Design Optimization of a Single-Sided Disk-Rotor Induction Motor for the FST In-Wheel Traction

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Abstract

The purpose of this thesis is the construction of an axial-flux concentrated winding disk induction motor and its electromechanical characterization, including a lumped parameter equivalent electric circuit description. This machine uses an innovative geometry as well as an isotropic soft magnetic composite material called Somaloy. Axial-flux machines are especially used when the short axial length of the machine is advantageous, so vehicle applications can benefit from this research, using a single-sided rotor design operating in a direct drive system.

After the typical tests for obtaining the lumped parameters failed to give an accurate set of values, a trust region algorithm was used to estimate the parameters that best fit the experimental data. These results were validated with a method developed in a previous thesis.

The results obtained prove that the developed machine does not have enough torque to speed up to near the synchronous speed so the machine’s efficiency is low. The algorithm used to compute the equivalent machine’s parameter has provided a satisfactory fit to the data. Several recommendations are given in order to improve the machine’s performance.

Keywords: Induction machine; axial-flux machine; electrical machine design;

1. Introduction

The present work continues the research developed in the master thesis [], where an in-wheel, single-sided disk-rotor induction motor was initially developed for a student car competition. In the previous work, it was verified that the machine’s developed torque was insufficient to meet the race car requirements. The designed machine, however, proved promising for high torque and low speed applications where a short axial length is required. As such, the proposed design, a Somaloy based induction machine working prototype using the axial flux topology, was implemented in this thesis with the construction of a prototype and tested to characterized its electromechanical characteristic and lumped-parameter model.

Axial-field electrical machines (AFMs) differ from conventional machines in that the air-gap flux is in the axial direction, while the effective conductors are radially positioned, and the stator and rotor cores are of disc type []. The special features of an axial flux machine, such as its planar and adjustable air-gap, better power to weight and diameter to length ratio, compact construction and better efficiency, especially in a machine with a large number of poles, makes it an attractive alternative over conventional machines.

AFM and new discoid topologies are now being used in electric vehicle propulsion systems, mostly as wheel-direct coupling motors, replacing a mechanical gearbox by an electric one and attach the load machinery directly on the motor shaft, removing the losses brought by the mechanical gearbox and giving a full speed control to the drive.

AF induction motors, moreover, have been used in applications where short axial length geometry is advantageous. Indeed, for an electric vehicle drive motor, it is often required that it combines high power and torque density and efficiency with a light structure; this geometry achieves this with advantages in terms of size and increased economy in the use of raw materials. A solid-rotor induction motor is also claimed to offer advantages in terms of mechanical strength over conventional cage-induction motors especially when an elevated rotational speed is needed.

This geometry has its drawbacks, however, especially due to the difficulty associated with the production of the magnetic cores, given the axial force acting on the rotor and bearings caused by one-sided magnetic pull. Powder composites, however, offer advantages in the construction and man-
ufacture of the magnetic cores, easing the construction of build-up structures, otherwise difficult to build from traditional materials, though they are not as efficient as magnetic strips or sheet. They contribute to fringing by allowing the main flux in stator cores to stray in the r-dimension, being isotropic, while their low permeability strengthens these stray parts.

2. Background
An analytic model solving for the magnetic vector potential requires the machine to be reduced to a single homogeneous domain. The equivalent model is constructed in the following way:

a) One single homogeneous zone (the equivalent air-gap);

b) Two thin layers at the boundary points of the domain: one current density layer representing the stator windings and characterized by a parameter $j_{eq}$, and an electrical conducting layer, being characterized by a parameter $\sigma_{eq}$

c) A material of high permeability $\mu_r$ capable of closing the magnetic circuit, passing from the stator to the rotor and back again.

The thin layers described must maintain an equivalence with the real stator windings and rotor disk parameters. This is to say that the total current and Joule losses have to be the same in the real and equivalent machines and therefore a relation must be established between the real 2D current density and the 1D thin layer: the equivalent rotor layer conductivity and stator layer current density are as follows:

$$\sigma_{eq} = \sigma_{al} h_{al}$$ \hspace{1cm} (1)

$$j_s = \frac{I_n}{S_c} h_{sc} \frac{\theta}{2\pi}$$ \hspace{1cm} (2)

where $I_n$ is the rated current, $S_c$ is the cross-section of the conductor, $\theta$ is the angular displacement of the coil between two stator teeth, $\sigma_{al}$ is the aluminum electrical conductivity and $h_{al}$ and $h_{sc}$ are the thickness of the aluminum disk and copper coils, respectively. This approximation is possible because the air-gap in the real model is very small compared with the diameter and thickness of the electrical machine; in the equivalent model, however, the aluminum height adds to the air-gap, forming the magnetic air gap, as this material has non-magnetic characteristics. Because the flux lines do not cross the copper windings they are not accounted for in the equivalent air-gap length:

$$\delta_{eq} = \delta + h_{al}$$ \hspace{1cm} (3)

