rigid body; the fins are flat plates.

Since the designed rocket does not present a transition, the contribution of this section was not considered. For the notation used in these equations see Figure 3.13. The contribution of the nose cone and body are grouped, having a normal force coefficient of $(C_N)_{cone} = 2$ and a center of pressure located at $CP_{cone} = \frac{2}{3} L_{cone}$ from the nose cone tip. The Barrowman Equations for the fins are shown in Equations 3.66 and 3.67.

$$(C_N)_{fins} = \left[ 1 + \frac{R}{S + R} \right] \left[ \frac{4N(S_0)^2}{1 + \sqrt{1 + \left( \frac{2L_F}{C_R + C_T} \right)^2}} \right] \quad (3.66)$$

$$ CP_{fins} = x_B + \frac{x_R}{3} \frac{(C_R + 2C_T)}{C_R + C_T} + \frac{1}{6} \left[ (C_R + C_T) - \frac{C_R C_T}{C_R + C_T} \right] \quad (3.67) $$
3.4.4 Static Margin

To measure the stability of the rocket a variable called static margin \( SM \) is created, relating the position of the center of pressure and center of gravity, according to the nose cone tip, to the rocket external diameter, as shown in Equation 3.69.

\[
SM = \frac{CP - CG}{D_{ex}}
\]  

(3.69)

3.5 Model Limitations

As a product of the approximations adopted by the theoretical models, some physical effects are not considered in this model. Since the project is related to design optimization, a more sophisticated model would result in a much longer simulation, being the usage of the simplifications more suitable for conceptual design. Some of the limitations of the model are:

- Heat Transfer on the combustion chamber and nozzle;
• Axial and radial variation on the combustion chamber properties;
• Transient effects on the oxidizer tank and combustion instability;
• Non-chemical sources of combustion efficiency, such as fuel additives and pre- and post-combustion chamber;
• One-dimensional trajectory analysis;
• Non-optimization of aerodynamic structures shapes, namely nose cone and fins;
• Estimation of recovery, avionics, nozzle lengths and masses.
Chapter 4

Multidisciplinary Design Optimization

In the last decades, with the increasing technological advances, numerical models have been developed to predict the performance and efficiency of an engineering project. For this reason, the research for the design with optimal performance has been gaining an increasing importance. While single discipline optimization has a vast background of solutions, a system that involves multiple disciplines has additional issues in the analysis and design optimization [48].

Multidisciplinary design optimization (MDO) is a research field that aims at finding the optimal design for a system involving different disciplines, using numerical methods. A complex system model needs to ensure not only the performance of each subsystem but also their interactions, making an MDO approach a key component to improve the design while saving time and costs of a design cycle [48].

In this chapter, the MDO architecture and the Individual Discipline Feasible are explained, showing the structure of the disciplines arrange, the design variable, state variables and the constraints imposed on the system.

4.1 Multidisiplinary Structure

On this hybrid rocket model, four disciplines were mentioned in Chapter 3: aerodynamics, propulsion, stability and mass & sizing. The first step for the MDO is to identify the inputs and outputs of each discipline. The variables that are only inputs of a discipline are known as design variables. However, if a variable is an output of discipline and an input to another, it is known as a state variable. These variables are the ones responsible for the optimization and therefore shall be further analyzed. Each subsystem inputs and outputs are shown in Table 4.1.

The next step is to define the system's constraints. The constraints are important to ensure that the theoretical model respects the physical limitations of a real system. With the constraints, design and state variables defined it is necessary to select the output that shall be optimized, named as the objective function.

After the definition of the objective function, the subsystems must be arranged in such a way that optimizes the inputs and outputs that are necessary for each component. In Table 4.1 it is possible to
Table 4.1: Inputs and Outputs for the optimization problem

<table>
<thead>
<tr>
<th>Propulsion</th>
<th>Mass &amp; Sizing</th>
<th>Aerodynamics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input</td>
<td>Output</td>
<td>Input</td>
</tr>
<tr>
<td>$V_{OT}$</td>
<td>$r_{CCin}$</td>
<td>$V_{OT}$</td>
</tr>
<tr>
<td>$C_{inj}$</td>
<td>$m_{fuel}$</td>
<td>$r_{OT}$</td>
</tr>
<tr>
<td>$L_{f}$</td>
<td>$m_{ax}$</td>
<td>$r_{CCin}$</td>
</tr>
<tr>
<td>$D_{port}$</td>
<td>$P_{OT}$</td>
<td>$m_{fus}$</td>
</tr>
<tr>
<td>$D_{th}$</td>
<td>$P_{CC}$</td>
<td>$m_{ax}$</td>
</tr>
<tr>
<td>$A_{nata}$</td>
<td>$G$</td>
<td>$L_{CC}$</td>
</tr>
<tr>
<td>$r_{OT}$</td>
<td>$l_{ip}$</td>
<td>$S_{fin}$</td>
</tr>
<tr>
<td>$D_{ex}$</td>
<td>$P_{feed}$</td>
<td>$B_{fin}$</td>
</tr>
<tr>
<td>$L_{OT}$</td>
<td>$L_{body}$</td>
<td>$b_{fin}$</td>
</tr>
<tr>
<td>$r_{CC}$</td>
<td>$L_{rec}$</td>
<td>$m_{addcone}$</td>
</tr>
<tr>
<td>$d_{OT}$</td>
<td>$m_{addcone}$</td>
<td>$L_{av}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

notice that the inputs of Mass & Sizing that are state variables come from the Propulsion subsystem. Considering also that the inputs of the Aerodynamic subsystem are highly dependent on the outputs of the other subsystems, the Propulsion subsystem was selected as the first on the simulation line, followed by the Mass & Sizing and Aerodynamics subsystems, as shown in Figure 4.1. The convergence between inputs and outputs should be ensured as described in Sections 4.3, 4.4 and 4.5.

After listing each subsystem’s inputs and outputs the design and state variables were defined. The system’s design variables are the oxidizer tank volume $V_{OT}$, the effective injection area $C_{inj}$, the fuel grain length $L_{f}$ and initial port diameter $D_{port}$, the nozzle throat diameter $D_{th}$ and area ratio $A_{ratio}$, the combustion chamber length $L_{CC}$, the fin span $S_{fin}$, root chord $B_{fin}$ and tip chord $b_{fin}$, the oxidizer tank radius $r_{OT}$ and the additional masses at the nose cone $m_{addcone}$ and at the recovery bay $m_{add}$.

The system’s state variables are the rocket mass $m_{rocket}$ and external diameter $D_{ex}$, external structure length $L_{body}$ and mass $m_{body}$, combustion chamber mass $m_{CC}$, oxidizer tank mass $m_{OT}$ and length $L_{OT}$, recovery bay length $L_{rec}$ and mass $m_{rec}$, avionics bay length $L_{av}$ and mass $m_{av}$, the mass of the fins $m_{fins}$, the nozzle $m_{nozzle}$, the nose cone $m_{cone}$, fuel $m_{fuel}$ and oxidizer $m_{ox}$, the distance of the oxidizer tank to the nose cone tip $d_{OT}$ and the combustion chamber internal $r_{CCin}$ and external radius $r_{CC}$.

The definition of the model constants is equally necessary for the model to work properly. The typical launch site for the competition is Spaceport America, located between the city of Las Cruces and Truth or Consequences, New Mexico. The location lies at an elevation of 1400m above sea level and have an
average temperature of 301 K during the month of June when the competition occurs [67]. According to the International Standard Atmosphere, the pressure at a height 1400m above sea level is 85.6 kPa[58]. The discharge correction factor, characteristic velocity correction factor and thrust coefficient correction factor were settled as 1.05, 0.9 and 0.9, respectively, as recommended by Sutton and Biblar [6].

For the propulsion subsystem, a few parameters need to be defined as well. The tank liquid fill level was settled to 60% for a more conservative approach, taking into account the possible differences in the theoretical and real model, allowing the designer to increase the fill level if necessary. For the initial oxidizer tank pressure, a value of 5 MPa was set, since the average temperature at the launch site can guarantee that this pressure is always higher than the vapor pressure of N\textsubscript{2}O. The feed system pressure drop was considered 0.1 MPa, as in [13]. To establish the paraffin wax density, the NIST calculator [68] was used, settling it to 930 kg/m\textsuperscript{3}. Being the regression rate fundamental for the operation, the definition of the regression rate coefficient and the exponent is essential. The constants presented in McCormick et al. [39] for an aluminized paraffin wax - N\textsubscript{2}O were used, settling the regression rate coefficient to 0.155 mm/s and exponent to 0.5. A summary of these parameters is shown in Table 4.2.

4.2 Design Variables

As described before in Section 4.1, the design variables are the actual inputs of the system, being the ones that needs to be taken into account at the project design stage, since they will directly influence the objective function. The set of design variables are assembled into a vector \(\{x\}\).

To select a physically compatible set of design variables, the definition of lower and upper boundaries are necessary. The design variables of the hybrid rocket model are shown in Table 4.3.

Initially, six design variables were considered for the system. However, after further testing, the com-
Table 4.2: Parameters of the simulation

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time step (dt)</td>
<td>0.01 s</td>
</tr>
<tr>
<td>Tank Fill Level</td>
<td>60%</td>
</tr>
<tr>
<td>Initial Oxidizer Tank Pressure (P_{OT,0})</td>
<td>5 Mpa</td>
</tr>
<tr>
<td>Feed System Pressure Drop (P_{feed})</td>
<td>0.1 Mpa</td>
</tr>
<tr>
<td>Fuel Density (\rho_f)</td>
<td>930 kg/m³</td>
</tr>
<tr>
<td>Regression Rate Coefficient (a)</td>
<td>0.155 mm/s</td>
</tr>
<tr>
<td>Regression Rate Exponent (n)</td>
<td>0.5</td>
</tr>
<tr>
<td>Launch Elevation (alt_0)</td>
<td>1400 m</td>
</tr>
<tr>
<td>Ambient Pressure (P_{amb})</td>
<td>85.6 kPa</td>
</tr>
<tr>
<td>Ambient Temperature (T_{amb})</td>
<td>301 K</td>
</tr>
<tr>
<td>Discharge Correction Factor (\epsilon_d)</td>
<td>1.05</td>
</tr>
<tr>
<td>Characteristic Velocity Correction Factor (c_c)</td>
<td>0.9</td>
</tr>
<tr>
<td>Thrust Coefficient Correction Factor (\zeta_m)</td>
<td>0.9</td>
</tr>
</tbody>
</table>

Table 4.3: Design Variables (x)

<table>
<thead>
<tr>
<th>x</th>
<th>Parameter</th>
<th>Description</th>
<th>Lower Bound</th>
<th>Upper Bound</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Oxidizer Tank Volume (V_{OT})</td>
<td>Controls altitude</td>
<td>4 (10^{-3}) m³</td>
<td>14 (10^{-3}) m³</td>
</tr>
<tr>
<td>2</td>
<td>Effective Injection Area (G_{ini})</td>
<td>Controls O/F, chamber pressure, thrust</td>
<td>1 (10^{-5}) m²</td>
<td>4 (10^{-5}) m²</td>
</tr>
<tr>
<td>3</td>
<td>Fuel Grain Length (L_f)</td>
<td>Controls O/F</td>
<td>0.2 m</td>
<td>0.7 m</td>
</tr>
<tr>
<td>4</td>
<td>Fuel Grain Initial Port Diameter (D_{port})</td>
<td>Controls mass flux, limits altitude</td>
<td>3 (10^{-2}) m</td>
<td>8 (10^{-2}) m</td>
</tr>
<tr>
<td>5</td>
<td>Nozzle Throat Diameter (D_{th})</td>
<td>Controls thrust, chamber pressure</td>
<td>2 (10^{-2}) m</td>
<td>5 (10^{-2}) m</td>
</tr>
<tr>
<td>6</td>
<td>Nozzle Area Ratio (A_{ratio})</td>
<td>Controls thrust</td>
<td>3</td>
<td>10</td>
</tr>
<tr>
<td>7</td>
<td>Combustion Chamber Length (L_{CC})</td>
<td>Controls rocket length, combustion stability</td>
<td>0.5 m</td>
<td>3 m</td>
</tr>
<tr>
<td>8</td>
<td>Fin Span (S_{fin})</td>
<td>Controls drag, stability</td>
<td>0.01 m</td>
<td>0.4 m</td>
</tr>
<tr>
<td>9</td>
<td>Oxidizer Tank Radius (r_{OT})</td>
<td>Controls rocket diameter</td>
<td>0.01 m</td>
<td>0.09 m</td>
</tr>
<tr>
<td>10</td>
<td>Fin Root Chord (B_{fin})</td>
<td>Controls stability</td>
<td>0.01 m</td>
<td>0.3 m</td>
</tr>
<tr>
<td>11</td>
<td>Fin Tip Chord (b_{fin})</td>
<td>Controls stability</td>
<td>0.01 m</td>
<td>0.225 m</td>
</tr>
<tr>
<td>12</td>
<td>Additional Mass at Reovery bay (m_{add})</td>
<td>Controls stability, rocket mass</td>
<td>0 kg</td>
<td>1.5 kg</td>
</tr>
<tr>
<td>13</td>
<td>Additional Mass at Nosecone (m_{addcone})</td>
<td>Controls stability, rocket mass</td>
<td>0 kg</td>
<td>4.5 kg</td>
</tr>
</tbody>
</table>

bustion chamber length, the fins dimensions and the oxidizer tank radius, which was defined previously as a function of other rockets dimensions, were added to the design variable vector, adding corresponding constraints to ensure physical compatibility. Since the program was having difficulties to stabilize the rocket, two additional masses were added to the nose cone and the recovery bay, adding two more design variables.

4.3 State Variables

With the definition of the design variables, it is necessary to ensure the coherence between the subsystems. The state variables of the system express the measurement of this coherence, since they are both inputs and output the convergence of the simulation depends on the convergence of these
variables. The set of state variables are assembled into a vector \( \{y\} \).

Similar to the design variables, the state variables need also to have their lower and upper boundaries defined, to ensure physical compatibility. The state variables of the hybrid rocket model and their boundaries are shown in Table 4.4.

<table>
<thead>
<tr>
<th>( y )</th>
<th>Parameter</th>
<th>Lower Bound</th>
<th>Upper Bound</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Rocket Mass ( (m_{rocket}) )</td>
<td>10 kg</td>
<td>50 kg</td>
</tr>
<tr>
<td>2</td>
<td>External structure Length ( (L_{body}) )</td>
<td>1.5 m</td>
<td>4 m</td>
</tr>
<tr>
<td>3</td>
<td>Rocket External Diameter ( (D_{ext}) )</td>
<td>0.03 m</td>
<td>0.2 m</td>
</tr>
<tr>
<td>4</td>
<td>Combustion Chamber Mass ( (m_{CC}) )</td>
<td>1 kg</td>
<td>10 kg</td>
</tr>
<tr>
<td>5</td>
<td>Oxidizer Tank Mass ( (m_{OT}) )</td>
<td>3 kg</td>
<td>10 kg</td>
</tr>
<tr>
<td>6</td>
<td>External structure Mass ( (m_{body}) )</td>
<td>2 kg</td>
<td>6 kg</td>
</tr>
<tr>
<td>7</td>
<td>Oxidizer Tank Length ( (L_{OT}) )</td>
<td>0.3 m</td>
<td>3 m</td>
</tr>
<tr>
<td>8</td>
<td>Recovery Bay Length ( (L_{rec}) )</td>
<td>0.25 m</td>
<td>1 m</td>
</tr>
<tr>
<td>9</td>
<td>Avionics Bay Length ( (L_{av}) )</td>
<td>0.1 m</td>
<td>1 m</td>
</tr>
<tr>
<td>10</td>
<td>Avionics Bay Mass ( (m_{av}) )</td>
<td>0.5 kg</td>
<td>6 kg</td>
</tr>
<tr>
<td>11</td>
<td>Recovery Bay Mass ( (m_{rec}) )</td>
<td>0.5 kg</td>
<td>12 kg</td>
</tr>
<tr>
<td>12</td>
<td>Fins Mass ( (m_{fins}) )</td>
<td>0.1 kg</td>
<td>3 kg</td>
</tr>
<tr>
<td>13</td>
<td>Nozzle Mass ( (m_{nozzle}) )</td>
<td>0.1 kg</td>
<td>3 kg</td>
</tr>
<tr>
<td>14</td>
<td>Nosecone Mass ( (m_{cone}) )</td>
<td>0.1 kg</td>
<td>5 kg</td>
</tr>
<tr>
<td>15</td>
<td>Oxidizer Tank Distance to Tip ( (d_{OT}) )</td>
<td>1.1 m</td>
<td>4 m</td>
</tr>
<tr>
<td>16</td>
<td>Combustion Chamber Internal Radius ( (r_{CCint}) )</td>
<td>0.002 m</td>
<td>0.09 m</td>
</tr>
<tr>
<td>17</td>
<td>Fuel Mass ( (m_{fuel}) )</td>
<td>0.1 kg</td>
<td>3 kg</td>
</tr>
<tr>
<td>18</td>
<td>Oxidizer Mass ( (m_{ox}) )</td>
<td>1 kg</td>
<td>7 kg</td>
</tr>
<tr>
<td>19</td>
<td>Combustion Chamber External Radius ( (r_{CC}) )</td>
<td>0.015 m</td>
<td>0.1 m</td>
</tr>
</tbody>
</table>

Table 4.4: State Variables \( (y) \)

As it will be further discussed in Sections 4.4 and 4.5, two distinct state variables vectors are created: the vector of targets state variables \( \{y^t\} \), containing the inputs for the analysis which is included in the design variables set; and the vector of responses state variables \( \{y\} \), containing the outputs from the analysis. To ensure the compatibility between target and response state variables, an equality constraint \( \{y^t\} - \{y\} = 0 \) should be added to the optimization problem.

