Static and Dynamic Analysis of Building Structures

Models with 3 Degrees of Freedom Per Story

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Abstract

The motivation for this dissertation comes from the desire to create a piece of software that enables structural analysis to be performed in early design stages, namely conceptual design. The goal is to write a code that can perform both static and dynamic analysis in an expeditious manner; capturing the most essential components of the behavior of structures, while not being overly complex, in order to extend the space of possibilities for any given project, which could help in the efficient design of buildings.

Keywords: Structural analysis of Buildings, Stress Resultant Interaction, Wall modelling, static condensation, sub-structures.

1- Introduction

In the project of a building, the phase of conceptual design is the part with most impact in the outcome. Every decision taken can severely influence aspects of functionality, economy and safety. However, not many the choices regarding the structural part of the project are taken during this phase [1]. This hinders the potential of any project and can happen for a series of reasons, one of them being the lack of flexible software which can be used alongside designing teams during the aforementioned conceptual design phase, which has a very heuristic nature.

The present work seeks to implement a piece of software which bridges the gap between the conceptual and structural design by allowing several analyses to be carried out without consuming too much time or resources, so that the structural design can accompany the conceptual design as it evolves and consider all possibilities discussed in real time, instead of doing in-depth analyses once much of the decisions regarding the conceptual design have already been taken.

2- Formulation and Description of the Models

The first main simplification was to assume that the building which are modelled will have three main degrees of freedom per floor: two translations and one rotation. The motivation behind this decision was that the dynamic behavior was one of the main aspects which were intended to analyze. Its validity hinges on the assumption that slabs will have a diaphragm behavior.

The second main simplification was to create three types of sub-structures which can be used as building blocks to create a more complicated structure. Thus, all possible structures are thought of as being a combination of these three sub-structures: 2D Frames, 3D Frames and Core walls. An example of a model assembled from these three types of sub-structure is given in Figure 1:

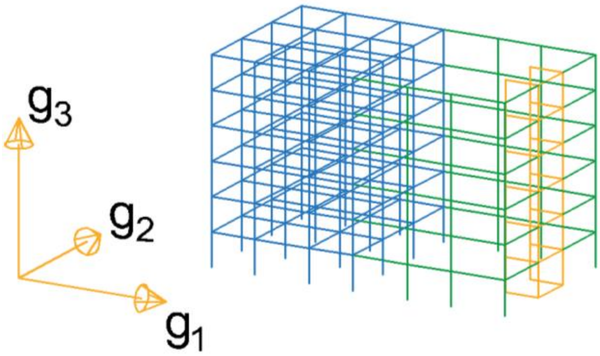


Figure 1 – Example of a model assembled from the three types of sub-structures considered.

In order to define these sub-structures, a global reference frame is considered, with respect to which the 3 degrees of freedom per story are defined, as well as the position of all sub-structures.

The interaction between the several sub-structures needs to be taken into account, and as such a distinction is made between the degrees of freedom of every sub-structure:

- "Slave" degrees of freedom, which are dependent directly on the three global degrees of freedom per story, meaning they can be written as a combination of the global displacements of the floor they belong to. These "Slave" degrees of freedom consist of horizontal displacements and rotations in horizontal planes and will be referred to as $\boldsymbol{q}_s$.
- "Local" degrees of freedom, which do not depend directly on these three global degrees of freedom, and as such can only be written directly in terms of the other displacements in the structure. These are the degrees of freedom associated to vertical displacements and rotations in non-horizontal planes, written as $\boldsymbol{q}_l$.

With this distinction in mind, the method of static condensation is utilized in order to isolate the terms that correspond to "Slave" degrees of freedom in the static system of equations of each sub-structure. This is achieved by making a partition on the system of equations:

$$\begin{bmatrix} \boldsymbol{K}_{ss} & \boldsymbol{K}_{sl} \\ \boldsymbol{K}_{ls} & \boldsymbol{K}_{ll} \end{bmatrix} \begin{bmatrix} \boldsymbol{q}_s \\ \boldsymbol{q}_l \end{bmatrix} = \begin{bmatrix} \boldsymbol{f}_s \\ \boldsymbol{f}_l \end{bmatrix}$$