In order to obtain a relationship between the parameters $\sigma_{eq}$ and $j_s$ and the magnetic vector potential in the equivalent air-gap, Ampres law is used. As there is no current density in the air gap, Ampres law is written as:

$$\nabla \times \mathbf{H} = 0$$ \hspace{1cm} (4)

Recalling the magnetic flux density as the curl of the potential vector $\mathbf{A}$:

$$\nabla \times \mathbf{A} = B_z + B_\theta = \frac{\partial A_r}{\partial z} \hat{e}_\theta - \frac{1}{r} \frac{\partial A_r}{\partial \theta} \hat{e}_z$$ \hspace{1cm} (5)

and taking again the curl in the previous expression, the equation of a wave is obtained:

$$\nabla \times (\nabla \times \mathbf{A}) = \left( -\frac{1}{r^2} \frac{\partial^2 A_r}{\partial \theta^2} - \frac{\partial^2 A_r}{\partial z^2} \right) = 0$$ \hspace{1cm} (6)

This equation is only valid on the air gap, where the curl of the magnetic flux density, i.e., the current density, is equal to zero. As the fields are traveling fields and have harmonic content, it is necessary to define the magnetic vector potential as a function of its harmonics, so the $n^{th}$ harmonic of the magnetic vector potential $\mathbf{A}$ can be written as:

$$\mathbf{A}_n(\theta,t) = A_n(z) e^{i(n\theta - \omega t)} \hat{e}_r$$ \hspace{1cm} (7)

where the subscript $n$ denotes the harmonic rank and $k$ denotes the number of pair of poles of the machine. Substituting in the wave equation it is possible to write each harmonic term as a sum of a forward and backward travelling wave, where $C_1$ and $C_2$ are constants to be found for each harmonic:

$$A_n(z) = C_1 e^{\frac{kn}{r}} + C_2 e^{-\frac{kn}{r}}$$ \hspace{1cm} (8)

These constants are obtained through two boundary conditions. The first boundary condition is in the stator, where the equivalent stator current density $j_s$ is related with the magnetic field component $H_\theta$ via the closed path $C_s$; similarly, Ampres law is used in the rotor path $C_r$, relating the magnetic field with the equivalent induced current density $j_i$. This is represented schematically in figure (1):

Figure 1: Ampre’s law application in the stator and in the rotor
The induced current density \( j_i \) can be defined in terms of the electromotive force and the magnetic field, both function of the potential vector \( A_r \):

\[
j_i = \sigma \varepsilon_0 (E + \mathbf{v} \times \mathbf{B}_z)
\]

and substituting the electric field and the rotor velocity by the following equations:

\[
E = -\frac{dA}{dt} \mathbf{e}_r
\]

\[
\mathbf{v} = \tau \omega_r \mathbf{e}_\theta
\]

the induced current can be written as:

\[
j_i = -j \sigma \varepsilon_0 \mathbf{A}_r \cdot (\omega - \omega_r n k) \mathbf{e}_r
\]

Equation (13) and (14) result from using Ampere’s law in the two boundaries defined in figure (1), where \( z_0 \) and \( z_{\text{rotor}} \) denote, respectively, the \( z \)-coordinate of the thin current layer on the top of the stator and the height of the thin electrical conducting material layer on the bottom of the rotor.

\[
\mathbf{H}_\theta(z = z_{\text{rotor}}) = j_i
\]

\[
\mathbf{H}_\theta(z = z_0) = -j_s
\]

It is now necessary to write the magnetic field as a function of the potential vector, which is done using the equation above:

\[
\mathbf{H}_\theta = \frac{1}{\mu} (\nabla \times \mathbf{A}) \cdot \mathbf{e}_\theta = \frac{1}{\mu \sigma} \frac{\partial A_r}{\partial z}
\]