### 4.4 Constraints

With the inputs and outputs fully defined, it is necessary to specify the system constraints to ensure physical coherence, model convergence, safety and mission limitations. To prevent combustion instability caused by the injector backflow failure mechanism, the combustion chamber needs to be limited by a fraction of the upstream injector pressure as recommended by Waxman [69]. Another constraint used to prevent combustion instability is to restrict the mass flow through the grain port to \( 500\text{ kg/m}^3 \), as stated in Fraters and Cervone [70].
To take into account the recommendations of the competition, the rocket’s off-the-rail velocity was restricted to be higher than $30\text{m/s}$, to ensure aerodynamic stability, and the acceleration to be lower than $100\text{m/s}^2$, to reduce the aerodynamic loads.

For an aerodynamic vehicle to be considered stable, its static margin must be positive. On a sounding rocket, an ideal static margin is between 1 and 2 [71]. For this reason, a constraint for the stability was imposed. If the static margin is higher than 2, the model is denominated overstable. An overstable rocket is more sensitive to wind effects, tending to turn to the wind direction, with the possibility to go horizontal in the worst scenario [72]. This model is not considered as infeasible, however, the wind effects must be carefully taken into account. A constraint to limit the maximum static margin is therefore imposed on the system. The combustion chamber length can be divided into three parts: the pre-chamber, fuel grain and post-chamber. The pre-chamber main function is to allow the inject gas expansion, making it cover the port surface completely and ensure even burn. The post-chamber creates extra space for the completion of the combustion before the nozzle exhaust [12]. To consider this a constraint was imposed on the system. To take into account the geometry of the system as stated in Subsection 3.3.3, constraints for the dimensions of the oxidizer tank radius and the fins were created. The constraints applied to the system are shown in Table 4.5.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Constraint</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chamber Pressure</td>
<td>Avoid Combustion instability and backflow failure mode</td>
<td>$p_{cc} - 0.8(p_{or} - p_{feed}) &lt; 0$</td>
</tr>
<tr>
<td>Mass flux</td>
<td>High mass flow associated with combustion instability</td>
<td>$G - 500 &lt; 0$</td>
</tr>
<tr>
<td>Off-the-rail velocity</td>
<td>Required for aerodynamic stability</td>
<td>$30 - V_{off_rail} &lt; 0$</td>
</tr>
<tr>
<td>Acceleration</td>
<td>Reduces Loads</td>
<td>$accel_{max} - 100 &lt; 0$</td>
</tr>
<tr>
<td>Stability</td>
<td>Required for rocket stability</td>
<td>$1 - SM &lt; 0$</td>
</tr>
<tr>
<td>Stability</td>
<td>Limit static margin</td>
<td>$SM - 3 &lt; 0$</td>
</tr>
<tr>
<td>Oxidizer tank radius</td>
<td>Rocket diameter related to combustion chamber diameter</td>
<td>$r_{OT} - r_{CC} &lt; 0$</td>
</tr>
<tr>
<td>Combustion Chamber Length</td>
<td>Pre and post combustion length</td>
<td>$L_{fuel} + D_{en} - L_{CC} &lt; 0$</td>
</tr>
<tr>
<td>Fins Length</td>
<td>Restrict the fin’s dimensions</td>
<td>$b_{fin} - 0.9b_{fin} &lt; 0$</td>
</tr>
</tbody>
</table>

Table 4.5: Inequality Constraints (g)

In addition to the inequality constraints, some equality constraints are also applied to the system. Since the rocket mission is to reach an apogee as close as possible to 10000 ft, a constraint for the model maximum altitude was imposed [60]. As mentioned before, an equality constraint for the state variables was imposed. The equality constraints are shown in Table 4.6

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Constraint</th>
</tr>
</thead>
<tbody>
<tr>
<td>Altitude</td>
<td>Set the rocket altitude for 10000 ft</td>
<td>$alt - 3050 = 0$</td>
</tr>
<tr>
<td>State Variables</td>
<td>Ensure the model consistency</td>
<td>$y' - y = 0$</td>
</tr>
</tbody>
</table>

Table 4.6: Equality Constraint (h) and Consistency Constraints (c)
4.5 Individual Discipline Feasible

As previously discussed, the hybrid rocket model is composed of highly interdependent subsystems, making the objective function and constraints a function of the design and state variables. Taking this interdependence into account can lead to a more accurate response of the system. To manage this behavior, a monolithic architecture, that solves the MDO as a single optimization problem, was selected for simplicity and efficiency matters. There are three widely used monolithic architectures: Multidisciplinary Feasible (MDF), Individual Discipline Feasible (IDF) and Simultaneous Analysis and Design (SAND) [48].

The chosen architecture was the Individual Discipline Feasible (IDF), since it allows the analysis of each discipline in parallel, meaning that it is much smaller than a SAND architecture and it runs each analysis once per iteration, being much simpler to implement than an MDF [48]. The IDF separates the state variables into two distinct vectors: state variables targets \( \{y^t\} \), that are the inputs of the system and set as design variables, and state variables responses \( \{y\} \), that are outputs of the system, as in Section 4.3. As a consequence, this formulation removes all the state variables and disciplines analysis equations from the problem statement, making the system inputs \( \{x\} \) and \( \{y^t\} \), that results in the output \( \{y\} \) at each iteration. To ensure the coherence of the model, a set of consistency constraints \( c \) is created, as already stated in Section 4.4. The selected optimization algorithm was the Sequential quadratic programming (SQP), that solves the optimization problems using gradient-based methods [48].

To establish the optimization problem, it is necessary to define the objective function that needed to be minimized. On this work, one selected objective function was to maximize the specific impulse \( I_{sp} \), since it is an indicator of the engine efficiency, calculated as a function of the total impulse \( I_{tot} \) and propellant mass \( m_{prop} \), as shown in Equation 4.1. To implement a maximization problem instead of a minimization, it describes the objective function as the negative value of the specific impulse. The optimization problem is shown in Equation 4.2.

\[
I_{sp} = \frac{I_{tot}}{m_{prop}}
\]  

Maximize \( I_{sp}(x, y(x, y^t)) \)

w.r.t. \( x_i, y^t_j \) for \( i = 1, ..., 13 \) \( j = 1, ..., 19 \)

subject to \( g_k(x, y(x, y^t)) \leq 0 \) for \( k = 1, ..., 9 \)

\( h_l(x, y(x, y^t)) = 0 \) for \( l = 1 \)

\( c_j = y^t_j - y_j = 0 \) for \( j = 1, ..., 19 \)

The second selected objective function was the rocket mass since mass minimization can lead to minimal fuel, oxidizer, and structural material usage, reducing the project costs. The minimization problem is shown in Equation 4.3.
Minimize \( m_{\text{rocket}}(x, y(x, y^t)) \)

w.r.t. \( x_i, y_j^t \) for \( i = 1, \ldots, 13 \) \( j = 1, \ldots, 19 \) \( (4.3) \)

subject to \( g_k(x, y(x, y^t)) \leq 0 \) for \( k = 1, \ldots, 9 \)

\( h_l(x, y(x, y^t)) = 0 \) for \( l = 1 \)

\( e_j = y_j^t - y_j = 0 \) for \( j = 1, \ldots, 19 \)

A visual representation of the IDF architecture can be useful as a communication medium among practitioners and those new to the field. For this reason, a diagram for visualizing the MDO process was developed by Martins [48], called extended design structure matrix (XDSM). The IDF architecture in an XDSM is shown in Figure 4.2.

![Figure 4.2: IDF architecture in a XDSM](image)

The order of execution of the components is defined by its numbering. If a loop, denoted by \( m \rightarrow n \), where \( n < m \), is present, the sequence returns to component \( n \) until the convergence conditions are satisfied, proceeding then to the component \( m+1 \) [48]. The inputs and outputs are represented by vertical and horizontal lines, respectively, and the data flow follows a clockwise direction. The disciplines use some of the state variables coming from the others disciplines analyzed in previous steps. The sequence of operations is the following:

- Input initial data in RocketOptimization.
- Evaluate all the disciplines separately.
- Compute objective function and constraints.
• Compute new design point. If optimization has not converged, return to 1; otherwise, return optimal solution.
Chapter 5

Simulation Code

Based on the theoretical model described in Chapter 3, a computational code was written. The selected software to implement the simulation was Matlab since it already has an implemented optimization function, fmincon. This chapter presents a software overview, describes the developed code used in the simulations and the numerical methods implemented. The source code is included in Appendix B.

5.1 Software Overview

The Matlab code can be divided into three groups: the individual disciplines, each one implemented as a separated function, Multidisciplinar2, which couples the individual disciplines, and RocketOptimization, that calls Multidisciplinar2 and applies the fmincon to optimize the system.

5.1.1 Multidisciplinar2

Multidisciplinar2 is the main Matlab function for the hybrid rocket model. It receives a vector with the design and target state variables as an input, performs the simulation on each discipline sub-function, leading to the objective function and the structure variable "state", that contains all the relevant simulation data, as the function output. The other simulation parameters, mentioned in Section 4.1, are hardcoded into the function.

Being the main function, Multidisciplinar2 provides all the input data necessary for the simulations, as the CEA lookup table, the N2O saturated properties table and the drag coefficients. By using the structure variable "state", the number of free variables is minimized, leading to a more organized and simpler code. The simulation starts by adding the design and target state variables, input in the function, and the constant parameters to the "state" parameter. Afterward, this structure variable is used as an input to Propulsion, that adds the motor simulation results to the "state" variable; leading to the Mass%Sizing simulation, adding all the masses and dimensions of the rocket model to the structure variable; finishing the simulation by calling the function Aerodynamics, that adds the stability and trajectory analysis to the "state"; and by selecting the objective function.
If any physically incoherent value or error occurs during the simulation, all the variables in "state" are set to not-a-number(NaN). This set is important for the optimization function to discard this solution and run more fluidly.

5.1.2 Disciplines

Using the theoretical models described in Chapter 3, three separated functions were implemented in Matlab: Propulsion, Mass & Sizing and Aerodynamics.

Propulsion is a sub-function implemented in Matlab that simulates the hybrid rocket motor described in Section 3.1. The function receives a structure variable "state", using it as input for the simulations. To create the lookup table for the combustion properties, a separated sub-function uses the CEA information to create a table for the paraffin wax-nitrous oxide combination within a \( O/F \) and \( p_{CC} \) range expected for the hybrid rocket operation. After the simulation, the outputs thrust, fuel mass and oxidizer mass are added to the "state" variable to be used in the other disciplines and constraints.

Mass & Sizing is a sub-function that estimates and calculates the rocket's masses, as described in Section 3.3. The function uses the structure variable "state" as input, using outputs from Propulsion as fuel and oxidizer mass for the model calculation. The system outputs are then added to the "state" variable, to be used in the Aerodynamics function.

Aerodynamics is a sub-function that calculates the hybrid rocket trajectory and stability analyzes, as described in Sections 3.2 and 3.3. The input is the structure variable "state", using outputs from Propulsion and Mass & Sizing, as the masses and lengths of the rockets compartments. The outputs are added for the "state" variable.

5.1.3 RocketOptimization

RocketOptimization is a Matlab script that call Multidisciplinar2 for the optimization process. Two sub functions were created for the optimization function fmincon: Constraints and Initial Guess.

Constraints is a sub-function that follows the same simulation path as Multidisciplinar2 to calculate the constraints described in Section 4.4. The inputs for this function are the design and target state variables to calculate each constraint imposed on the system and store the inequality constraints in the vector \( c \) and the consistency constraints in the vector \( c_{eq} \). Constraints output are the \( c \) and \( c_{eq} \) vectors to be used in the optimization problem.

Initial Guess is a sub-function that receives the inputs for Propulsion and uses the same simulation path as Multidisciplinar2 to estimate a set of design variables and target state variables to be used as initial guess \( x_0 \) at the simulation process.

In the RocketOptimization, the lower and upper boundaries for the design and target state variables are defined, as in Tables 4.3 and 4.4. To increase the efficiency of the simulation, the function Multistart (which creates different sets of initial guesses \( x_0 \) to be simulated in a row) is used. A diagram of the simulation is shown in Figure 5.1.
5.2 Numerical Methods

For the propulsion subsystem, a few numerical approximations were done by Klammer [13]. Linear interpolation is used on the N$_2$O saturation table and a bilinear interpolation for the CEA lookup table. As for the injector model, the average between the current and previous step is used to calculate the mass flow, to deal with numerical instabilities, modeling the damping of the mass flow that occurs in a real injector, where the chamber pressure cannot change abruptly.

To perform numerical integration of the differential equations, an explicit Euler method was used, for its simplicity and computation speed. The Euler discretization is shown in Equation 5.1.

$$x_{t+\Delta t} = x_t + \frac{dx_t}{dt} \Delta t$$ (5.1)

To find the temperature of the oxidizer tank and Mach number on the nozzle exit, a secant method for root finding is applied. Since this iterative solving is time-intensive, it is only applied when the error between the calculated and the reference value is greater than 1\%. The secant method is shown in Equation 5.2.

$$x_n = x_{n-1} + f(x_{n-1}) \frac{x_{n-1} - x_{n-2}}{f(x_{n-1}) - f(x_{n-2})}$$ (5.2)
Chapter 6

Model Validation

To ensure the coherence of the model to an actual hybrid rocket, a validation using experimental data was performed. Since the estimation of rocket mass and size is already based directly on the data of real hybrid rockets, the model validation is unnecessary. For the aerodynamic subsystem, taking into account that the calculation of the rocket center of gravity and center of pressure is done by applying the definitions of these geometrical properties, the validation of these models is not mandatory. However, the trajectory analysis needs to be tested since it applies a simplified one-dimensional model with numerical approximations, which can lead to an inaccurate model. The validation was performed by Pearson [57] against the data from 28 distinct suborbital rockets with an average error of 5.66%.

To validate the propulsion subsystem, experimental data using blown-down N\textsubscript{2}O tanks with paraffin-based fuels were considered. Three different hybrid rockets were considered: Deliverance II from the University of Toronto[31], launched on the 2017 IREC; Boundless from the University of Washington[20], launched on the 2018 IREC; and Phoenix 1A from the University of KwaZulu-Natal [35]. The rockets design parameters are shown in Table 6.1. The time-curves from the literature have been smoothed by Klammer [13]. The oscillations on the experimental data are related to combustion instabilities and external noises. In addition to the experimental data, a comparison between the theoretical model developed by Klammer [13] and the one presented in this work is done. The regression rate model was then validated against experimental data from Karabeyoglu et al. [41] by Klammer [13]. The results have shown an average error of 5.08% for the O/F ratio, 1.26% and 3% for the final port diameter fore and aft, respectively.