One can solve the second line of the previous system for $\boldsymbol{q}_l$ and then introduce it into the first line:

$$\boldsymbol{q}_l = \boldsymbol{K}_{ll}^{-1} \boldsymbol{f}_l - \boldsymbol{K}_{ll}^{-1} \boldsymbol{K}_{ls} \boldsymbol{q}_s \qquad \Rightarrow \qquad (\boldsymbol{K}_{ss} - \boldsymbol{K}_{sl} \boldsymbol{K}_{ll}^{-1} \boldsymbol{K}_{ls})\, \boldsymbol{q}_s = \boldsymbol{f}_s - \boldsymbol{K}_{sl} \boldsymbol{K}_{ll}^{-1} \boldsymbol{f}_l$$

Thus arriving to a modified stiffness matrix and force vector ($\boldsymbol{K}_{ss}^*$ and $\boldsymbol{f}_s^*$), defining a system which depends only on $\boldsymbol{q}_s$:

$$K_{ss}^{*} = K_{ss} - K_{sl}\, K_{ll}^{-1}\, K_{ls} \qquad ; \qquad f_s^{*} = f_s - K_{sl}\, K_{ll}^{-1}\, f_l$$

This is a new static system of equations which depends only on the “Slave” displacements. Now, since these can be written in terms of the global degrees of freedom by means of a matrix A_{sg}, this system can be transformed into one which depends only on the global displacements:

$$K_{gg} = A_{sg}^{T}\, K_{ss}^{*}\, A_{sg} \qquad ; \qquad f_g = A_{sg}^{T}\, f_s^{*}$$

This gives a system of equations for the i^{th} substructure of the form:

$$K_{gg}^{(i)}\, q_g = f_g^{(i)}$$

This equation gives the stiffness matrix and the force vector with respect to the global degrees of freedom, which are common for all sub-structures, and as such, the matrices and vectors need only be summed in order to assemble a system which describes the whole structure.

3- Sub-structures

2D Frames are conceived as structures which act only on their plane, meaning they have no stiffness for displacements in other directions, while 3D Frames are a generalization of the previous concept, with all 6 degrees of freedom in each node, and elements are modelled with a full 3D element stiffness matrix.

Core Walls are thought of as being composed of walls which act like linear elements with stiffness associated only to displacements within their plane. Its displacements q_p can be written in terms of the displacements of the four nodes it connects.

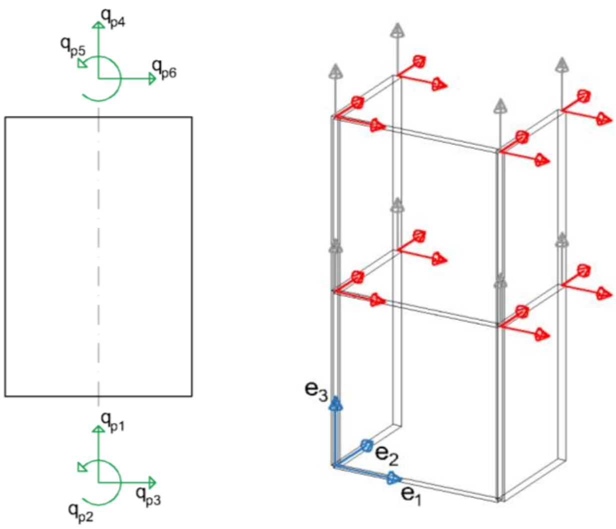


Figure 2 – Displacements a wall, and distinction between “Slave” and “Local” degrees of freedom in a wall

As such, there exists a matrix which relates the displacements of the wall as a linear element, $\boldsymbol{q}_p$, with the displacements of its nodes, $\boldsymbol{q}_e^w$:

$$\boldsymbol{q}_p = \boldsymbol{A}_w \, \boldsymbol{q}_e^w$$

This gives a way to write the stiffness matrix of the wall (the same as a 2D frame element), in terms of those nodal displacements:

$$\boldsymbol{K}_e^w = \boldsymbol{A}_w^T \, \boldsymbol{K}_p \, \boldsymbol{A}_w$$

From here, the stiffness matrix and force vector of the whole core wall sub-structure can be assembled in the standard manner.