The model can be simplified setting \( z_0 = 0 \). The model is unchanged by making the stator top as the origin of the \( z \)-axis, since only the relative distance between the two layers is relevant for the calculations:

\[
-\mu \sigma j_s = \frac{kn}{r} (C_1 - C_2)
\]

Solving now for the two constants it is possible to obtain:

\[
C_1 = \frac{\mu_0 j_s r}{K} \left( 1 - j \frac{\mu_0 \sigma \varepsilon_0 S}{K} \right) e^{2Kz_{\text{rotor}}} \left( 1 + j \frac{\mu_0 \sigma \varepsilon_0 S}{K} \right)
\]

\[
C_2 = \frac{\mu_0 j_s r}{K} \left( 1 + j \frac{\mu_0 \sigma \varepsilon_0 S}{K} \right) e^{2Kz_{\text{rotor}}} \left( 1 - j \frac{\mu_0 \sigma \varepsilon_0 S}{K} \right)
\]

where

\[
K = kn
\]

\[
S = (\omega - \omega_r K)
\]

Since \( K \) depends on the harmonic rank, each harmonic will have its set of \( C_1 \) and \( C_2 \) constants. Substituting these constants for each harmonic rank in equation (8) yields the magnetic vector potential for that rank. It is possible to decompose the spatial distribution of the traveling current density using the FFT and from it describe this signal as a sum of its relevant harmonic components traveling either in the positive or negative direction. Equation (8) states that the value of the magnetic vector potential for each harmonic depends only on these two values and on the \( z \)-value. At the boundaries, the potential vector is equal to:

\[
A_n(z = 0) = C_1 + C_2
\]

\[
A_n(z = z_{\text{rotor}}) = C_1 e^{Kz_{\text{rotor}}} + C_2 e^{-Kz_{\text{rotor}}}
\]

Having the vector potential defined at the boundary points, it is now possible to obtain the induced current density. The machine torque is calculated by integrating the force density on the volume of the rotor conductive thin layer. To compute the total force it is necessary to integrate the force density over the rotor area:

\[
d\mathbf{F} = \mathbf{j}_i \times \mathbf{B}_z
\]

\[
F = \int_0^{2\pi} \int_0^{r_{\text{avg}}} dF dr d\theta = 2\pi r_{\text{avg}} dF
\]

\[
T = rF = 2\pi r_{\text{avg}}^2 dF
\]

To estimate the efficiency of the machine, the output power is first defined based on the developed torque and the Joule losses in both the stator and rotor disk are computed. No iron losses are considered in this model:

\[
P = T \omega_r
\]

\[
P_j = R_s I_s^2 + \int_0^{2\pi} \int_0^{\frac{2\pi}{2\sigma_{\text{al}}} \frac{dl}{dh}} \int_0^{\frac{j_s}{2\sigma_{\text{al}}}} \frac{1}{dh} dF d\theta
\]

Equation (27) represents the Joule losses both in the stator windings and in the rotor disk. This last quantity is calculated integrating the power density over the rotor volume.

\[
\eta = \frac{P}{P + P_j}
\]
3. Implementation

Following the design proposed on [1], the task was to build a circular bottom part with an external radius of 85 mm and an internal radius of 35 mm from a soft magnetic composite (SMC) material called Somaloy; this material is sold in wafer-shaped pieces with a fixed height of 20 mm and a fixed radius of 60 mm. This meant that it was necessary to cut and glue several pieces in order to obtain the required shape; however, it was decided from the start that only straight cuts were to be made, so the circular arc of the wafer was used.

A high number of cut wafers glued together would make the process expensive as well as create a number of air gaps inside the stator base, increasing the magnetic reluctance; using a high number of wafers, however, has the advantage of better fitting the original design. Four models were developed, with 5, 6, 8 and 12 parts, respectively.

A comparison between the circular area and the area obtained using the cut wafers is done in table (1), where the extra area error $\epsilon$ is also calculated and it can be seen that, as expected, $\epsilon$ decreases as $n$ increases.