The measurement of the model average absolute error $e$ was normalized using the mean value of the analyzed variable, allowing an easier interpretation and analysis of the results. The model error is shown in Equation 6.1.

$$ e = \frac{\sum |X_i - X_{model,i}|}{X_i} \quad (6.1) $$
To validate the hybrid rocket model it is necessary to compare it to experimental data from actual hybrid rockets. However, some of the inputs necessary for the Propulsion subsystem, such as the correction factors and regression rate coefficients, are not available, since these types of variables are used to simulate the physical behavior of the system and therefore are not estimated by the designers.

For all three models, the correction factors for the discharge $\zeta_d$, combustion efficiency $\zeta_c$ and thrust $\zeta_t$ were based on commonly used values for initial design, as stated by Sutton and Biblar [6]. The expected values for $\zeta_d$, $\zeta_c$ and $\zeta_t$ are 1.07, 0.95 and 0.90, respectively. The ambient properties were set to the standard values, setting the temperature at $20^\circ C$ and the pressure at 1 atm.

The most difficult factor to determine is the injector discharge coefficient $C_d$. Since the oxidizer mass flow depends directly on $C_d$, as shown in Equation 3.12, it is important to properly define this constant to ensure that the model can describe the real behavior of the injector. The coefficient was normalized by using the liquid run out time of the $\text{N}_2\text{O}$ by Klammer [13] to estimate its value. The inputs implemented in each simulation are shown in Table 6.2.

### Table 6.1: Reference rockets information

<table>
<thead>
<tr>
<th>Deliverance II</th>
<th>Boundless</th>
<th>Phoenix 1A</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length (m)</td>
<td>2.95</td>
<td>4.32</td>
</tr>
<tr>
<td>Outer Diameter (m)</td>
<td>0.14</td>
<td>0.20</td>
</tr>
<tr>
<td>Lift Off Weight (kg)</td>
<td>39.7</td>
<td>80.7</td>
</tr>
<tr>
<td>Thrust (kN)</td>
<td>3.56</td>
<td>5.34</td>
</tr>
<tr>
<td>Total Impulse (N s)</td>
<td>18620</td>
<td>40960</td>
</tr>
<tr>
<td>Predicted Apogee (km)</td>
<td>7.10</td>
<td>8.17</td>
</tr>
</tbody>
</table>

### Table 6.2: Reference rockets inputs

<table>
<thead>
<tr>
<th></th>
<th>Deliverance II</th>
<th>Boundless</th>
<th>Phoenix 1A</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tank Volume (L)</td>
<td>12</td>
<td>25.9</td>
<td>43</td>
</tr>
<tr>
<td>Initial Oxidizer Mass (kg)</td>
<td>8.2</td>
<td>15.4</td>
<td>20.8</td>
</tr>
<tr>
<td>Initial Oxidizer Tank Pressure (MPa)</td>
<td>3.92</td>
<td>4.93</td>
<td>6.50</td>
</tr>
<tr>
<td>Feed System Pressure Drop (MPa)</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>Injector Orifice Diameter (mm)</td>
<td>1.50</td>
<td>3.05</td>
<td>2.000</td>
</tr>
<tr>
<td>Number of Injection Orifice</td>
<td>37</td>
<td>12</td>
<td>17</td>
</tr>
<tr>
<td>Fitted Discharge Coefficient ($C_d$)</td>
<td>0.41</td>
<td>0.30</td>
<td>0.45</td>
</tr>
<tr>
<td>Fuel Grain Length (m)</td>
<td>0.508</td>
<td>0.419</td>
<td>0.400</td>
</tr>
<tr>
<td>Fuel Grain Initial Port Diameter (mm)</td>
<td>87.4</td>
<td>55.0</td>
<td>50.0</td>
</tr>
<tr>
<td>Fuel Grain Outer Diameter (mm)</td>
<td>108</td>
<td>171</td>
<td>156</td>
</tr>
<tr>
<td>Fuel Density (kg/m$^3$)</td>
<td>930</td>
<td>964</td>
<td>848</td>
</tr>
<tr>
<td>Regression Rate Coefficient (mm/s)</td>
<td>0.155</td>
<td>0.155</td>
<td>0.155</td>
</tr>
<tr>
<td>Regression Rate Exponent (n)</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>Nozzle Throat Diameter (mm)</td>
<td>36.8</td>
<td>40.7</td>
<td>29.8</td>
</tr>
<tr>
<td>Nozzle Area Ratio ($A_{ratio}$)</td>
<td>4.96</td>
<td>5.00</td>
<td>5.99</td>
</tr>
</tbody>
</table>
6.1.1 Deliverance II

Deliverance II is a hybrid rocket designed at the University of Toronto that was launched at the 2017 edition of the Spaceport America Cup [31]. The rocket carries a drag measurement system as the payload and uses aluminum 6061-T6 as the oxidizer tank material, just as assumed in the mass calculation, in Subsection 3.3.2.

The mass of the oxidizer used during the tests was not given in the report, and so the oxidizer mass used in the flight motor design was an approximation. In the engine simulation performed by the University of Toronto team, the values for the regression rate coefficient and exponent were the same as the expected in the propulsion model. Since the model from the team shows a similar result to the experiment, these constants were maintained.

Although the composition of the fuel was 10\% tar, this additive was not considered in the fuel density or the CEA combustion code, since its composition is not well-defined. The fuel port diameter was defined from scaled drawings since it was not specified.

6.1.2 Boundless

Boundless is a hybrid rocket developed by the Society for Advanced Rocket Propulsion team at the University of Washington that was launched at the 2018 Spaceport America Cup [20]. It carries an exploratory rover vehicle that collects the surroundings data as a payload, using aluminum 6061-T6 as the material for both the combustion chamber and oxidizer tank.

The paraffin wax used by the Advanced Rocket Propulsion team had 4\% stearic acid and 5\% aluminum powder, that can increase the regression rate, as addictive. Since the stearic acid is not defined in the CEA thermodynamic library it was not included in the combustion code. As for the aluminum powder, since no regression rate data is given in the report and the percentage of aluminum powder is less than 10\%, the standard values for regression rate coefficients of the paraffin wax-N\textsubscript{2}O were used. However, this composition was taken into account for the density of the fuel.

6.1.3 Phoenix 1A

Phoenix 1A was the model with better-documented parameters [35]. The oxidizer tank of Phoenix 1A was pressurized with helium, to guarantee that the N\textsubscript{2}O pressure is slightly above saturation. The injector information was taken from Genevieve [73], which was published before the tests for the Phoenix 1A’s motor.

Since the Phoenix 1A used a conical nozzle, which does not have as much propellant flow as a bell-shaped nozzle, used on Deliverance II and Boundless, a correction factor of 0.983 was applied to the thrust equation.
6.2 Simulation and Comparison

To evaluate the model coherence with the experimental data, four different comparisons were done: the oxidizer tank pressure to evaluate the tank model; the combustion chamber to evaluate the chamber model; the thrust since it is a key factor to measure the rocket performance; and the tank temperature to compare the models with and without the heat transfer being considered. For the Phoenix 1A, the oxidizer tank pressure was not compared since the data is not available.

6.2.1 Deliverance II

The Deliverance II model shows good coherence with the experimental data. The oxidizer tank pressure fits very closely with the experimental data. The oscillation on the first two seconds can be related to the sudden start of the oxidizer injection, which occurs faster than the vaporization of the N\textsubscript{2}O, causing the pressure to drop. With this decrease in the pressure, the mass flow slows down until the vaporization rates catch up, making the behavior of the tank pressure to adjust to the theoretical model. The abrupt change in the theoretical curve slope occurring in the 6 seconds mark is related to the simplification of immediate change from liquid N\textsubscript{2}O to gaseous N\textsubscript{2}O when the tank runs out of liquid oxidizer, a phenomenon that occurs more smoothly on the physical system.

As in the tank pressure, the combustion chamber pressure also shows a good agreement with the experimental data. The oscillations present in the experiment pressure can be related to imperfections in the fuel grain, such as air bubbles or slight variations on the shape. This validation is a key factor for the rocket performance and safety reasons since a high combustion chamber pressure can cause a back-flow to the oxidizer tank, causing possibly detonation if the N\textsubscript{2}O overheats [12]. The oxidizer tank and combustion chamber pressure are shown in Figure 6.1.

![Figure 6.1: Deliverance II - theoretical and real pressure data comparison](image)

The thrust curve from the propulsion simulation shows a lower value in comparison to the experimental data. This can be related to the conservative assumption from the oxidizer tank fill level, shown in Section 4.1; from the tar additive from the fuel composition, that was not considered but can enhance
the motor performance; or some other consideration is shown in Subsection 6.1.1. The previously mentioned oxidizer tank oscillation at the beginning of the process causes the thrust to be higher until the equilibrium of the oxidizer mass flow and N₂O vaporization is achieved. The transition from the liquid N₂O to gaseous N₂O has a direct influence at the thrust, being a smoother transition on the experimental data at the 5 to 6 seconds mark and more abrupt in the theoretical model.

In the previously analyzed curves, the difference between the older model “MuleSim3” by Klammer [13], without the heat transfer, and the “Propulsion” model, with heat transfer, is not noticeable. This shows that the heat transfer occurring during the process is not high enough to influence the system. By comparing the oxidizer tank temperature, it is clear that the heat transfer only affects the system after the oxidizer tank runs out of liquid N₂O, after the 6 seconds mark, since before this event the difference between the tank temperature and ambient temperature is not high enough for the heat transfer to be noticed. Since the whole process has a short duration and the difference in the tank temperature between models is almost negligible (1.15%), the results of both models look the same. The thrust curves and oxidizer tank temperature are shown in Figure 6.2.

6.2.2 Boundless

Boundless shows a higher tank pressure than the experimental data. These errors can be expected since the saturated equilibrium model for the N₂O is a simplified model. However, the use of a more sophisticated model would result in a higher computational expense, which would not be suitable for conceptual design. The initial oscillation on the first 2 seconds in the tank pressure and the pressure drop can also be seen in this vehicle.

The combustion chamber pressure is underestimated, since the overestimation in the tank pressure causes an increased oxidizer mass flow, and consequent fuel mass flow, resulting in a chamber pressure lower than the experimental data. The oxidizer tank pressure and combustion chamber pressure plots are shown in Figure 6.3.

The thrust curve from Boundless is underestimated by the model, related to the combustion chamber
underestimation. As in Deliverance II, the difference between the theoretical models can not be detected, since the amount of heat transferred from the oxidizer tank is negligible, as can be noticed by the similar values of tank temperature. The thrust curve and tank temperature model for Boundless are shown in Figure 6.4.

### 6.2.3 Phoenix 1A

Phoenix 1A presents a sudden increase in the combustion chamber pressure between the 2 and 3 seconds mark. This variation could be related to the combustion instabilities presented for Deliverance II.

In the thrust curve, the same oscillations and sudden increase can be noticed. However, a good agreement for the thrust model can also be noticed. The combustion chamber and thrust for the Phoenix 1A are shown in Figure 6.5. The influence of heat transfer is also negligible, as shown in Figure 6.6.
6.2.4 Results

The model for the Deliverance II and Phoenix 1A shows an acceptable agreement to the experimental data, with errors lower than 15%. The highest error region for both models is influenced by the model oscillation at the first 2 seconds and the smooth transition from the liquid N₂O form the gaseous N₂O, not covered by the saturated equilibrium model.

The Boundless model presents a larger error than the other hybrid rockets simulated. The oxidizer tank pressure simplified model is incapable of simulating the complex behavior of the real system. However, to solve this problem, a significantly higher computational power is required, which is out of the range of the scope of the project. Since the errors in the combustion chamber and thrust are propagated from the oxidizer tank, with an error of 15% and taking into account that the model is used for conceptual design, the model was considered acceptable. The errors of each hybrid rocket model are shown in Table 6.3.
<table>
<thead>
<tr>
<th></th>
<th>Deliverance II</th>
<th>Boundless</th>
<th>Phoenix 1A</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tank Pressure error (%)</td>
<td>1.55</td>
<td>15</td>
<td>-</td>
</tr>
<tr>
<td>Combustion Chamber Pressure error (%)</td>
<td>10.7</td>
<td>25.7</td>
<td>12.3</td>
</tr>
<tr>
<td>Thrust (%)</td>
<td>7.55</td>
<td>32.80</td>
<td>12.20</td>
</tr>
</tbody>
</table>

Table 6.3: Model error
Chapter 7

Results

After a series of simulations, the optimization software generated results. To select the optimal solution, it is necessary to take into account the Spaceport America Cup rules, since the rocket apogee should be as close as possible to the 10000 ft mark [60]. Using the constraint described in Section 4.4 the design space was limited to the responses that meet the competition’s requirements.

As the simulations were run, some issues with the formulation were revealed. Some issues with complex numbers on the Propulsion were happening due to the Equation 3.12, where a negative square root happens when \( p_{CC} + p_{feed} > p_{OT} \). This would generate a false answer with a low mass and high specific impulse, since the low oxidizer mass flow was causing an increased burn time, allowing the system to avoid the maximum acceleration constraint and being lighter at the same time. Physically, this event would generate an oxidizer backflow, possibly causing a system failure. To ensure that this would not happen during the simulations, an \( if \) statement was added to the code, ensuring that the oxidizer mass flow is always a real number.

The results for the specific impulse maximization, gross weight minimization, and multi-objective function are shown in this chapter.

7.1 Maximization of the Specific Impulse

After over 50000 points simulated, the optimal design to maximize the specific impulse leads to a 191.51 s response. The lift-off mass of 25.12 kg and length of 3.37 m are expected, since the generated impulse is directly related to the propulsion subsystem, leading to a larger oxidizer tank and combustion chamber, as well as the extra fuel and oxidizer mass. Comparing it to Callisto’s 21.5 kg to reach a 2.884 km apogee the design’s weight seems to be in accordance with a physical hybrid rocket model.

Analyzing the Equation 4.1 the specific impulse is directly proportional to the total impulse and inversely proportional to the propellant mass. Since this objective function is not focused on the rocket’s mass, the optimization function prioritizes the maximization of the total impulse, resulting in a final solution with a higher weight than the other rocket designs, further analyzed. Since the vehicle mass is directly related to its costs and some of the teams projecting a sounding rocket have a limited amount of
expenses, focusing only on the maximum specific impulse is not ideal.

Another parameter that should be analyzed is the model’s static margin of 1.61. As stated in Section 4.4, the ideal range of values for the static margin is between 1 and 2, meaning that this design presents an optimized stability.

7.2 Minimization of the Rocket’s Weight

With the simulation of over 30000 new points in the design space the optimal design to minimize the lift-off weight lead to an 18.49 kg vehicle. By comparing the lift-off mass and length of 3.22 m to the maximized specific impulse response, it is clear that this configuration will present a lower cost to produce since the amount of fuel, oxidizer and structural material necessary are also lower. However, since the specific impulse is the main efficiency parameter for the propulsion system, the optimization of this variable is also of the designer’s interest.

The static margin of 2.74 means that the rocket is overstable. As stated in Section 3.4 an overstable design is still considered feasible, however it tends to present more stability problems in the presence of wind forces. For this reason, focusing only on the minimization of the rocket’s lift-off mass is not the optimal approach.

7.3 Multi-objective optimization

The optimization of the rocket specific impulse and mass are both important for an efficient design. However the simple maximization of the propulsion system efficiency and minimization of the rocket gross mass have negative effects, as previously discussed in Sections 7.1 and 7.2. To find the most effective rocket design, a multi-objective function is used to combine the benefits of each function.