Some configurations of core walls might exhibit unwanted behaviors when modelled in this way, such as an I-section core. The reason for this is that all nodes are considered as being independent of each other. Thus, in order to consider the “web” wall of the I-section and its connection to the “flange” walls, these have to be separated in two, which leads to some problems with the modelling when the wall is subject to axial stress:

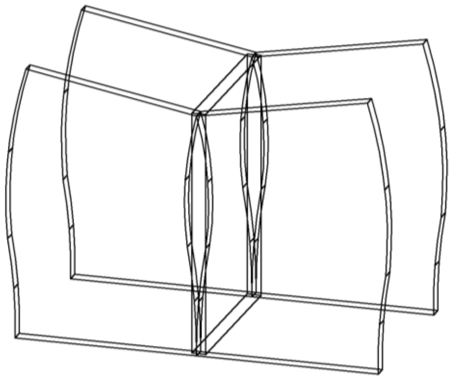


Figure 3 – Example of an unwanted effect in the regular wall model

This problem was overcome by the introduction of a new type of wall which will be referred to as a “Connected” wall. This type of wall has the property of having its nodal displacements dependent on those of the walls that it connects to, and that dependence is defined in terms of the distance of the connecting node to each of the nodes in the wall.

4- Analysis

The static analysis is done by solving the system of equations which was assembled from those of the sub-structures, with respect to the global degrees of freedom. All nodal displacements in the model are recovered using the relations between displacements defined previously for every sub-structure.

The dynamic analysis is performed by following the methods prescribed in Eurocode 8 [2]. Namely, a response spectrum is defined, and resorting to the spectral acceleration and participation factors of each mode, the modal forces are computed and then combined using the Complete Quadratic Combination (CQC). This is done for the effects of an earthquake in both directions, and the results are then combined using the Square Root of the Sum of Squares combination (SRSS).

Having found the combined stress resultants, and with the aim of evaluating the amount of reinforcement needed for each element, an interaction diagram taken from [3] is used. However, in order to implement it in the code, it needs to be discretized. This can be done by considering several families of planar equations, which approximate the curves in the diagram. These equations are of the form:

$$\omega = \vec{x} \cdot \vec{t} + C$$

with $\vec{x}$ being a vector in the μ- ν plane; and $\vec{t}$ and C being a vector and a scalar which define each family of equations. In the following figure, two such equations are shown:

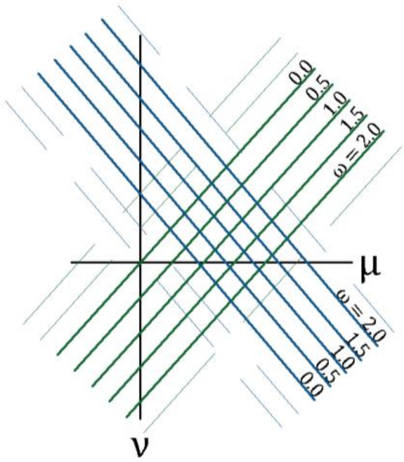


Figure 4 – Discretization of an interaction diagram using two families of equations

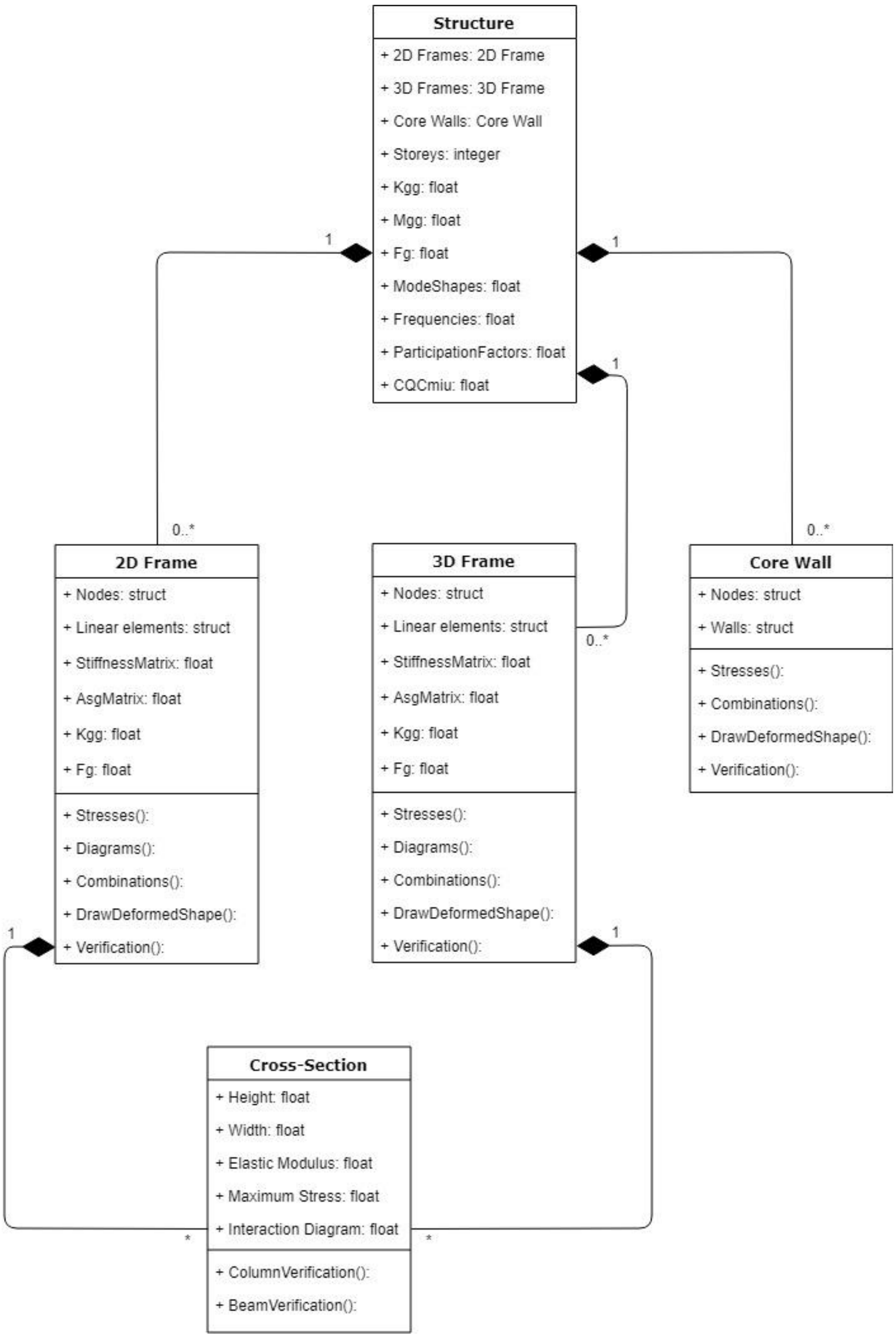
To find the value of ω, one simply needs to take the maximum result among all families of planes. This is particularly helpful in assisting an approximate design which takes into account the correlation of stress resultants.