<table>
<thead>
<tr>
<th>Model</th>
<th>Extra stator core area (%)</th>
<th>Needed wafers</th>
<th>$\Delta R_m$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>31</td>
<td>9</td>
<td>21.3</td>
</tr>
<tr>
<td>6</td>
<td>17.9</td>
<td>9</td>
<td>25.6</td>
</tr>
<tr>
<td>8</td>
<td>8.9</td>
<td>8</td>
<td>34.1</td>
</tr>
<tr>
<td>12</td>
<td>5.4</td>
<td>9</td>
<td>51.1</td>
</tr>
</tbody>
</table>

Table 1: Cost and magnetic reluctance comparison of the four models

From the results of table (2), the following conclusions can be made: i) the 8-part model is the cheapest although by a small margin; ii) increasing the number of glued parts decreases the extra stator core area, which has a small influence on the motors geometry and an increase in its weight; on the other hand, increasing $n$ also has the negative effect of adding extra magnetic reluctances. As the second objective has a much bigger influence in the machines performance than the first, the 5-part model was chosen. The type a and b CAD designs are represented in figure (2) and two complete views of the final stator design are shown in figure (3).

![Figure 2: Type (a) and type (b) parts](image1)

![Figure 3: Two views of the stator final design](image2)

The final design, complete with a shaft to connect the induction machine to a DC machine acting as load, three windings with $N=66$ turns, a stainless steel vertical support and a pillow block bearing is shown in figure (4).

![Figure 4: Final assembly](image3)

3.1. Results

In order to obtain both the machines nominal current and its thermal time constant, a temperature
test was performed with the rotor blocked. As no information was available on the maximum temperature the insulation band is capable of withstanding, a threshold of 90°C was established for these tests.

The temperature inside the stator coils was measured using a logger thermometer. The temperature curve over time for the stator coil closer to the rotor when a 10 A rms AC current with a frequency \( f = 50 \text{ Hz} \) was applied to the three phases is shown in figure (5). The test was interrupted at the temperature threshold as the temperature was not yet in its steady-state value. A second test was immediately performed: the motor was left to chill by natural convention with no stator currents applied. In this case, the steady-state temperature is equal to the room temperature \( T_{\text{room}} = 18^\circ \text{C} \).

By varying the steady state temperature value and the thermal time constant it is possible to fit a theoretical curve to the experimental data; however, it was not possible to obtain a small error between the two curves using only one time constant. A new exponential term was added; this has allowed for a better fit (the residual, defined as the sum of the squared difference between experimental data and the theoretical curve, decreased from \( 10^5 \) to 20). Two thermal time constants may indicate that the heat propagation is not isotropic, but is composed instead of two directions, radial and axial, with different heat transfer coefficients.

One reason why the one time constant fit is not satisfactory for the experimental data is the relatively short time of the experiment, compared with the thermal time constants of the motor. Assuming now that the curve with two time constants is a good representation of the temperature dynamic of the stator coil, an exponential with only one time constant was fitted to that function, evaluated for a much longer time than in the experiment. This is shown in figure (6), where it can be seen that one time constant fairly approximates the real temperature curve, with a maximum error of 10°C and a steady state error of less than 5°C. The thermal constant of the machine, using the one thermal time constant model, is then \( \tau = 50 \text{ mins} \).

Given the relatively high thermal constant of the machine, it was considered that even though the steady-state temperature is above the threshold temperature, the machine will never operate more than two hours, so the applied current can be defined as the maximum current.

3.2. No load and blocked rotor tests
The equivalent circuit parameters of the machine can be obtained performing two tests: a blocked rotor test and a no-load test. In the blocked rotor test, an iron bar blocks the rotor movement, causing the slip to be equal to 1, resulting in the following expression for the total impedance seen from the stator:

\[
R_{\text{im}} + jX_{\text{im}} = R_s + jX_s + (jX_m/(R_r' + jX_r'))
\]

where the two slashes denote a parallel of two impedances. The variable resistance \( R_r' \) is here at its minimum, therefore, as the magnetizing impedance is expected to be much higher than the rotor branch this expression takes the following approximated form:

\[
R_{\text{im}} + jX_{\text{im}} = R_s + R_r' + j(X_s + X_r)
\]

meaning that the magnetizing branch can be neglected. Figure (6) (a) shows the voltage and current waveforms taken in such conditions.

![Figure 5: Temperature dynamics: heating (a) and cooling (b)](image)

![Figure 6: Voltage and current waveforms for the blocked rotor (a) and no load (b) tests](image)
Calculating the total impedance and taking the real and imaginary part yields the following results: 
\[ R_s + R'_r = 0.77 \Omega \] and \[ X_s + X_r = 2.76 \Omega \]. As the stator resistance is known \( R_s = 0.31 \Omega \), the rotor resistance is obtained from the equation above and is equal to \( R'_r = 0.46 \Omega \). Since neither the stator or the rotor reactances are known, these quantities cannot be decoupled from the total reactance value.