To define a multi-objective function, a weighted sum is applied to the maximization of specific impulse and minimization of the lift-off weight as shown in Equation 7.1. Each of the weights varies between 0 and 1 and the sum of them is equal to 1.

\[
\text{obj} = w_1 \text{obj}_1 + w_2 \text{obj}_2
\]

Since the specific impulse and rocket mass have a different order of magnitude, being \(10^2\) for the specific impulse and \(10^1\) for the lift-off mass, a parameterization is necessary. Using the values of the optimal design proposed by Klammer [13], the objective functions were defined as shown in Equations 7.2 and 7.3.

\[
\text{obj}_1 = -\frac{I_{sp}}{201.8}
\]

\[
\text{obj}_2 = \frac{M_{\text{rocket}}}{34.8}
\]

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By varying the weights and running the optimization program, new intermediate points are generated. Combining these new points with the two old models for the maximum specific impulse and minimum lift-off mass, a trend curve of the optimal solutions, known as the Pareto frontier, is plotted using the lift-off mass in the x-axis and the specific impulse in the y-axis. The Pareto Optimality is used when the objective functions are conflicting, meaning that by lowering one the other will increase, which is the case with the specific impulse and lift-off weight. This strategy allows the project designer to select the optimal solution according to the project’s priorities [74].

The initial approach was to vary the weights with a step size of 0.1. However, the results for \( w_1 \) equal to 0.1, 0.2 and 0.3 leads to the same result as the minimized weight. This behavior is related to the fact that the minimization of the lift-off mass causes the propellant mass to be reduced and consequent increase in the specific impulse, meaning that a lower rocket’s gross mass can increase the specific impulse, which can be seen in the specific impulse of the minimal mass response 187.18 s being not much lower than 191.51 s of the maximized response. However, this behavior highlights the importance of the usage of Pareto Optimality, since it will guarantee that the optimal model selected has minimal mass possible.

By varying the weight \( w_1 \) with values near 1 and simulating nearly 25000 points per weight new optimal designs were obtained. It is important to notice that some of the responses found presented a similar specific impulse to the maximized response while presenting a lower mass. This behavior can be related to the specific impulse equation (Equation 4.1) since in the multiobjective case the optimization software prioritizes the reduction of the propellant, once it will benefit both objective functions, in contrast with the maximum specific impulse situation where the software used the total impulse increase as the solution.

The Pareto frontier is shown in Figure 7.1. The plot presents a higher slope for the lower masses, from 18.3 to 19 kg, presenting a significant increase of the specific impulse as the lift-off mass grows. For designs with a mass higher than 19 kg, the curve slope begins to decrease as the lift-off mass increases, reaching the maximum specific impulse of 191.5 s. If the designer prioritizes the specific impulse maximization, the response with a mass of 20.8 kg and a specific impulse of 191.36 s or the next with a lift-off mass of 23.8 kg and specific impulse of 191.48 s are the optimal solution, since the further increase of the rocket’s weight does not increase the specific impulse. However, for a university team, a design with a lower lift-off mass and cost is more beneficial. For this reason, the selected design was the rocket with a mass of 20.01 kg and a specific impulse of 191.04 s. The other designs can be consulted in Appendix A.

The design variables of the maximized specific impulse, minimized lift-off mass and final configurations are shown in Table 7.1. Comparing the oxidizer tank dimensions of the responses, it is possible to notice that the minimum mass and final design presents a smaller tank volume than the maximized specific impulse design since this variable is directly related to the oxidizer mass consumed by the motor, being minimized if the lift-off mass reduction is taken into account. However, it is important to notice that the optimal design is achieved by increasing the oxidizer tank’s length since the final design presents the smallest tank’s radius between the three responses. The increase of the oxidizer tank’s length con-
tributes to lower the static margin, reducing the overstability problems presented by the minimized lift-off mass response.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Max Isp</th>
<th>Min Mass</th>
<th>Final Design</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oxidizer Tank Volume</td>
<td>$7.40 \times 10^{-3} m^3$</td>
<td>$5.60 \times 10^{-3} m^3$</td>
<td>$5.92 \times 10^{-3} m^3$</td>
</tr>
<tr>
<td>Effective Injection Area</td>
<td>$1.98 \times 10^{-5} m^3$</td>
<td>$1.09 \times 10^{-5} m^3$</td>
<td>$1.34 \times 10^{-5} m^3$</td>
</tr>
<tr>
<td>Fuel Grain Length</td>
<td>0.237 m</td>
<td>0.200 m</td>
<td>0.205 m</td>
</tr>
<tr>
<td>Fuel Grain Initial Port Diameter</td>
<td>$7.11 \times 10^{-2} m$</td>
<td>$5.03 \times 10^{-2} m$</td>
<td>$5.70 \times 10^{-2} m$</td>
</tr>
<tr>
<td>Nozzle Throat Diameter</td>
<td>$2.46 \times 10^{-2} m$</td>
<td>$2.12 \times 10^{-2} m$</td>
<td>$2.04 \times 10^{-2} m$</td>
</tr>
<tr>
<td>Nozzle Area Ratio</td>
<td>5.71</td>
<td>6.42</td>
<td>5.25</td>
</tr>
<tr>
<td>Combustion Chamber Length</td>
<td>0.671 m</td>
<td>0.500 m</td>
<td>0.500 m</td>
</tr>
<tr>
<td>Fin Span</td>
<td>$4.79 \times 10^{-2} m$</td>
<td>$4.66 \times 10^{-2} m$</td>
<td>$4.43 \times 10^{-2} m$</td>
</tr>
<tr>
<td>Oxidizer Tank Radius</td>
<td>$4.89 \times 10^{-2} m$</td>
<td>$4.10 \times 10^{-2} m$</td>
<td>$3.82 \times 10^{-2} m$</td>
</tr>
<tr>
<td>Fin Root chord Dimension</td>
<td>$3.00 \times 10^{-1} m$</td>
<td>$1.78 \times 10^{-1} m$</td>
<td>$2.06 \times 10^{-1} m$</td>
</tr>
<tr>
<td>Fin Tip chord Dimension</td>
<td>$6.67 \times 10^{-2} m$</td>
<td>$1.58 \times 10^{-1} m$</td>
<td>$1.85 \times 10^{-1} m$</td>
</tr>
<tr>
<td>Additional Mass at Recovery bay</td>
<td>0.99 kg</td>
<td>0 kg</td>
<td>0 kg</td>
</tr>
<tr>
<td>Additional Mass at Nose cone</td>
<td>2.02 kg</td>
<td>0 kg</td>
<td>0 kg</td>
</tr>
</tbody>
</table>

Table 7.1: Design Variables for the Different Designs

The final design presents the smallest fin span between the three configurations. This shows that the optimal design is directly related to the reduction of the cross-section area and drag force, as expected. It is important to notice that the final design achieved a stable configuration without the addition of masses on the recovery bay or the nose cone, meaning that the vehicle will not need the extra non-functional weights.

The state variables of the maximized specific impulse, minimized lift-off mass and final configurations are shown in Table 7.2. As stated before, the optimal design is associated with the minimization of the rocket’s cross-section, which is highlighted by its length being higher than the other two configurations, attending the volume constraints of the oxidizer tank and combustion chamber by increasing their lengths.
instead of their diameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Max isp</th>
<th>Min Mass</th>
<th>Final Design</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rocket Mass</td>
<td>25.12 kg</td>
<td>18.49 kg</td>
<td>20.01 kg</td>
</tr>
<tr>
<td>External structure Length</td>
<td>2.665 m</td>
<td>2.51 m</td>
<td>2.896 m</td>
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<tr>
<td>Rocket External Diameter</td>
<td>0.104 m</td>
<td>0.088 m</td>
<td>0.093 m</td>
</tr>
<tr>
<td>Combustion Chamber Mass</td>
<td>1.71 kg</td>
<td>1.06 kg</td>
<td>1.12 kg</td>
</tr>
<tr>
<td>Oxidizer Tank Mass</td>
<td>4.60 kg</td>
<td>4.11 kg</td>
<td>4.63 kg</td>
</tr>
<tr>
<td>External structure Mass</td>
<td>4.05 kg</td>
<td>3.21 kg</td>
<td>3.92 kg</td>
</tr>
<tr>
<td>Oxidizer Tank Length</td>
<td>0.98 m</td>
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<td>1.29 m</td>
</tr>
<tr>
<td>Recovery Bay Length</td>
<td>3.92 10^{-1} m</td>
<td>3.54 10^{-1} m</td>
<td>4.47 10^{-1} m</td>
</tr>
<tr>
<td>Avionics Bay Length</td>
<td>1.91 10^{-2} m</td>
<td>1.66 10^{-1} m</td>
<td>2.30 10^{-2} m</td>
</tr>
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<td>Avionics Bay Mass</td>
<td>0.62 kg</td>
<td>0.50 kg</td>
<td>0.50 kg</td>
</tr>
<tr>
<td>Recovery Bay Mass</td>
<td>1.48 kg</td>
<td>1.20 kg</td>
<td>1.20 kg</td>
</tr>
<tr>
<td>Fins Mass</td>
<td>0.11 kg</td>
<td>0.10 kg</td>
<td>0.11 kg</td>
</tr>
<tr>
<td>Nozzle Mass</td>
<td>0.37 kg</td>
<td>0.30 kg</td>
<td>0.30 kg</td>
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<tr>
<td>Nosecone Mass</td>
<td>0.65 kg</td>
<td>0.54 kg</td>
<td>0.58 kg</td>
</tr>
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<td>Oxidizer Tank Distance to Tip</td>
<td>1.58 m</td>
<td>1.52 m</td>
<td>1.68 m</td>
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<tr>
<td>Combustion Chamber Internal Radius</td>
<td>4.58 10^{-2} m</td>
<td>3.78 10^{-2} m</td>
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<td>Fuel Mass</td>
<td>0.573 kg</td>
<td>0.465 kg</td>
<td>0.481 kg</td>
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<td>3.96 kg</td>
<td>3.00 kg</td>
<td>3.17 kg</td>
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<tr>
<td>Combustion Chamber External Radius</td>
<td>4.89 10^{-2} m</td>
<td>4.10 10^{-2} m</td>
<td>4.34 10^{-2} m</td>
</tr>
</tbody>
</table>

Table 7.2: State Variables for the Different Designs

The configurations outputs are shown in Table 7.3. The final design’s static margin is 1.23 that is lower than the other two designs, while still being in the ideal range. Since the fuel and oxidizer located near the outflow are burned out during the flight, the center of gravity of the rocket tends to move in the direction of the nose cone tip. With the center of pressure maintained, the static margin of the system will increase, ensuring its stability.

To analyze the stress on the oxidizer tank and combustion chamber, the thin-walled pressure vessel theory is applied. The circumferential stress \( \sigma \) is calculated as a function of the chamber pressure \( P \), thickness \( t \) and radius \( r \) as shown in Equation 7.4 [75].

\[
\sigma = \frac{Pr}{t}
\]  

(7.4)

For the maximum specific impulse configuration, the maximum stress in the tank and chamber are 39.5 and 61.80 MPa respectively. Using the yield stress of the Aluminum 6061-T6 of 276 MPa [76] as the reference value, it is possible to estimate a safety factor of 6.99 and 4.47 for the components. In the minimum lift-off mass design, the stresses in the oxidizer tank and combustion chamber are 31.67 and 67.88 MPa, respectively. This result leads to safety factors of 8.71 for the tank and 4.07 for the chamber. For the optimal design, the stresses in the oxidizer tank and combustion chamber are
Table 7.3: Outputs for the Different Designs

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Max Isp</th>
<th>Min Mass</th>
<th>Final Design</th>
</tr>
</thead>
<tbody>
<tr>
<td>Specific Impulse</td>
<td>191.51 s</td>
<td>187.18 s</td>
<td>191.04 s</td>
</tr>
<tr>
<td>Rocket Length</td>
<td>3.37 m</td>
<td>3.22 m</td>
<td>3.6 m</td>
</tr>
<tr>
<td>Maximum Thrust</td>
<td>2743.8 N</td>
<td>1527.9 N</td>
<td>1868.9 N</td>
</tr>
<tr>
<td>Maximum Chamber Pressure</td>
<td>3.92 MPa</td>
<td>3.01 MPa</td>
<td>3.87 MPa</td>
</tr>
<tr>
<td>Maximum Tank Pressure</td>
<td>5 MPa</td>
<td>5 MPa</td>
<td>5 MPa</td>
</tr>
<tr>
<td>Static Margin</td>
<td>1.61</td>
<td>2.74</td>
<td>1.23</td>
</tr>
<tr>
<td>Apogee</td>
<td>3049 m</td>
<td>3050 m</td>
<td>3050 m</td>
</tr>
<tr>
<td>Velocity off the rail</td>
<td>37.77 m/s</td>
<td>34.5 m/s</td>
<td>34.73 m/s</td>
</tr>
<tr>
<td>Burn Time</td>
<td>3.72 s</td>
<td>4.41 s</td>
<td>4.35 s</td>
</tr>
<tr>
<td>Impulse</td>
<td>8135.4 N s</td>
<td>6082 N s</td>
<td>6539.5 N s</td>
</tr>
</tbody>
</table>

29.33 MPa and 53.92 MPa, respectively. The resulting safety factor of 9.41 for the oxidizer tank and 5.12 for the combustion chamber. These results show that the final design presents the highest safety factor between the three configurations. It is important to highlight that a structural analysis using finite elements is recommended to ensure that the structures will not fail.

The pressure in the oxidizer tank and combustion chamber are shown in Figure 7.2. It is possible to notice that the final configuration has a decrease in the oxidizer tank's pressure similar to the minimized mass response. However, the values of the optimal response for the combustion chamber pressure are higher than the minimum mass response and similar to the maximum specific impulse configuration. This behavior is important to regulate the mass flow rate, allowing the design to have a slower fuel consumption and higher burn time.

![Oxidizer Tank Pressure Over Time for the Different Designs](image1)

![Combustion Chamber Pressure Over Time for the Different Designs](image2)

Figure 7.2: Vessel pressure for the Different Designs

The thrust force and the rocket's acceleration are shown in Figure 7.3. The final design's thrust curve
presents a similar behavior to the minimized lift-off mass configuration, presenting almost the same burn time while maintaining a higher thrust. The higher burn time is important to optimize the rocket’s propulsion system since it will directly influence the total generated impulse by maintaining an elevated thrust force for a longer period.

![Thrust vs Time](image)

**Figure 7.3: Thrust Force and Acceleration for the Different Designs**

The rocket’s altitude and velocity are shown in Figure 7.4. It is noticeable that the three designs present a very similar trajectory. The maximum specific impulse configuration reaches the apogee earlier than the other two designs, which is evident in the velocity graphic, when the rockets reach their maximum velocity.

![Altitude vs Time](image)

**Figure 7.4: Altitude and Velocity for the Different Designs**

Using the results from the simulation a scheme for each rocket was designed, as shown in Figure 7.5. The rockets’ length to diameter ratio were calculated, leading to a ratio of 25.63 for the maximum specific impulse design, 36.63 for the minimum mass design and 38.8 for the final design. This behavior is related to the cross-section area reduction stated previously.
Figure 7.5: Rocket model and cross section for each configuration
Chapter 8

Conclusions

The model presented in this work reached its goals and developed a multi-objective optimization of a hybrid rocket model. The conclusions taken from the project are presented in this chapter.

8.1 Achievements

This work developed a theoretical model for the conceptual design of a hybrid rocket. The model covered the main disciplines involving a sounding rocket manufacture, incorporating the major components of each system. In the Propulsion area the theoretical equations of the oxidizer tank with heat transfer, injector, combustion chamber containing a single fuel port cylindrical fuel grain and nozzle were implemented. For the Aerodynamics system, an analysis of the rocket trajectory and stability is performed. A mass and sizing estimation is performed by the usage of empirical models developed with experimental data and theoretical models.

These models were implemented in a Matlab program and validated against experimental data, with average errors of 17.5% for the thrust estimation, 8.275% for the oxidizer tank pressure and 16.23% for the combustion chamber pressure.