5- Implementation

In order to implement the discussed methods as a computer program, the language chosen was MATLAB, and an object-oriented approach was taken. This means that several classes of data were created, namely (i) Cross-section Class, (ii) 2D Frame Class, (iii) 3D Frame Class, (iv) Core Wall Class and (v) Structure Class.

These allow to have specific procedures defined for each class, which perform operations on the data they contain, and can interact with other classes. This approach was chosen mainly due to the nature of the problem at hand, since it easier to develop each sub-structure separately, and then create routines which can gather their information and make them interact. In particular, it leads to a more flexible and updatable program, since each class is sealed off from the rest of the program, making it feasible in the future to introduce optimizations or even new classes which can interact with the previous ones.

The following class diagram presents a visualization of the concepts just introduced:



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6- Tests and analysis of results

In order to get a first glimpse of the performance of the program, some simple models were tested.

Firstly, a regular building was modelled both as a 3D Frame, and then as a combination of 2D Frames, so that their analyses could be compared:

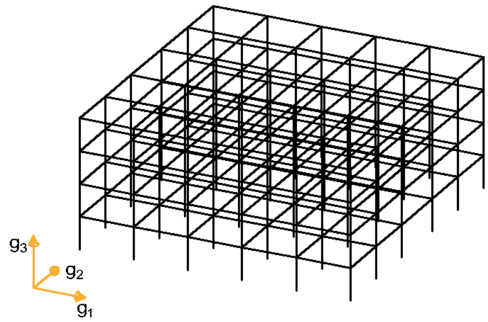

Figure 6 – Regular building being modelled

The dynamic analysis achieved very similar results for both models, whereas the static analysis gave significantly different figures. This was due to the fact that in the model assembled from 2D frames, the columns are double counted since they appear in one frame in each direction. This had the effect of halving the axial stress in each column, as well as separating the two components of bending moment, each being captured by one of the two frames which contained said column. This separation of moments was not apparent in the dynamic analysis since the effects of the earthquake were already divided in the two possible directions.

This problem was solved by adding a procedure to the code which identifies columns from different 2D frames as being the same whenever they have the same coordinates. From there, the results were almost equal, which suggested that most regular structures might be able to be modelled solely as a combination of 2D frames.

Afterwards, an I-section core wall with 6 stories of square slabs was modelled using both the regular and “Connected” walls formulations, and their results were compared against a continuous vertical beam model.