In order to perform the no-load test, the machine has to attain a speed near the synchronous speed. This is not the case with this machine, which is unable to develop a torque bigger than the static torque for high speeds. To perform the no-load test, therefore, additional torque provided by a DC machine was necessary to make the rotor disk accelerate near synchronous speed. The machines topology changes now in the following way: as the slip approaches zero, the rotor resistance increases as
\[ \lim_{s \to 0} \frac{R'_r}{s} \to \infty \]  
so the rotor current is approximately zero. The result of this experiment is shown in figure (6) (b). The impedance seen from the stator is then:
\[ R_{im} + jX_{im} = R_s + j(X_s + X_m) \]  
and again taking the impedance imaginary part the following relation can be obtained: \[ X_s + X_m = 3.24 \Omega \].

The results obtained show that the magnetizing inductance is not, as previously expected, much bigger than the other inductances. This explains the high currents drawn from the motor even at the no-load test. It also shows that the approximation that the magnetizing impedance is bigger than the rotor branch, made in equation (30), is not valid. The typical tests and their common assumptions, therefore, revealed not to be a valid way of characterizing the tested machines equivalent electrical circuit parameters. Further in this chapter a means of obtaining these parameters is developed.

3.3. Testing the induction machine symmetry
A test was conducted to see if the magnetic circuit is symmetrical: feeding only one phase, an induced voltage is expected in the two other phases and it is given by equation (??). The magnetic circuit is considered symmetrical if:
\[ M_{ik} = M_{ki} = M, i, k := 1 : 3 \]  
or, equivalently,
\[ e_{ik} = e_{ki} = e, i, k := 1 : 3 \]  
for the same excitation current \( i_k \). From the results of this experiment, it can be seen that the induced voltage is not the same in all three phases for the same current value. This result shows that the stator leakage inductance is not the same for all the phases and that the machine will be working on an unbalanced condition. The machine is not entirely symmetrical, both due to a variation of the winding position in the z-dimension and the presence of a stator core on only one side.

In order to work in a more balanced condition, each phase is separately fed by a single phase transformer with the same current value. This has shown to improve the machine’s performance, making it achieve higher speeds as the torque ripple decreases.

3.4. DC machine characterization
A DC generator can be used to measure the torque developed by the induction machine: since the angular acceleration on the rotor disk is equal to zero in steady state operation, the sum of torques acting on the rotor disk has to be zero, so a relationship can be established between the developed torques in the two machines:
\[ T_{dc} = k\phi i_a \]  
\[ T_{im} = -(k\phi i_a + T_s + T_{s,\omega}) \]  
where \( k\phi = f(i_f) \). It is possible to characterize the static torque, here defined as:
\[ T_f = T_s + T_{s,\omega} \]  
by allowing only the DC machine to turn the rotor assembly, implying that
\[ T_{im} = 0 \]  
and that
\[ T_f = -k\phi (i_f) i_a \]

By defining \( K = k\phi \), it is possible to obtain a relationship between the DC motor speed and the electromotive force and represent its variation alongside the field current.

This relationship was obtained for a number of field current values by measuring the motor speed at various armature voltage values. The electromotive force was calculated using equation (??), with an armature resistance value \( R_a = 5.3 \Omega \). For each field current value, a linear regression was performed to obtain the line slope which represents the constant \( K \) with dimension \([V/\text{rad s}^{-1}]\).

Since the rated field current is close to the saturation point of the magnetization curve, the machine is expected to saturate for current values bigger than the rated value. This is represented in figure (7), where \( K \) is plotted as a function of the field current.

The rated field current for this DC machine is equal to 0.37A, and as expected, the DC machine
seems to enter the saturation zone for a value of field current bigger than 0.4\text{A}. The rated field current is close to the saturation point on the magnetization curve since this ensures that the flux/field current ratio is the highest, allowing for the maximum power to weight ratio out of a machine. From this point on, a fairly large increase in field current will be necessary in order to obtain only a small increase in the induced voltage.

Figure 7 represents a plot of the developed torque versus machine speed for all the acquired points, showing a dependency of the static torque with speed. A linear regression allows to describe the static torque as a constant value plus a speed variable term:

\[
T_f = 0.16 + 8.378 \times 10^{-5} n
\]  
(40)

where \( n \) denotes rotational speed in rotations per minute.