Multidisciplinary Design Optimization was conducted selecting the minimization of the lift-off weight and maximization of the specific impulse as the objective functions. The selected design variables to be optimized were the oxidizer tank volume, effective injection area, fuel grain length, fuel grain initial port diameter, nozzle throat diameter and ratio, combustion chamber length, oxidizer tank radius, fin span, root and tip chord, and the additional masses on the recovery bay and nosecone. To ensure the coherence between each discipline the state variables selected were the rocket mass and external diameter, external structure length and mass, combustion chamber mass, internal and external radius, the oxidizer tank mass and length, the recovery bay mass and length, the avionics section mass and length, the mass of the fins, nozzle, fuel and oxidizer, and the distance of the oxidizer tank to the nosecone tip. Several constraints were imposed on the system to simulate the physical restrictions of the system and that the target apogee is reached. An IDF architecture was selected to optimize the system combined with the multistart Matlab function to increase the number of initial points simulated to
search for the optimal solution.

The maximized specific impulse design presented a 25.12 kg lift-off mass, 3.37 m length, 10.4 cm diameter, 191.51 s specific impulse and reached the target apogee of 3049m. The design also presented a static margin of 1.61. The minimized lift-off mass design presented an 18.49 kg lift-off mass, 3.22 m length, 8.8 cm diameter, 187.18 s specific impulse and reached the target apogee of 3050m. The model presented a static margin of 2.74, being considered overstable.

Multiobjective optimization was then performed to combine the benefits of the mass minimization and specific impulse maximization. A weighted objective function was created and several simulations were performed. The results were plotted to form a Pareto frontier to select the final design. The Pareto optimality has shown to be essential for the optimization software to reach the final solution, since the specific impulse maximization is dependent on the total impulse maximization and propellant minimization simultaneously, being necessary to impose a multiobjective function to properly select the optimal solution. The final design presented a 20.01 kg lift-off mass, 3.6 m length, 9.3 cm diameter, 191.04 s specific impulse and reached the target apogee of 3050m. The design presented a static margin of 1.23, being in the ideal range for this variable.

8.2 Future Work

The model created in this work is a tool for the conceptual design of hybrid rockets. For this reason, several model simplifications were applied to reduce computational efforts. However, to develop a more detailed design using this computational code as a base would be beneficial to the users.

For future development, some improvements can be made to this model, such as:

- **Guide User Interface (GUI):** create a GUI to allow any user to run the code properly;
- **Recovery:** implement a recovery discipline, creating a more realistic model and allowing for a more precise mass and length estimation;
- **Inputs of the system:** implement a second path to the program that allows the input of all the systems information, such as the dimensions and weights of the nose cone, nozzle, recovery and avionics system, improving the simulation accuracy;
- **Material selection:** allow the user to select the materials of the rocket’s components, such as the external tube, nose cone, nozzle, oxidizer tank, combustion chamber, fuel and oxidizer;
- **Sensitivity analysis:** perform a sensitivity analysis, allowing identification of the system inputs contribution to uncertainty in performance;
- **Aerodynamics:** the performance of a CFD simulation, allowing the designer to estimate more accurately the vehicle drag coefficient. With these simulations, a proper selection of the nose cone and fins airfoils shape can be performed. A more accurate estimation of the vehicle’s drag allows the trajectory simulation to be more precise;
• Propulsion: the improvement of the oxidizer tank model could be beneficial for the system, however, it is important to take into account the computational efficiency of the used model. With more experimental data on the propellants, a heat transfer system for the combustion chamber and nozzle could be implemented into the system;

• Structural: a structural simulation with finite elements would certificate that the model will not present structural failure, especially on high-pressure vessels as the oxidizer tank and combustion chamber as well as in zones of impact during land, as the nose cone, fins and external body;

• Financial: for university teams with a limited amount of spending, the estimation of the costs is essential to the project development. With the mass estimation presented in the model, the price can be easily adjusted according to each group’s needs;
Bibliography


# Appendix A

## Pareto Frontier Points

### Table A.1: Pareto Frontier Points

<table>
<thead>
<tr>
<th>Parameter</th>
<th>x1</th>
<th>x2</th>
<th>x3</th>
<th>x4</th>
<th>x5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oxidizer Tank Volume (m³)</td>
<td>0.00560344</td>
<td>0.00563421</td>
<td>0.00547676</td>
<td>0.00566564</td>
<td>0.00592034</td>
</tr>
<tr>
<td>Effective Injection Area (m²)</td>
<td>1.09E-05</td>
<td>1.47E-05</td>
<td>1.29E-05</td>
<td>1.50E-05</td>
<td>1.34E-05</td>
</tr>
<tr>
<td>Fuel Grain Length (m)</td>
<td>0.2</td>
<td>0.2</td>
<td>0.21146074</td>
<td>0.2</td>
<td>0.20486202</td>
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<td>0.05029159</td>
<td>0.05432862</td>
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<td>0.05703911</td>
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<td>0.0211949</td>
<td>0.02313415</td>
<td>0.02077791</td>
<td>0.02160312</td>
<td>0.0204553</td>
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<td>18.495678</td>
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<td>20.0324951</td>
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<td>External structure Mass (kg)</td>
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<td>0.5</td>
<td>0.5</td>
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<tr>
<td>Fin Span (m)</td>
<td>0.04661477</td>
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### Table A.2: Pareto Frontier points

<table>
<thead>
<tr>
<th>Parameter</th>
<th>x1</th>
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<th>x3</th>
<th>x4</th>
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<td>Oxidizer Tank Radius (m)</td>
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<td>0.039254699</td>
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<td>0.038184754</td>
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<td>Fin Root Chord Dimension (m)</td>
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<td>0.20645776</td>
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<td>Additional Mass at Recovery bay (kg)</td>
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Table A.3: Pareto Frontier points

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<tr>
<td>Fuel Grain Initial Port Diameter (m)</td>
<td>0.05853213</td>
<td>0.06595102</td>
<td>0.06591665</td>
<td>0.06718644</td>
<td>0.0711123</td>
</tr>
<tr>
<td>Nozzle Throat Diameter (m)</td>
<td>0.02077777</td>
<td>0.02394311</td>
<td>0.02148134</td>
<td>0.02128517</td>
<td>0.02456378</td>
</tr>
<tr>
<td>Nozzle Area Ratio</td>
<td>6.50820842</td>
<td>5.97382864</td>
<td>6.371156</td>
<td>5.96654686</td>
<td>5.70624682</td>
</tr>
<tr>
<td>Rocket Mass (kg)</td>
<td>20.79999991</td>
<td>23.8</td>
<td>24.5067296</td>
<td>24.8020004</td>
<td>25.1241382</td>
</tr>
<tr>
<td>External structure Length (m)</td>
<td>2.84888014</td>
<td>2.4320179</td>
<td>3.56569636</td>
<td>3.50135742</td>
<td>2.66541475</td>
</tr>
<tr>
<td>Rocket External Diameter (m)</td>
<td>0.09406635</td>
<td>0.09880555</td>
<td>0.10249873</td>
<td>0.10492308</td>
<td>0.10384548</td>
</tr>
<tr>
<td>Combustion Chamber Mass (kg)</td>
<td>1.15392253</td>
<td>1.23131436</td>
<td>1.2590512</td>
<td>1.28734703</td>
<td>1.70948592</td>
</tr>
<tr>
<td>Oxidizer Tank Mass (kg)</td>
<td>4.65002436</td>
<td>4.44325014</td>
<td>5.88529535</td>
<td>6.03843209</td>
<td>4.59817783</td>
</tr>
<tr>
<td>External structure Mass (kg)</td>
<td>3.91218051</td>
<td>3.51356944</td>
<td>5.34998403</td>
<td>5.3614595</td>
<td>4.0533346</td>
</tr>
<tr>
<td>Oxidizer Tank Length (m)</td>
<td>1.25870607</td>
<td>1.00574617</td>
<td>1.68824974</td>
<td>1.65174638</td>
<td>0.96373126</td>
</tr>
<tr>
<td>Combustion Chamber Length (m)</td>
<td>0.50505072</td>
<td>0.51087631</td>
<td>0.50171013</td>
<td>0.5</td>
<td>0.67151501</td>
</tr>
<tr>
<td>Fin Span (m)</td>
<td>0.04361921</td>
<td>0.04080353</td>
<td>0.05284838</td>
<td>0.06379901</td>
<td>0.04786897</td>
</tr>
</tbody>
</table>

Table A.4: Pareto Frontier points

<table>
<thead>
<tr>
<th>Parameter</th>
<th>x6</th>
<th>x7</th>
<th>x8</th>
<th>x9</th>
<th>x10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oxidizer Tank Radius (m)</td>
<td>0.039294057</td>
<td>0.040402774</td>
<td>0.037248202</td>
<td>0.03891533</td>
<td>0.048921319</td>
</tr>
<tr>
<td>Fin Root Chord Dimension (m)</td>
<td>0.245840334</td>
<td>0.207553969</td>
<td>0.294274971</td>
<td>0.2</td>
<td>0.29805554</td>
</tr>
<tr>
<td>Fin Tip Chord Dimension (m)</td>
<td>0.173855051</td>
<td>0.18652452</td>
<td>0.08363777</td>
<td>0.01</td>
<td>0.066740018</td>
</tr>
<tr>
<td>Recovery Bay Length (m)</td>
<td>0.435774288</td>
<td>0.335560606</td>
<td>0.608096007</td>
<td>0.59262987</td>
<td>0.39169603</td>
</tr>
<tr>
<td>Avionics Bay Length (m)</td>
<td>0.221892062</td>
<td>0.152834814</td>
<td>0.340639579</td>
<td>0.329981214</td>
<td>0.191499284</td>
</tr>
<tr>
<td>Avionics Bay Mass (kg)</td>
<td>0.549065423</td>
<td>0.5000813216</td>
<td>0.500567843</td>
<td>0.627845823</td>
<td>0.61652361</td>
</tr>
<tr>
<td>Recovery Bay Mass (kg)</td>
<td>1.317757026</td>
<td>1.201951712</td>
<td>1.201362822</td>
<td>1.50829975</td>
<td>1.479008065</td>
</tr>
<tr>
<td>Fins Mass (kg)</td>
<td>0.117167169</td>
<td>0.102915399</td>
<td>0.112500233</td>
<td>0.126577228</td>
<td>0.11229473</td>
</tr>
<tr>
<td>Nozzle Mass (kg)</td>
<td>0.329439257</td>
<td>0.300487929</td>
<td>0.300340706</td>
<td>0.376707494</td>
<td>0.369752016</td>
</tr>
<tr>
<td>Nosecone Mass (kg)</td>
<td>0.584231046</td>
<td>0.615853065</td>
<td>0.640515323</td>
<td>0.656714371</td>
<td>0.649513102</td>
</tr>
<tr>
<td>Oxidizer Tank Distance to Tip (m)</td>
<td>1.65766635</td>
<td>1.488395419</td>
<td>1.948736485</td>
<td>1.92261041</td>
<td>1.583168451</td>
</tr>
<tr>
<td>Combustion Chamber Internal Radius (m)</td>
<td>0.040863176</td>
<td>0.043232774</td>
<td>0.045079364</td>
<td>0.046291538</td>
<td>0.04575274</td>
</tr>
<tr>
<td>Fuel Mass (kg)</td>
<td>0.477422014</td>
<td>0.518233843</td>
<td>0.575542436</td>
<td>0.597020255</td>
<td>0.572666993</td>
</tr>
<tr>
<td>Oxidizer Mass (kg)</td>
<td>3.268850564</td>
<td>3.642440657</td>
<td>3.939706383</td>
<td>4.207384652</td>
<td>3.95994426</td>
</tr>
<tr>
<td>Combustion Chamber External Radius (m)</td>
<td>0.040033176</td>
<td>0.046027744</td>
<td>0.048249364</td>
<td>0.049461538</td>
<td>0.04822747</td>
</tr>
<tr>
<td>Additional Mass at Recovery bay (kg)</td>
<td>0.322132845</td>
<td>1.5</td>
<td>0.741375129</td>
<td>2.76E-19</td>
<td>0.985042625</td>
</tr>
<tr>
<td>Additional Mass at Nosecone (kg)</td>
<td>0.117806831</td>
<td>2.229170232</td>
<td>0.000488093</td>
<td>1.65E-19</td>
<td>2.018861088</td>
</tr>
</tbody>
</table>
Appendix B

Simulation Code

function [obj, state] = Multidisciplinary2(x)
%
Multidisciplinary2 is the main function that uses
% the individual disciplines to create an optimal
% design of a hybrid rocket.
%
% DESIGN VARIABLES + STATE VARIABLES

% PROPULSION
% 'V_tank' – oxidizer tank volume (m^3)
% 'C_inj' – effective injector area (m^2)
% 'L' – fuel grain length (m)
% 'd_port_init' – initial fuel grain port diameter(m)
% 'd_th' – nozzle throat diameter (m)
% 'A_ratio_nozzle' – nozzle area ratio as a fraction
% 'ROT' – oxidizer tank radius (m)
% 'RCC_in' – combustion chamber internal radius (m)
% 'Mfuel' – fuel mass (kg)
% 'Mox' – fuel mass (kg)

% MASS&SIZING
% 'V_tank' – oxidizer tank volume (m^3)
% 'ROT' – oxidizer tank radius (m)
% 'LCC' – combustion chamber length (m)
% 'D_fin' – fin span (m)
% 'B_fin' – fin root chord (m)
% 'bfin' – fin tip chord (m)
% 'Madd' – additional mass in the recovery bay(kg)
% 'Madd_cone' – additional mass in the nose cone(kg)
% 'M_rocket' – rocket’s mass (kg)
% 'Lfuel' – fuel length (m)
% 'De' – rocket’s external diameter (m)
% 'MCC' – combustion chamber mass (kg)
% 'MOT' – oxidizer tank mass (kg)
% 'Mtube' – external structure mass (kg)
% 'LOT' – oxidizer tank length (m)
% 'Lrec' – recovery system length (m)
% 'Lav' – avionics system length (m)
% 'Mrec' – recovery system mass (kg)
% 'Mav' – avionics system mass (kg)
% 'Mfins' – fins mass (kg)
% 'Mnozzle' – nozzle mass (kg)
% 'Mcone' – cone mass (kg)
% 'RCC' – combustion chamber radius (m)

% AERODYNAMICS
% 'd_th' – nozzle throat diameter (m)
% 'A_ratio_nozzle' – nozzle area ratio as a fraction
% 'Dfin' – fin span (m)
% 'Bfin' – fin root chord (m)
% 'bfin' – fin tip chord (m)
% 'LCC' – combustion chamber length (m)
% 'd_OT,j' – distance between nose cone tip and oxidizer tank (m)

% STABILITY
% 'CP' – center of pressure
% 'CG' – center of gravity
% 'SM' – static margin

% CONSTANTS
% 'p_feed' – feed system pressure drop (Pa)
% 'rho_f' – solid fuel density (kg/m^3)
% 'a' – regression rate coefficient (m/s)
% 'n' – regression rate exponent
% 'launchAlt' – altitude of launch site (m)
% 'p_amb' – ambient pressure at launch site (Pa)
% 'T_amb' – ambient temperature at launch site (K)
% 'zeta_d' – discharge correction factor
% 'zeta_cstar' – characteristic velocity correction factor
% 'zeta_CF' – thrust coefficient correction factor
% 'C_D' – drag coefficient variability

% Inputs%
Load input files
state.CEA = load('MuleSim3CEA.mat'); % Combustion product lookup table
state.N2Osat = load('N2Osat.mat'); % Nitrous oxide saturation properties
CdA = load('Cdavg.mat'); CdA = CdA.Cdavg; % Coefficients of drag versus mach number

% Declare additional non-design model input
del_time = 0.01; % (s) Time step
time_max = 30; % (s) Maximum model run time
fill_level = 0.6; % (%) Initial liquid volume fraction in tank
p_tank_init = 5000000; % (Pa) Initial tank pressure
p_feed = 100000; % (Pa) Feed system pressure drop
rho_f = 930; % (kg/m$^3$) Fuel density
a = 0.000155; % Regression rate constant
n = 0.5; % Regression rate exponent
launchAlt = 1400; % (m) Elevation of Truth or Consequences, New Mexico
T_amb = 301; % (K) 28degC, average of high and low temperatures for Truth or Consequences, New Mexico in June
p_amb = 85600; % (Pa) Pressure at 1400m ASL (Truth or Consequences, New Mexico)
zeta_d = 1.05; % Nozzle discharge correction factor
zeta_cstar = 0.90; % Characteristic velocity correction factor
zeta_CF = 0.90; % Nozzle coefficient correction factor
zeta_Cd = 1; % Drag coefficient variability