When considering the modes along the direction of least inertia of the I-section, the “Connected” walls and the continuous beam models gave identical results, with the regular walls approach showing some deviation from the values of the other two. In particular, it had lower frequencies, which was expected due to its lower stiffness.

When considering the modes acting in the direction of highest inertia, some differences were registered between the three models with the “Connected” walls being slightly closer to the continuous beam model than the regular walls were.

As for the rotational modes, there were very clear differences between all three models. As expected, the “Connected” walls model had higher frequencies than the regular walls, while the frequencies of the continuous beam model were considerably smaller. This was attributed to the fact that the torsion of a core is not well modelled by a regular beam, since it doesn’t account for effects such as restrained warping.

Finally, with the intent of doing a more ambitious analysis, an irregular building was modelled as being composed of 2D frames:

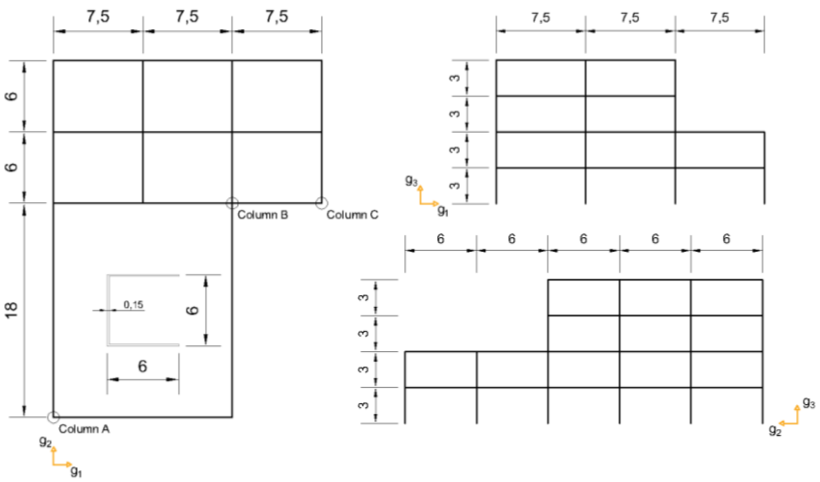

Figure 7 – Dimensions of the irregular building considered.

Exhibiting both vertical and plan irregularities, and not having large cross-sections, the values of stresses and reinforcement were understandably high in some elements, especially in the corner columns and those closer to the irregular zones:

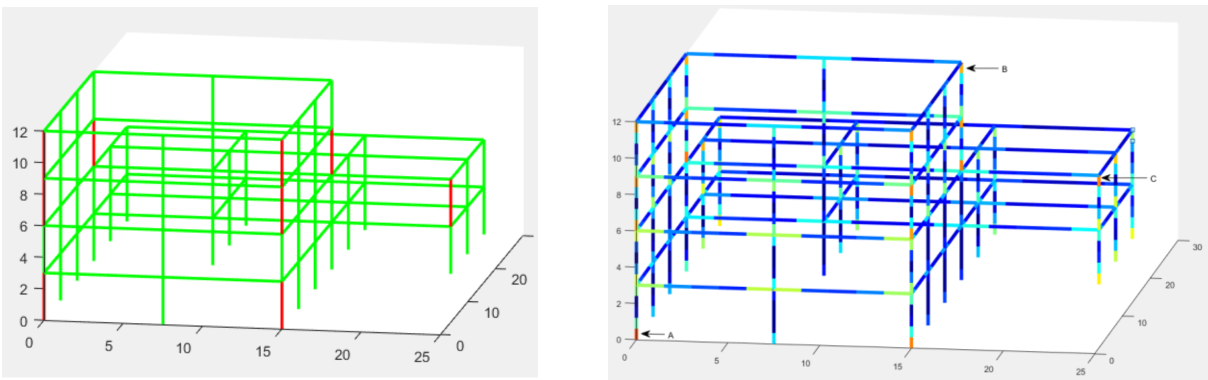

Figure 8 – Output plots showing the over-stressed elements in red on the left, and the levels of mechanical reinforcement ratio on the right

In a second stage, a U-Section core wall was added to the model, which had two very evident repercussions: the values of stresses in the 2D frames were greatly reduced, and the frequencies increased significantly. These effects were indeed expected due to the higher stiffness of the core when compared to the 2D frames.