Figure 9: Torque/speed characteristic for low speed values

To obtain an estimation of the developed torque and efficiency for high speeds, a new set up is used where both the induction machine and the DC machine are working as motors, and where they only have to overcome the friction torque. Since the friction torque dependency with rotational speed is known, the torque developed by the induction machine can be obtained for a number of shaft speed points. This is represented in figure (9). As the friction torque alone is bigger than the induction machine torque at high speed, this experience can only acquire data for low speed points. The field current, moreover, was kept at a minimum, as the opposing DC machine torque is proportional to \( K \).

Hence, by changing the armature resistance and by adding the friction torque to the calculated DC generator torque, the value of the induction machine induced torque can be obtained for a number of shaft speed points. This is represented in figure (9). As the friction torque alone is bigger than the induction machine torque at high speed, this experience can only acquire data for low speed points. The field current, moreover, was kept at a minimum, as the opposing DC machine torque is proportional to \( K \).

Figure 8: Measured static torque as a function of rotational speed

3.5. Induction machine under load

The induction machine developed torque can now be obtained from the DC machine armature current and the value of \( K \) computed from the field current characteristic for each operating point. When the DC machine is used as a generator and the induction machine is used as a motor, spinning the assembly shaft, the DC machine electromotive force is proportional to the shaft speed and this constant is equal to \( K(i_f) \), determined in the previous section.

The armature circuit can either be left open, meaning that the induction machine only has to overcome the friction torque or an additional resistance can be added between the armature terminals. As this resistance is lowered, the armature current for a given shaft speed starts to increase; this in turn increases the developed torque by the DC machine, opposing the developed torque by the induction motor and decreasing the shaft speed. This means that a decrease in the armature resistance leads to a new operation point with higher torque and lower speed.

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\[ P_{out} = T \omega_m \]  

(41)

from where its efficiency can be calculated as:

\[ \eta = \frac{P_{out}}{P_{in}} \]  

(42)

where the input power is the sum of the three phases real power. The efficiency curve is shown in figure (12). It can be seen that the calculated torque does not follow accurately the theoretical model; this might be due to miscalibration of the material used to obtain the DC developed torque. The error propagates into the efficiency plot as well, as the torque value is used as a measure of the output power.

3.6. Parameter fitting
The data acquired in the last experiment, where both the DC and the induction machine are working as motors, is used as input data for a fitting equation alongside with the equations which describe the equivalent electrical circuit of the induction machine. This data consists of the stator voltage and currents, as well as the active and reactive power, obtained using a FLUKE power analyzer. The \textit{lsqcurvefit} function, a trust region algorithm already defined in section (??), can estimate the parameters of the equivalent electrical circuit by giving it both the input data and the equations that the data must fit.

Three equations are given which describe the topology of the equivalent circuit: an outward mesh comprising the stator and rotor branches and an inward mesh comprising the magnetizing and the rotor branches, where all the voltage drops sum to zero, and an implicit equation, which allows the magnetizing current to be written as the difference between stator and rotor current. Finally, two equations relate the input real and reactive power with the power consumed by each lumped parameter.

This function can be made to return the value of the squared norm of the residual at the solution point, so if several solutions returned by \textit{lsqcurvefit} are obtained by changing the initial value vector \( x_0 \) the squared norm of the residual is used to retain the vector \( x \) which creates the estimation with the smallest deviation to the input.

From the structure of the equations above, \( y_{\text{data}} \) is a \((i \times 7)\) null matrix, where \( i \) denotes the number of experimental data points. The trust-region-reflective algorithm used by the trust region algorithm requires the number of equations (the row dimension of \( f \)) to be equal or greater than the number of variables, i.e., it does not solve undetermined systems.