% Surrocket
x_test = x(1:6);
{x, Surrocket_out} = Surrocket(x_test);

% Inputs arrangement
% Place values into struct for function handling
state.design = x(1:6);
state.parameters = [del_time, time_max, fill_level, p_tank_init, p_feed, rho_f, a, n, T_amb, p_amb, zeta_d, zeta_cstar, zeta_CF];
state.CdA = CdA* zeta_Cd;
state.launchAlt = launchAlt;
state.p_amb = p_amb;
state.Mrocket = x(7);
state.Ltube = x(8);
state.De = x(9);
state.MCC = x(10);
state.MOT = x(11);
state.Mtube = x(12);
state.LOT = x(13);
state.LCC = x(14);
state.Dfin = x(15);
state.ROT = x(16);
state.Bfin = x(17);
state.bfin = x(18);
state.Lrec = x(19);
state.Lav = x(20);
state.Mav = x(21);
state.Mrec = x(22);
state.Mfins = x(23);
state.Mnozzle = x(24);
state.d_OT_i = x(25);
state.RCC_in = x(27);
state.Mfuel = x(28);
state.Max = x(29);
state.RCC = x(30);
state.Madd = x(31);  
state.Madd_cone = x(32);  
state.vel = Surrocket_out.vel;  
state.rail_length = 9;  % (m) Launch rail length

% Create Constants
% Length
Lcone = 0.6;  % average value from references  
Lpay = 0.4;  % 30 cm from 3U cubesat + tolerance  
Lnozzle = 0.135;  % average value from reference
state.length = [Lcone,Lpay,Lnozzle];

% Mass payload
state.payload = 4;  % (kg) it's a payload according to the rules

% Model Calculations
% Propulsion
try
    state = Propulsion(state);
catch ME
    state.ME = ME;  % If error gets thrown, return NaN  
    state.ISP = NaN;
    state.apogee = NaN;
    state.pcc_max = NaN;
    state.Thrust_max = NaN;
    obj = NaN;
    return
end
state = Mass_sizing(state);  
state = Aerodynamics(state);

if max(isnan(state.alt)) == 0
    si = find(state.alt ~= 0, 1, 'first');  % First index that is not zero
    ei = find(state.alt > 9, 1, 'first');  % First index that is greater than nine
    state.vel_off_rail = interp1(state.alt(si:ei), state.vel(si:ei), state.rail_length);  % Calculate off-the-rail velocity
else
    state.vel_off_rail = NaN;
end

% Calculate Objective Function
if isnan(state.ISP) == 0
    obj_1 = real((1000 - state.ISP)/(1000 - 201.8));  
    obj_2 = real(state.Mrocket/34.8);  
    weight = 0.93;
    obj = weight*obj_1 + (1-weight)*obj_2;
else
    obj = NaN;
end
function [state] = Propulsion(state)
% PROPULSION
% Propulsion is a function that simulates the hybrid rocket motor. It is used for design optimization
% using "Multidisciplinar2" as the main function.
%
% All calculations performed in metric units

% INPUTS %
CEA = state.CEA; % Combustion product lookup table
N2Osat = state.N2Osat; % Nitrous oxide saturation properties
design = num2cell(state.design); % Distribute design vector
[Vtank, C_inj, L, d_port_init, d_th, A_ratio_nozzle] = design{;};
parameters = num2cell(state.parameters); % Distribute parameter vector
[del_time, time_max, fill_level, p_tank_init, p_feed, rho_f, a, ...
 n, T_amb, p_amb, zeta_d, zeta_cstar, zeta_CF] = parameters{;};
De = state.De;
ROT = state.ROT;
LOT = state.LOT;
RCC = state.RCC;
d_OT_i = state.d_OT_i;
vel = state.vel;
iteration_tolerance = 0.01; % Fraction tolerances on iterative solvers
tic; % Start timing function

% INITIAL CALCULATIONS %
% Calculate initial thermodynamic properties from tank pressure
thermo_sat_init = thermoSat(p_tank_init, 'p', {'T', 'rho_liq', 'rho_vap', 'u_liq', 'u_vap'});
thermo_sat_init = num2cell(thermo_sat_init);
[Ttank, rho_liq, rho_vap, u_liq, u_vap] = thermo_sat_init{;};
if ~exist('m_ox_tank_init', 'var') % If oxidizer mass is not given, calculate it using fill level
 m_ox_tank_init = V_tank*(rho_liq*fill_level + (1-fill_level)*rho_vap); % Calculate initial oxidizer mass based on fill level
end
x_tank = (V_tank/m_ox_tank_init - 1/rho_liq)/(1/rho_vap - 1/rho_liq); % Calculate vapour mass fraction
u_tank = (x_tank*u_vap + (1-x_tank)*u_liq); % (J/kg*K) Calculate specific internal energy
U_tank = m_ox_tank_init*u_tank; % (J) Calculate total internal energy

% Calculation of constants
R_const = 8.3144598; % (J/mol*K) Gas constant
A_th = pi()*(d_th/2)^2; % (m^2) Nozzle Throat Area
V_tank_eps = iteration_tolerance*V_tank; % (m^3) Set acceptable tank volume error to 1% of tank volume
u_tank_eps = iteration_tolerance*u_tank; % (m^3) Set acceptable tank specific internal energy error to 1% of initial tank specific internal energy
A\_ratio\_nozzle\_eps = iteration\_tolerance\_A\_ratio\_nozzle; % (m^3) Set acceptable nozzle area ratio error to 1% of nozzle area ratio

if exist('d\_f', 'var')
    m\_f\_init = rho\_f\_L\_pi();/4*(d\_f\_2 - d\_port\_init\_2); % (kg) Calculate initial fuel mass from grain geometry
    m\_tot = m\_ox\_tank\_init + m\_f\_init; % Total mass
else
    m\_tot = 0; % If outer diameter not known, add total mass on after simulation
end

% Setting initial conditions
m\_ox\_tank = m\_ox\_tank\_init; % (kg) Mass of oxidizer in tank
r\_cc = d\_port\_init/2; % (m) Combustion chamber port radius
p\_cc = p\_amb; % (Pa) Initial Combustion chamber pressure
T\_cc = T\_amb; % (K) Combustion chamber temperature
T\_stag = T\_amb; % (K) Combustion chamber stagnation temperature
R\_cc = R\_const/0.02897; % (J/kg*K) Molar mass of air at 298K
k\_cc = 1.4; % Specific heat ratio of air at 298K
M\_a = 3; % Initialize the mach number near solution to ensure convergence
burn\_time = time\_max; % Initialize burn time to max time

% MODEL CALCULATIONS %

\begin{verbatim}
t = 1;
try % Start try block to catch errors that occur in the main loop (for debugging)
    while 1 % Run until break
        if isnan(T\_stag(t)) == 1 || isnan(vel(t)) == 1
            T\_stag(t+1) = NaN;
            break
        else
            if x\_tank(t) < 1
                % OXIDIZER TANK CALCULATIONS FOR SATURATED LIQUID AND VAPOUR%
                Vin\_U = U\_tank(t);
                Vin\_m = m\_ox\_tank(t);
                Vin\_V = V\_tank;
                if abs(Verro(T\_tank(t), Vin)) > V\_tank\_eps
                    T\_tank(t) = secant(@Verror, T\_tank(t), Vin);
                end
                \end{verbatim}

% Calculation of tank thermo properties
\begin{verbatim}
thermo\_sat\_tank = thermoSat(T\_tank(t), 'T', {'p', 'h\_liq', ...
    'h\_vap', 'rho\_liq', 'rho\_vap', 'u\_liq', 'u\_vap'});
thermo\_sat\_tank = num2cell(thermo\_sat\_tank);
[p\_tank(t), h\_liq, h\_vap, rho\_liq, rho\_vap, u\_liq, u\_vap] = thermo\_sat\_tank %
\end{verbatim}
\begin{verbatim}
{;
    x\_tank(t) = (U\_tank(t)/m\_ox\_tank(t) - u\_liq)/(u\_vap - u\_liq);
    u\_tank(t) = x\_tank(t)*u\_vap + (1-x\_tank(t))*u\_liq;
    h\_tank(t) = x\_tank(t)*h\_vap + (1-x\_tank(t))*h\_liq;
    rho\_tank(t) = 1/(x\_tank(t)/rho\_vap + (1-x\_tank(t))/rho\_liq);
\end{verbatim}
\[
\text{rho discharge} = \text{rho liq} ; \\
\text{h discharge} = \text{h liq} ; \\
\]

\% Record burn time to be the time at which the tank runs out of oxidizer 
if \(x_{\text{tank}}(t) > 1\)
   \[\text{burn time} = t \times \text{del time} ;\]
end 
else 
\% OXIDIZER TANK CALCULATIONS FOR VAPOUR ONLY \%
\[\text{rho tank}(t) = \text{m ox tank}(t)/V_{\text{tank}};\]
\[\text{u tank}(t) = U_{\text{tank}}(t)/\text{m ox tank}(t);\]
\[\text{uin.rho} = \text{rho tank}(t) ; \]
\[\text{uin.u} = \text{u tank}(t) ;\]
if abs(\text{uerror}(T_{\text{tank}}(t), \text{uin})) > \text{u tank eps}
   \[T_{\text{tank}}(t) = \secant(@\text{uerror}, T_{\text{tank}}(t), \text{uin});\]
end 

\[\text{thermo span tank} = \text{thermoSpanWagner(rho tank}(t), T_{\text{tank}}(t), \{\text{"p"}, \text{"h"})\};\]
\[\text{p tank}(t) = \text{thermo span tank}(1) ;\]
\[\text{h tank}(t) = \text{thermo span tank}(2) + 7.3397e+05 ; \%	ext{ Convert from Span–Wagner enthalpy convention to NIST}\]
\[\text{h discharge} = \text{h tank}(t) ;\]
\[\text{rho discharge} = \text{rho tank}(t) ;\]
end 

\[\text{m dot ox in}(t) = C_{\text{inj}}\times\sqrt{2 \times \text{rho discharge} \times (p_{\text{tank}}(t)-p_{\text{cc}}(t)-p_{\text{feed}})} ; \%	ext{ Incompressible fluid assumption (better than nothing)}\]
if \(t == 1\) % Take average over last two time periods to attenuate numerical instability
   \[\text{m dot ox in} = 1/2 \times \text{m dot ox in} ;\]
else 
   \[\text{m dot ox in}(t) = 1/2 \times \text{m dot ox in}(t) + 1/2 \times \text{m dot ox in}(t-1) ;\]
end 

\[\text{del m ox tank} = -\text{m dot ox in}(t) \times \text{del time} ; \%	ext{ Oxidizer tank mass differential equation}\]
\[\text{Q tank} = \text{HeatOT(De, ROT, T tank}(t),\text{LOT, RCC, d OT_i, vel}(t)) ; \%	ext{ Oxidizer tank heat transfer}\]
\[\text{del U tank} = (-\text{m dot ox in}(t) \times \text{h discharge} - \text{Q tank}) \times \text{del time} ;\%	ext{ Oxidizer tank energy differential equation}\]

\% COMBUSTION CHAMBER CALCULATIONS \%
\[\text{A cc}(t) = \pi() \times r_{cc}(t)^2 ; \%	ext{ Port geometry}\]
\[\text{G}(t) = \text{m dot ox in}(t)/\text{A cc}(t) ; \%	ext{ Mass flux}\]
\[\text{del r cc}(t) = a \times \text{G}(t)^n \times \text{del time} ; \%	ext{ Regression rate law}\]
\[\text{m dot f}(t) = 2 \times \pi() \times r_{cc}(t)^2 \times \text{rho f} \times (\text{del r cc}(t)/\text{del time}) ; \%	ext{ Fuel mass flow rate from geometry}\]
\% Iteratively solve for \text{m dot f}
k = 0;

m_dot_f_temp = 0;
while abs(m_dot_f_temp - m_dot_f(t)) > 0.01 * m_dot_f(t) && k < 100

converges

m_dot_f_temp = m_dot_f(t);
m_dot_cc(t) = m_dot_f(t) + m_dot_ox_in(t); % Total mass flow in

G(t) = (m_dot_ox_in(t) + m_dot_cc(t)) / (2 * A_cc(t)); % Average mass flux accross entire port
del_r_CC(t) = a * G(t) * del_time; % Regression rate law using updated average mass flux

m_dot_f(t) = 2 * pi() * r_CC(t) * L * rho_f * (del_r_CC(t) / del_time); % Fuel mass flow rate from geometry

k = k + 1;
end

OF(t) = m_dot_ox_in(t) / m_dot_f(t); % Calculate Oxidizer–Fuel Ratio

p_stag(t) = m_dot_cc(t) / (rho_CC(t) * A_th) * sqrt(T_stag(t) * R_CC(t) / k_CC(t) * ((k_CC(t) + 1) / 2) * ((k_CC(t) + 1) / (k_CC(t) - 1))); % Steady state choked flow expression for pressure

if t > 1 % First time step is too cold for accurate velocity correction

vel_CC(t) = m_dot_CC(t) / (rho_CC(t) * A_CC(t));
vel_CC(t) = 1/2 * vel_CC(t) + 1/2 * vel_CC(t-1); % Average velocity over last time step for numerical stability

T_CC(t) = T_stag(t) - vel_CC(t)^2 / (2 * cp_CC(t));

end

T_CC(t) = T_stag(t);
p_CC(t) = p_stag(t) * (((T_CC(t)/T_stag(t)))^((k_CC(t)/(k_CC(t)-1)))); % Velocity correction for stagnation pressure

thermo_comb = CEAProp(OF(t), p_CC(t), {'T', 'rho', 'cp', 'k', 'M'});

thermo_comb = num2cell(thermo_comb);

[T_stag(t+1), rho_CC(t+1), cp_CC(t+1), k_CC(t+1), M_CC(t+1)] = thermo_comb{:}; % Calculate chamber properties at next time step

T_stag(t+1) = T_stag(t) * zeta_cstar^2; % Correct temperature with cstar efficiency

R_CC(t+1) = R_const / M_CC(t+1);

% NOZZLE AND THRUST CALCULATIONS %

Ain.k = k_CC(t);

Ain.A = A_ratio_nozzle;

if abs(Aerror(Ma(t), Ain)) > A_ratio_nozzle_eps % Nozzle mach number solver

Ma(t) = secant(@Aerror, Ma(t), Ain); % Supersonic solution

end

p_exit(t) = p_stag(t) / (1 + (k_CC(t)-1) / 2 * Ma(t)^2 * (k_CC(t)/(k_CC(t)-1))); % Nozzle exit pressure

if x_tank(t) >= 1 % Physically wrong, but the condition is used to hide the transition that results from overexpansion

p_exit(t) = p_stag(t) * (2 / (k_CC(t)+1)) * (k_CC(t)/(k_CC(t)-1)); % Throat pressure

vel_exit(t) = sqrt(2 * k_CC(t) * R_CC(t) * T_stag(t) / (k_CC(t)+1)); % Throat velocity

A_ratio_nozzle_eff = 1; % Throat area ratio

else
\[
T_{\text{exit}}(t) = T_{\text{stag}}(t)/(1+(k_{cc}(t)−1)/2 \cdot Ma(t)^2); \quad \text{\% (K) Temperature of gas at exit of nozzle}
\]
\[
\text{vel}_{\text{exit}}(t) = Ma(t) \cdot \sqrt{k_{cc}(t) \cdot R_{cc}(t) \cdot T_{\text{exit}}(t)); \quad \text{\% (m/s) Velocity of gas at exit of nozzle}
\]
\[
A_{\text{ratio_nozzle_eff}} = A_{\text{ratio_nozzle; \% Effective nozzle area ratio is actual nozzle area ratio}}
\]

\[
F_{\text{thrust}}(t) = zeta_{CF} \cdot (m_{\dot{\text{cc}}}(t) \cdot \text{vel}_{\text{exit}}(t) + (p_{\text{exit}}(t)−p_{\text{amb}}) \cdot A_{\text{th}} \cdot A_{\text{ratio_nozzle_eff}}; \quad \text{\% (N) Rocket Motor Thrust!!!}
\]