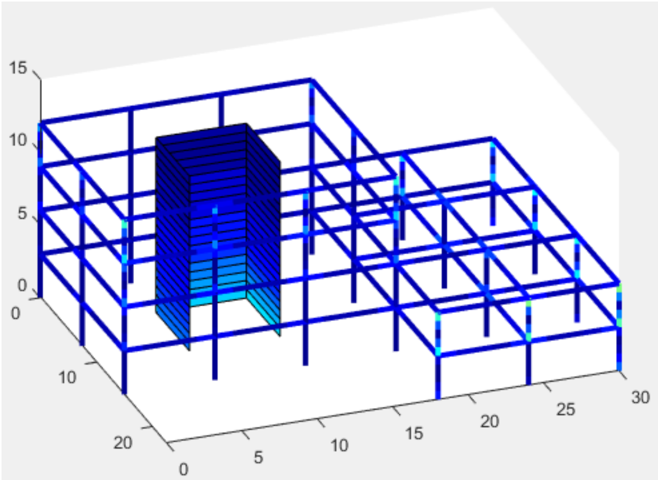

Figure 9 – Plot showing the amount of mechanical reinforcement ratio after adding a core wall

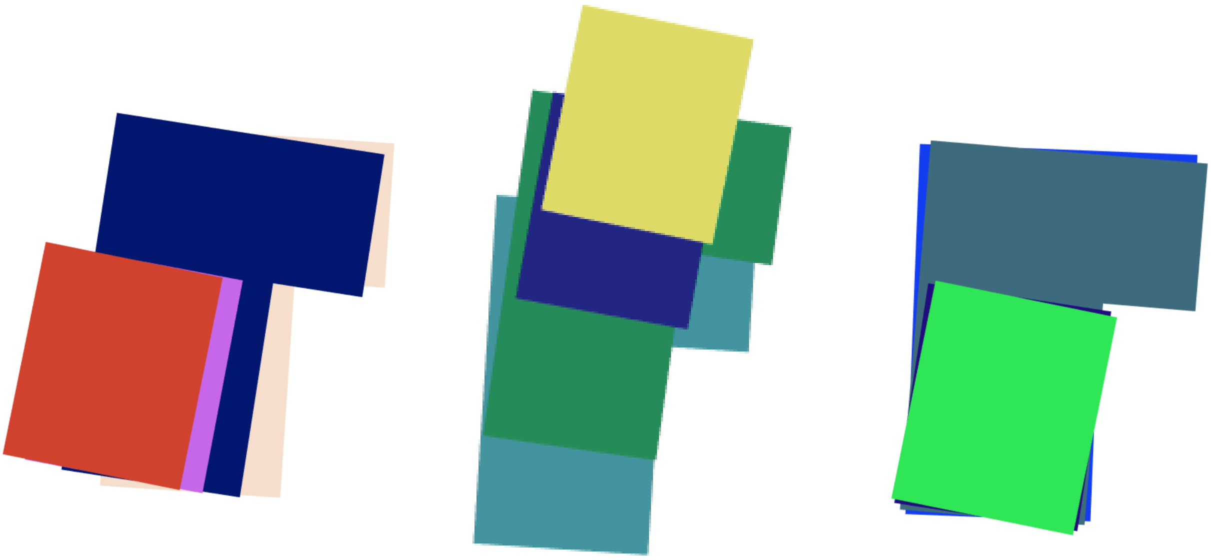

Figure 10 – Screen capture showing the animations of the modal displacements for three different modes

7- Conclusion

The objective of the dissertation was to implement a program which could do a simple but reliable analysis of the structural behavior of a given model, making use of a few key simplifications.

The concept of three degrees of freedom per story proved to be a reasonable simplification, since most of the dynamic effects were captured in the models which were tested. The division into sub-structures was an important decision as well, since it allowed for an intuitive way of conceiving and modelling a structure, which can often be the most complicated part when working with a structural analysis software. This was an aspect which was much sought-after, since the program is intended to be used as an aid to conceptual design.

A method of modelling walls was introduced, which despite having a very simple formulation, was able to model some complex behaviors, such as restrained warping. This helped to enlarge the possibilities for the modelling, thus making the program applicable to a much wider variety of situations.

The method for discretizing the axial force/bending moment interaction diagrams greatly improved the capabilities of the software, which when equipped with it was able to perform an evaluation of over-stressed elements in a swift manner. This was one of the main points of the work, since this type of information may not be immediately obvious when conceiving a building, yet it can greatly influence the outcome of the project later on.

The introduction of graphic output in the form of plots was also quite helpful, since it allows for information to be conveyed in a much more direct fashion than by outputting arrays or tables with numeric values. This also adds to the ease of use of the program, which was the main overarching principle throughout its conception.

Bibliography

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