Both the stator and the rotor resistances change their values as their temperature increases due to Joule losses. Therefore, the electric resistances of the equivalent circuit change from one data point to another, unlike the remaining parameters that do not change with temperature. This means that \textit{lsqcurvefit} will require two extra factors \( \beta_s \) and \( \beta_r \) defined as:

\[ \beta_s = 1 + \alpha_{Cu}(T_{sk} - T_0) \]  

(43)
\( \beta_r = 1 + \alpha_{AI}(T_{rk} - T_0) \)  
\( (44) \)

where \( \alpha_{Cu} = 0.00404 \) and \( \alpha_{AI} = 0.0039 \) denote, respectively, the copper and aluminum temperature coefficients of resistance and where \( T_0 = 20^\circ \text{C} \) is considered the reference temperature for the equivalent circuit. Both the average temperature of the three coils and the rotor is known, using a thermometer, so the stator and rotor resistances used in the \( f \) function fed to the program are now modified to:

\[
R_s^* = \beta_s R_s
\]

\( (45) \)

\[
R_r^* = \beta_r R_r
\]

\( (46) \)

The input data given to the program \( x_{\text{data}} \) is a matrix with \( i \) rows and 5 columns; the \( k \)th row of this matrix is: \( \{s_k, V_{n_k}, I_{n_k} [e^{-j\phi_k}, R_s, \beta_s]\} \), where \( s_k \) is the slip on the \( k \)th point. As only the slip of the machine is changed in this test, the frequency is kept constant. This program then returns the parameter vector \( \{\omega L_s, R_r, \omega L_r, \omega L_m, R_c\} \) with the lowest residual value from a set of initial points. The parameters obtained are used to calculate an estimation of the input data and a comparison between the two is made in terms of stator current, real and reactive power. These are presented in figures (13) and (14), where the fit is shown to be satisfactory. Finally, the equivalent circuit of the induction machine is shown in figure (??).

\[ \alpha = \frac{T}{T_0} \]

\( (47) \)

Typical stator winding temperatures range from \( \alpha = 1 \) to \( \alpha = 4 \) during the motor operation. The experimental temperature curve of the rotor disk follows approximately the stator coil temperature curve, but with a positive offset of 20C. This allows for the calculation of the equivalent stator and rotor resistances for each value of \( \alpha \). The influence of the operating motor temperature on the torque speed characteristic and on the efficiency of the motor are shown in figures (15) and (16), obtained by calculating these values using \( \alpha R_s \) and \( \alpha R_r \). These figures show that another negative consequence of a low magnetizing impedance is the strong temperature influence on the developed torque and thus on the efficiency.

These figures are closer to the expected by the theoretical model, reinforcing the idea that the torque measurements are not precise enough. The parameter estimation does not use the torque values, but only the input power and the losses on the stator resistance. The rated torque, efficiency and current values for the machine can now be obtained.
from the maximum efficiency point and are shown in table (4)

3.7. V/f experiment
A similar experiment to that of section (3.5) was made, where the torque value for different speeds was obtained by changing the armature resistance of dc machine working as a generator. In figure (17) the result of section (3.5) is shown alongside two new curves, obtained with a V/f ratio of 0.8 and 1, respectively and where it can be seen that for low speeds the maximum torque value increases with a higher flux. As the flux has a quadratic relation to the number of stator turns, this figure shows that increasing $N$ would result in a better performance.

<table>
<thead>
<tr>
<th></th>
<th>Rated value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n$</td>
<td>2000 rpm</td>
</tr>
<tr>
<td>$\eta$</td>
<td>0.25</td>
</tr>
<tr>
<td>$T$</td>
<td>0.222</td>
</tr>
<tr>
<td>$I_s$</td>
<td>10.1 A</td>
</tr>
</tbody>
</table>

Table 4: Induction machine rated values

4. Conclusions
In this master thesis, the construction of a axial flux machine was carried out. This included the design of both stator core and teeth, which were then glued together, the design of a 3D housing bearing part, the design of a shaft, the construction and assembly of the stator windings and the construction of a workbench with a DC machine for running tests. There were some deviations in terms with the previous design, mostly made because of cost and time limitations, such as the use of an iron plate for the rotor magnetic part instead of replicating the stator design and a significant reduction of the number of turns in the stator windings.

Following the motor construction, a number of experiments testing for the relative permeability of the soft composite material, thermal analysis to find the maximum current and a electromechanical characterization of the machine using a DC machine as load was carried out. As the typical tests for finding the equivalent electrical circuit of the machine failed did not give accurate results, a method using a trust-region algorithm was developed and a satisfactory fit to the experimental data was achieved. Finally, changing the V/f ratio has proven that the machine can work with higher flux levels without saturating. A higher number of winding turns can then be used in further applications of this geometry.

Partly due to the high air-gap length, the magnetizing inductance is quite low. This has consequences in the maximum torque value and maximum efficiency, which have proven to be unsatisfactory.