% ITERATE FORWARD IN TIME %
if t < time\_max/\text{del_time} \&\& m_{\text{ox}}(t)/m_{\text{ox}}(_{\text{tank}}) > 0.05 \&\& p_1 > p_{\text{atm}} %
If less than max burn time, more than 5\% of oxidizer is left, and flow is choked, step forward in time
% Oxidizer Tank Values
m_{\text{ox}}(t+1) = m_{\text{ox}}(t) + \text{del} \cdot m_{\text{ox}}(_{\text{tank}});
U_{\text{tank}}(t+1) = U_{\text{tank}}(t) + \text{del} \cdot U_{\text{tank}};
x_{\text{tank}}(t+1) = x_{\text{tank}}(t);
T_{\text{tank}}(t+1) = T_{\text{tank}}(t);
m_{\dot{\text{ox}}}(t+1) = m_{\dot{\text{ox}}}(t);
% Combustion Chamber Values
m_{\text{tot}}(t+1) = m_{\text{tot}}(t) − m_{\dot{\text{cc}}}(t) \cdot \text{del_time};
r_{cc}(t+1) = r_{cc}(t) + \text{del} \cdot r_{cc}(t);
p_{cc}(t+1) = p_{cc}(t);
Ma(t+1) = Ma(t);
t = t + 1;
else
break % Exit loop if no more time or oxidizer
end
end
catch ME % Catch any errors
if isa(state, \text{'double'}) \&\& length(state)==6
state = NaN;
return
end
rethrow(ME)
end
time = 0:\text{del_time}:(\text{del_time} \cdot (t−1)); \% Create a time vector

% OUTPUTS %
if max(isnan(T_{stag})) == 0 \&\& isreal(m_{\dot{\text{ox}}}(t)) == 1
% Major output calculations
r_{\text{dot}} = \text{del} \cdot r_{cc}(t)/\text{del_time}; \% (m/s)
d_{\text{port}}(t) = (2 \cdot r_{cc}(t+1)); \% (m)
m_{\text{out}} = trapz(m_{\dot{\text{cc}}}(t), \text{del_time}); \% (kg)
m_{\text{tot}} = m_{\text{tot}} + m_{\text{out}};
m_f = (\text{trapz}(m_{\text{dot}}(t+1), \text{del_time}); \% (kg)
m_ox = m_out-m_f; % (kg)
l_tot = (trapz(F_thrust)*del_time); % (N*s) Just total impulse during
v_e = l_tot/m_out; % (m/s)
l_sp = (v_e/9.81); % (s)
c_star = mean(p_cc)*A_th/mean(m_dot_cc); % (m/s)
% Save metadata of simulation
time_sim = toc; % Save how long the simulation took (minus loading data and plotting
figures)
time_stamp = datetstr(datetime); % Save the current time and date
computer_name = computer; % Save which computer the code is running on
function_name = mfilename; % Save the name of the function that is being called
% Output thrust and mass vs time for trajectory analysis
TCurve = [l_tot, mean(F_thrust), m_out];
TCurve = vertcat(TCurve, [time; F_thrust; m_tot']);:
% Save all pertinent data to the state variable
state.TC = TCurve;
state.burn = burn_time/del_time;
state.mass_prop = m_out;
state.Mfuel = m_f;
state.Mox = m_ox_tank_init;
state.OF = OF;
state.RCC_in = d_port_f/2;
state.l_sp = l_sp;
state.Ath = A_th;
state.AR = A_ratio_nozzle;
state.Thrust_max = max(F_thrust);
state.pcc_max = max(p_cc);
state.meta = [time_sim, time_stamp, computer_name, function_name];
state.ptank = p_tank;
state.pfeed = p_feed;
state.pcc = p_cc;
state.G = G;
state.l_tot = l_tot;
state.pank_max = max(p_tank);
state.time_prop = time;
state.Thrust = F_thrust;
else
    state.TC = NaN;
    state.burn = NaN;
    state.mass_prop = NaN;
    state.Mfuel = NaN;
    state.Mox = NaN;
    state.OF = NaN;
    state.RCC_in = NaN;
    state.l_sp = NaN;
    state.Ath = NaN;
    state.AR = NaN;
    state.Thrust_max = NaN;
    state.pcc_max = NaN;
end
state.meta = [NaN, NaN, NaN, NaN];
state.plank = NaN;
state.pfeed = NaN;
state.pcc = NaN;
state.G = [NaN, NaN];
end

% SUBFUNCTIONS %

function [out] = CEAProp(OF, p, in)

% MASSFRAC Calculates the mass fractions of products for a given reaction
% 'OF' is the oxidizer to fuel weight ratio
% 'p' is the pressure of the combustion chamber
% 'alpha' is an array of weight fractions

if OF < CEA.OF(1) % Check that query is inside bounds
    OF = NaN;
    error('CEAProp:lowOF', 'Outside O/F range: \n OF = %f', OF)
elseif OF > CEA.OF(end)
    OF = NaN;
    error('CEAProp:highOF', 'Outside O/F range: \n OF = %f', OF)
elseif p < CEA.p(1)
    p = NaN;
    error('CEAProp:lowP', 'Outside pressure range: \n p = %f Pa', p)
elseif p > CEA.p(end)
    p = NaN;
    error('CEAProp:highP', 'Outside pressure range: \n p = %f Pa', p)
end

if isnan(OF) == 0
    for mm = 1:length(CEA.OF)
        if OF < CEA.OF(mm) % Find first index that is larger than OF
            break
        end
    end
end

if isnan(p) == 0
    for nn = 1:length(CEA.p)
        if p < CEA.p(nn) % Find first index that is larger than p
            break
        end
    end
end

out = zeros(size(in));

if isnan(OF) == 1 || isnan(p) == 1
    OF_int = NaN;
p_int = NaN;
C_int = NaN;
out_int = NaN;
for kk = 1:length(in)
    out(kk) = NaN;
end
else
for kk = 1:length(in) % Iterate through all products
    % Do some bilinear interpolation (equations from wikipedia)
    OF_int = [CEA.OF(mm)−OF, OF−CEA.OF(mm−1)];
    p_int = [CEA.p(nn)−p; p−CEA.p(nn−1)];
    C_int = 1/((CEA.OF(mm)−CEA.OF(mm−1))+(CEA.p(nn)−CEA.p(nn−1)));
    out_int = CEA.(in(kk))(mm−1:nn, nn−1:nn);
    out(kk) = C_int.*OF_int.*out_int.*p_int;
end
end
end

function [val_out] = thermoSat(val_in, in, out)
% THERMOSAT returns thermodynamic properties at saturation
% 'val_in' is a double specifying the value of thermodynamic property inputted
% 'in' is a string specifying the given input thermodynamic property
% 'out' as a cell array specifying the desired output thermodynamic property
% 'val_out' is a double specifying the value of thermodynamic property returned
val_out = zeros(size(out));
for mm = 1:length(out) % For each desired output
    % Locate which variable column is input, and which is output
    ii = find((in == N2Osat.meta(1,:)), 1, 'first');
    jj = find((out(mm) == N2Osat.meta(1,:)), 1, 'first');
    % Interpolate between the two values
    for kk = 1:length(N2Osat.data(:,ii))
        if val_in < N2Osat.data(kk,ii)
            break
        end
    end
    val_out(mm) = (val_in − N2Osat.data(kk−1,ii))./(N2Osat.data(kk−1,ii)−N2Osat.data(kk−1,ii)).*(N2Osat.data(kk, jj)−N2Osat.data(kk−1, jj)) + N2Osat.data(kk−1, jj);
end
end

function [out] = thermoSpanWagner(rho, T, in)
% THERMOSWAGNER Calculates thermodynamic properties for N2O as a non−ideal gas
% 'in' is a cell array containing the desired output parameters
% 'rho' is the density in kg/m^3
% 'T' is the temperature in K
% 'out' is an array containing the output values in the order listed in 'in'
% Hardcode in data for N2O (from "Modeling Feed System Flow Physics for Self−Pressurizing Propellants")
R = 8.3144598/44.0128*1000; % (J/kg*K) Gas constant
T_c = 309.52; % (K) Critical Temperature
rho_c = 452.0115; % (kg/m^3) Critical Density
n0 = [0.88045, −2.4235, 0.38237, 0.068917, 0.00020367, 0.13122, 0.46032, ...
     −0.0036985, −0.23263, −0.00042859, −0.042810, −0.023038];
n1 = n0(1:5); n2 = n0(6:12);
a1 = 10.7927224829;
a2 = -8.2418318753;
c0 = 3.5;
v0 = [2.1769, 1.6145, 0.48393];
u0 = [879, 2372, 5447];
t0 = [0.25, 1.125, 1.5, 0.25, 0.875, 2.375, 2, 2.125, 3.5, 6.5, 4.75, 12.5];
d0 = [1, 1, 1, 3, 7, 1, 2, 5, 1, 1, 4, 2];
P0 = [1, 1, 1, 2, 2, 2, 3];
t1 = t0(1:5);
t2 = t0(6:12);
d1 = d0(1:5);
d2 = d0(6:12);

% Calculate non-dimensional variables
tau = Tc / T;
delta = rho / rho_c;

% Calculate explicit helmholtz energy and derivatives (from
ao = a1 + a2*tau + log(delta) + (c0-1)*log(tau) + sum(v0.*log(1-exp(-u0.*tau./T_c))) + sum(n2.*tau.*d2.*exp(-delta.*P0)) +
ao_tau = a2 + (c0-1)/tau + sum(v0.*u0./T_c.*exp(-u0.*tau./T_c)./(1-exp(-u0.*tau./T_c)));

ar = sum(n1.*tau.*t1.*delta.*d1) + sum(n2.*t2.*tau.*d2.*exp(-delta.*P0));

ar_tau = sum(n1.*t1.*tau.*d1) + sum(n2.*t2.*tau.*d2.*exp(-delta.*P0));

ar_tautau = sum(n1.*t1.*d1.*d2.*exp(-delta.*P0));

ar_tautau = sum(n1.*d1.*d2.*exp(-delta.*P0));

ar_delta = sum(n1.*d1.*d2.*exp(-delta.*P0));

ar_deltadelta = sum(n1.*d1.*d2.*exp(-delta.*P0));

ar_deltatau = sum(n1.*d1.*d2.*tau.*d1) + sum(n2.*t2.*tau.*d2.*exp(-delta.*P0));

out = zeros(size(in));

for kk = 1:length(in)
    switch in(kk)
        case 'p'
            out(kk) = rho*R*T*(1+delta*ar_delta);
        case 'u'
            out(kk) = R*T*tau*(ao+ar_tau);
        case 's'
            out(kk) = R*(ao+ar_tau)-ar;
        case 'h'
            out(kk) = R*T*(1+delta+ar_delta);
        case 'cv'
            out(kk) = R*T*(1+delta+ar_delta+ar_deltadelta);
        case 'cp'
            out(kk) = R*T*(1+delta+ar_deltadelta);
        case 'a'
            out(kk) = sqrt(R*T*(1+delta+ar_deltadelta-1+delta+ar_deltatau-2*ar_deltadelta) - (1+delta+ar_deltadelta+ar_deltatau-2*ar_deltadelta));
    end
end
otherwise
    error('Invalid input')
end
end

function [x] = secant(fun, x1, in)
% SECANT is a zero-finding function based on the secant method
% 'fun' is the function handle for which the zero is desired
% 'x1' is the initial guess
% 'in' is a struct containing any additional inputs 'fun' might require
% 'x' is the value of the zero
x_eps = x1 * 0.005; % Set the tolerance to be 0.5% of initial guess
x2 = x1 - x1 * 0.01; % Set a second point 1% away from the original guess
F1 = fun(x1, in); % Evaluate function at x1
F2 = fun(x2, in); % Evaluate function at x2
kk = 1; % Set up counter
kk_max = 1000;
while abs(x2 - x1) >= x_eps && kk < kk_max % While error is too large and counter is less than max
    x3 = x2 - F2 * (x2 - x1) / (F2 - F1);
    x1 = x2; % Move everything forward
    x2 = x3;
    F1 = F2;
    F2 = fun(x2, in);
    kk = kk + 1;
end
x = x2;
end

% SET UP FUNCTIONS TO BE ITERATIVELY SOLVED %

function V = Verror(T, in) % Finds the difference between the estimated and actual tank volume
    thermo = thermoSat(T, 'T', {'rho_liq', 'rho_vap', 'u_liq', 'u_vap'});
    thermo = num2cell(thermo);
    [rho_l, rho_v, u_l, u_v] = thermo{:};
    x = (in.U / in.m - u_l) / (u_v - u_l);
    V = in.m * ((1 - x) / rho_l + x / rho_v) - in.V;
end

function U = uerror(T, in) % Finds the difference between the estimated and actual tank internal energy
    U = (thermoSpanWagner(in.rho, T, {'u'}) + 7.3397e+5) - in.u;
end

function A = Aerror(M, in) % Finds the difference between the estimated and actual nozzle ratio
    A = (1 / M^2) * (2 / (in.k + 1) + (1 + (in.k - 1) / 2 * M^2)) * (((in.k + 1) / (in.k - 1)) - in.A^2);
end
function Q = HeatOT(De,ROT,TOT,LOT,RCC,d_OT,i,vel)

% Constants
T_a = 301; % (K)
K_cond = 0.0264704; % (W/m K) thermal conductivity air 30C
g = 9.81;
beta = 0.0033; % (1/K) expansion coefficient air 30C
nu = 15.9668e−6; % (m^2/s) kinematic viscosity air 30C
Pr = 0.71; % Prandtl number
re = De/2;
alfa = 22.07e−6; %Thermal Diffusivity m^2/s

% For model validation
% nu = 14.96e−6; % (m^2/s) kinematic viscosity air 19C
% K_cond = 0.025618; % (W/m K) thermal conductivity air 19C

% R convection internal
L_heat = 2*(log(RCC/ROT)^(4/3))/(RCC^(-3/5) + ROT^(-3/5))^(5/3);
Gr = g*beta*abs(TOT−T_a)*L_heat^3/nu^3;
Ra = Gr*Pr;
K_eff = K_cond*0.386*(Pr/(Pr+0.861))^0.25*Ra^0.25;
if K_eff < K_cond
    K_eff = K_cond;
end
R_conv_in = log(RCC/ROT)/(2*pi*LOT*K_eff);

% R conduction
k_cond = 2; % (W/mK) thermal conductivity carbon fiber−epoxy
R_cond = log(re/RCC)/(2*pi*k_cond*LOT);

% R convection external
dx = LOT/1000;
x = d_OT,i;
cont = 1;
iterations = ceil((x−LOT)/dx);
h = zeros(iterations,1);
for i = 1:iterations;
    position = x + (i−1)*dx;
    Re = vel*position/nu;
    Nu = 0.0296*Re^(4/5)*Pr^(1/3);
    h(i) = Nu*K_cond/position;
end
h_average = trapz(h)*dx/LOT;
R_conv_ex = 1/(h_average/2*pi*re*LOT);
T = TOT;
Q = (TOT−T_a)/(R_conv_in+R_cond+R_conv_ex);
end

end
function [state] = Mass_sizing(state)
% MASS&SIZING
% Mass&sizing is a function that estimates the rocket
% mass and size. It is used for design optimization
% using "Multidisciplinar2" as the main function.

if isnan(state.RCC_in) == 0
    % INPUTS %
    length = num2cell(state.length);
    [Lcone, Lpay, Lnozzle] = length{:};

    Mpay = state.payload;
    Mfuel = state.Mfuel;
    Mox = state.Mox;
    Rcc_in = state.RCC_in;

    ROT = state.ROT;
    LCC = state.LCC;
    Dfin = state.Dfin;
    Bfin = state.Bfin;
    bfin = state.bfin;
    Madd = state.Madd;
    Madd_cone = state.Madd_cone;

    design = num2cell(state.design); % Distribute design vector
    [V_tank, ˜, ˜, ˜, ˜, ˜] = design{:};

    % Constants
    N_nozzle = 0.2; % percentage of nozzle length on the outside
    t_CC = 0.00317;
    t_OT = 0.006;
    t_tube = 0.003;
    t_cone = 0.003;
    t_fins = 0.002;
    rho_CC = 2700; % Aluminum 6061 T6 density
    rho_OT = 2700; % Aluminum 6061 T6 density
    rho_fins = 1600;
    rho_tube = 1600; % Carbon fiber epoxy density
    rho_cone = 1800; % Fiberglass epoxy density

    % Calculation
    % Diameter
    Rcc = Rcc_in + t_CC;
    r_OT = ROT - t_OT;
    R_tube = Rcc + t_tube;
\[ D_e = 2 \times R_{\text{tube}}; \]

% Length
\[ \text{LOT} = \frac{V_{\text{tank}}}{(\pi \cdot R_{\text{OT}}^2)}; \]
\[ L_{\text{tube}} = L_{\text{pay}} + \text{LOT} + L_{\text{CC}} + L_{\text{nozzle}} \times N_{\text{nozzle}}; \]
\[ L_{\text{rocket}} = \frac{(L_{\text{tube}} + L_{\text{cone}} + L_{\text{nozzle}} \times (1-N_{\text{nozzle}}))}{0.59394036 - 1.32444}; \text{% from the linear regression} \]
\[ L_{\text{rec}} = 0.2404 \times L_{\text{rocket}} - 0.4193; \]
\[ L_{\text{av}} = 0.6891 \times L_{\text{rec}} - 0.0784; \]
\[ L_{\text{tube}} = L_{\text{tube}} + L_{\text{rec}} + L_{\text{av}}; \]
\[ L_{\text{rocket}} = L_{\text{tube}} + L_{\text{cone}} + L_{\text{nozzle}} \times (1-N_{\text{nozzle}}); \]

% Transversal area
\[ A_{\text{trans}} = \pi \times \left( \frac{(D_e + D_{\text{fin}})}{2} \right)^2; \]

% Volume
\[ V_{\text{CC}} = L_{\text{CC}} \times \pi \times (R_{\text{CC}}^2 - R_{\text{CC,in}}^2); \]
\[ V_{\text{OT}} = \text{LOT} \times \pi \times (R_{\text{OT}}^2 - r_{\text{OT}}^2); \]
\[ V_{\text{tube}} = L_{\text{tube}} \times \pi \times (R_{\text{tube}}^2 - R_{\text{CC}}^2); \]

% Cylindrical components
\[ M_{\text{CC}} = \rho_{\text{CC}} \times V_{\text{CC}}; \]
\[ M_{\text{OT}} = \rho_{\text{OT}} \times V_{\text{OT}}; \]
\[ M_{\text{tube}} = \rho_{\text{tube}} \times V_{\text{tube}}; \]

% Nose cone
\[ g_{\text{cone}} = \sqrt{L_{\text{cone}}^2 + R_{\text{CC}}^2}; \]
\[ A_{\text{cone}} = \pi \times R_{\text{CC}} \times g_{\text{cone}}; \]
\[ \text{Factor}_{\text{cone}} = 1.3; \]
\[ M_{\text{cone}} = A_{\text{cone}} \times t_{\text{cone}} \times \rho_{\text{cone}} \times \text{Factor}_{\text{cone}}; \]
\[ M_{\text{cone}} = M_{\text{cone}} + \text{Madd}_{\text{cone}}; \text{% Add weight for stability} \]

% Fins
\[ A_{\text{fins}} = 4 \times (B_{\text{fin}} + b_{\text{fin}}) \times D_{\text{fin}} / 2; \]
\[ M_{\text{fins}} = A_{\text{fins}} \times t_{\text{fins}} \times \rho_{\text{fins}}; \]

% Mass estimation
\[ M_{\text{prop}} = M_{\text{fuel}} + M_{\text{oxy}}; \]
\[ \text{Mass}_{\text{structural}} = 0.434866 \times M_{\text{prop}} + 259.0147 \times D_e - 13.2621; \text{% Calculate structural mass from empirically fitted equation} \]
\[ \text{Mass}_{\text{rec,av,noise}} = \text{Mass}_{\text{structural}} - M_{\text{tube}} - M_{\text{cone}} - M_{\text{fins}} - M_{\text{CC}} - M_{\text{OT}}; \]
\[ M_{\text{nozzle}} = \text{real}(0.15 \times \text{Mass}_{\text{rec,av,noise}}); \]
\[ M_{\text{rec}} = \text{real}(0.60 \times \text{Mass}_{\text{rec,av,noise}}); \]
\[ M_{\text{av}} = \text{real}(0.25 \times \text{Mass}_{\text{rec,av,noise}}); \]
\[ M_{\text{rec}} = M_{\text{rec}} + \text{Madd}; \text{% Add weight for stability} \]
\[ M_{\text{aero}} = M_{\text{tube}} + M_{\text{cone}} + M_{\text{fins}} + M_{\text{CC}} + M_{\text{OT}} + M_{\text{nozzle}}; \]
\[ M_{\text{rocket}} = \text{real}(M_{\text{prop}} + M_{\text{aero}} + M_{\text{rec}} + M_{\text{av}} + M_{\text{pay}}); \]
%% outputs
state.Mrocket = M_rocket;
state.Ltube = L_tube;
state.Lrocket = L_rocket;
state.De = De;
state.Atrans = Atrans;
state.MCC = M_CC;
state.MOT = M_OT;
state.Mtube = M_tube;
state.LLOT = LOT;
state.Lrec = L_rec;
state.Lav = L_av;
state.Mav = M_av;
state.Mrec = M_rec;
state.Mfins = M_fins;
state.Mnozzle = M_nozzle;
state.Mcone = M_cone;
state.RCC = RCC;

else
    state.Mrocket = NaN;
    state.Ltube = NaN;
    state.Lrocket = NaN;
    state.De = NaN;
    state.Atrans = NaN;
    state.MCC = NaN;
    state.MOT = NaN;
    state.Mtube = NaN;
    state.LLOT = NaN;
    state.LCC = NaN;
    state.Dfin = NaN;
    state.ROF = NaN;
    state.Bfin = NaN;
    state.bfin = NaN;
    state.Lrec = NaN;
    state.Lav = NaN;
    state.Mav = NaN;
    state.Mrec = NaN;
    state.Mfins = NaN;
    state.Mnozzle = NaN;
    state.Mcone = NaN;
    state.RCC = NaN;
end
end

function [state] = Aerodynamics(state)

% AERODYNAMICS
% Aerodynamics is a function that simulates the rocket
% trajectory and stability. It is used for design
% optimization using "Multidisciplinary2" as the main function.

% INPUTS
parameters = num2cell(state.parameters); % Distribute parameter vector
[~, ~, ~, ~, a, ~, ~, p_amb, ~, ~] = parameters{::};
launchAlt = state.launchAlt;
Mrec = state.Mrec;
Mav = state.Mav;
Mfins = state.Mfins;
Mnozzle = state.Mnozzle;
Mcone = state.Mcone;
length_rocket = num2cell(state.length);
[Lcone, Lpay, Lnozzle] = length_rocket{::};
design = num2cell(state.design); % Distribute design vector
[~, ~, ~, d_th, A_ratio_nozzle] = design{::};
Ltube = state.Ltube;
De = state.De;
Dfin = state.Dfin;
Bfin = state.Bfin;
bfin = state.bfin;
Mtube = state.Mtube;
MOT = state.MOT;
MCC = state.MCC;
Mpay = state.payload;
Mox = state.Mox;
Mfuel = state.Mfuel;
Mtotal = state.Mrocket;
LOT = state.LOT;
LCC = state.LCC;
TC = state.TC;
Lrec = state.Lrec;
Lav = state.Lav;
Atrans = pi*(((De+Dfin)/2)^2);
A_th = pi*d_th^2/4;
AR = A_ratio_nozzle;

% CG and CP
%Areas
A_fin_tri = (Bfin-bfin)*Dfin/2; %trapezoidal
A_fin_quad = bfin*Dfin;
n_fins = 4; % number of fins
%Distances (with the nose cone as reference)
d_cone = 2*Lcone/3;
d_tube = (Lcone + Ltube/2);
d_fin_tri = Lcone+Ltube-bfin-(Bfin-bfin)/3;
d_fin_quad = (Lcone+Ltube-bfin/2);

%Center of pressure Barrowman Equation
CN_nose = 2;
R = De/2;
Lf = (Dfin^2 + (Bfin/2 - bfin/2)^2)^0.5;
\[ x_B = L_{cone} + L_{tube} - B_{fin}; \]
\[ x_R = B_{fin} - b_{fin}; \]
\[ CN_{fins} = \left( 1 + \frac{R}{(D_{fin} + R) \cdot \left( 4 \cdot n_{fins} \cdot (D_{fin} / De)^2 \right) / (1 + (2 + L_f / (B_{fin} + b_{fin}))^2 \cdot 0.5) \right) \times x_{fins} = \frac{x_B + x_R / 3 \cdot (B_{fin} + 2 \cdot b_{fin}) / (B_{fin} + b_{fin}) + (B_{fin} + b_{fin} - B_{fin} \cdot b_{fin} / (B_{fin} + b_{fin})) / 6}{(1 + R / (D_{fin} + R) \cdot \left( 4 \cdot n_{fins} \cdot (D_{fin} / De)^2 \right) / (1 + (2 + L_f / (B_{fin} + b_{fin}))^2 \cdot 0.5) \times x_{fins}}; \]
\[ CP = \text{real} \left( \frac{CN_{nose} \cdot d_{cone} + CN_{fins} \cdot x_{fins}}{CN_{nose} + CN_{fins}} \right); \]

% Additional distances
\[ d_{rec} = L_{cone} + L_{rec} / 2; \]
\[ d_{pay} = L_{cone} + L_{rec} + L_{pay} / 2; \]
\[ d_{av} = L_{cone} + L_{rec} + L_{pay} + L_{av} / 2; \]
\[ d_{OT} = L_{cone} + L_{rec} + L_{pay} + LOT / 2; \]
\[ d_{CC} = L_{cone} + L_{rec} + L_{pay} + LOT + LCC / 2; \]
\[ d_{nozzle} = L_{cone} + L_{rec} + L_{pay} + LOT + LCC + L_{nozzle} / 2; \]
\[ d_{OT, i} = d_{OT} - LOT / 2; \]

% Fuel and oxidizer on CC and OT
\[ M_{OT} = M_{OT} + M_{ox}; \]
\[ M_{CC} = M_{CC} + M_{fuel}; \]

% Center of gravity
\[ CG = \text{real} \left( \frac{(d_{cone} \cdot M_{cone} + d_{tube} \cdot M_{tube} + d_{fin, tri} \cdot A_{fin, tri} \cdot M_{fins} / (A_{fin, tri} + A_{fin, quad}) + d_{fin, quad} \cdot A_{fin, quad} \cdot M_{fins} / (A_{fin, tri} + A_{fin, quad}) + d_{rec} \cdot M_{rec} + d_{av} \cdot M_{av} + d_{pay} \cdot M_{pay} + d_{OT} \cdot M_{OT} + d_{CC} \cdot M_{CC} + d_{nozzle} \cdot M_{nozzle}) / M_{total}}{M_{total}} \right); \]

% Stability
\[ SM = \text{real} \left( \frac{(CP - CG)}{De} \right); \% \text{Static Margin}, \ CP \& \ CG \text{ measured according to the nose cone} \]

% Altitude
\[ CdA = \text{state.CdA}; \]
\[ mass = M_{total}; \]
\[ area = A_{trans}; \% \text{factor of 1.1 accounts for fins} \]
\[ time = 0; \% \text{time accounting} \]
\[ dt = 0; \% \text{time delta} \]
\[ gravloss = 0; \% \text{determines total drag loss to delta-v} \]
\[ onpower = 0; \% \text{for plot to show when engine is firing} \]
\[ stagesep = 0; \% \text{for plot to show when rocket is staging} \]
\[ A_{exit} = AR \cdot A_{th}; \% \text{Nozzle exit area for thrust correction} \]

if isnan(TC) == 0
    [alt1, vel1, accel1, time1] = power(launchAlt, 0, 0, TC, CdA);
    [alt2, vel2, accel2, time2] = coast(alt1(end), vel1(end), accel1(end), CdA, time1);

    alt = [alt1; alt2];
    vel = [vel1; vel2];
    accel = [accel1; accel2]; \% \text{Concatenate power and coasting array} \]
    alt = alt - launchAlt; \% \text{Subtract launch altitude to get AGL altitude} \]

% Outputs
state.CP = CP;
state.CG = CG;
state.alt = alt;
state.vel = vel;
state.accel = accel;
state.apogee = alt(end);
state.d_OT.i = d_OT.i;
state.SM = SM;
state.time_aero = time2;
else
    state.CP = NaN;
    state.CG = NaN;
    state.apogee = NaN;
    state.d_OT.i = NaN;
    state.vel = NaN;
    state.alt = NaN;
    state.accel = NaN;
    state.SM = NaN;
end

% Sub Functions

% Rocket motor producing thrust

function [alt, vel, accel, time] = power(alt, vel, accel, TCurve, CdA)
    dt = TCurve(end,1)/length(TCurve); % Assumes evenly spaced thrust curve
    plen = length(onpower);
    time = 0;
    for i = 1:(length(TCurve) - 2)
        %% calculate atmospheric conditions %%
        [~, a, p_act, rho] = atmosisa(alt(i));
        %% calculate drag force %%
        M = vel(i)/a;
        [~, index] = min(abs(CdA(:,1) - M)); % Find the index of the closest mach number in Cd table to actual mach number
        drag = CdA(index,2)*area*0.5*rho*vel(i)^2;
        %% calculate gravitational acceleration %%
        g = grav(alt(i)); % gravity decreases as you get farther from earth
        %% correction for sea-level thrust curves %%
        Thrust = TCurve(i,2) + (p_amb - p_act)*A_exit;
        %% calculate acceleration, velocity and altitude %%
        accel(i+1,1) = real((Thrust-drag)/mass); % F = ma
        vel(i+1,1) = real(accel(i)*dt+vel(i));
        alt(i+1,1) = real(vel(i)*dt+(accel(i)*dt^2)/2 +alt(i));
        if alt(i+1,1) < launchAlt % check if hit ground
            alt(i+1,1) = launchAlt;
            vel(i+1,1) = 0;
            accel(i+1,1) = 0;
        end
    end
    onpower(i+plen+1,1) = alt(i+1,1); % for plotting
    stagesep(i+plen+1,1) = NaN; % for plotting
    mass = mass - (TCurve(i+1,3)-TCurve(i+2,3)); % Michael's motor data mass data is in grams
    time = time + dt;
    gravloss = gravloss + (drag*dt/mass);
end
function [alt, vel, accel, time] = coast(alt, vel, accel, CdA, time)

i = 1;
plen = length(onpower);
while vel(i) > 0

% calculate atmospheric conditions
[~, a, ~, rho] = atmosisa(alt(i));

% calculate drag force
M = vel(i)/a;
[~, index] = min(abs(CdA(:,1)-M));
drag = CdA(index,3) * area * 0.5 * rho * vel(i)^2;

% calculate gravitational acceleration
% g = grav(alt(i)); % must take into account

% calculate acceleration, velocity and altitude
accel(i+1,1) = real(-drag/mass-g);
vel(i+1,1) = real(accel(i)*dt+vel(i));
alt(i+1,1) = real(vel(i)*dt+(accel(i)*dt^2/2)+alt(i));
if alt(i+1,1) < launchAlt % check if hit ground
    alt(i+1,1) = launchAlt;
    vel(i+1,1) = 0;
    accel(i+1,1) = 0;
end

onpower(i+plen,1) = NaN; % for plotting
stagesep(i+plen,1) = NaN; % for plotting
i = i + 1;
time = time + dt;

end
end % end coast
end % end main program

% Calculates gravitational acceleration at altitude
function g = grav(alt)
g = (6.67408e-11*5.972e24)/(6.371e6+alt)^2;
end