Design of transverse stiffeners of high strength steel plate girders

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Thesis to obtain the Master of Science Degree in Civil Engineering

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May 2019
DECLARATION

I declare that this document is an original work of my own authorship and that it fulfills all the requirements of the Code of Conduct and Good Practices of the Universidade de Lisboa.
ACKNOWLEDGEMENT

Throughout the writing of this dissertation I have received a great deal of support and assistance.

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Pedro Santos
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RESUMO

Os tabuleiros em laje vigada de pontes metálicas e mistas aço-betão são usualmente projetados utilizando reforços transversais, que permitem explorar a importante reserva de resistência pós-critica dos painéis de alma. Após a encurvadura do painel, as tensões de compressão permanecem constantes, enquanto que as tensões de tração continuam a aumentar, equilibradas por compressões verticais no reforço transversal.

Foi desenvolvido um estudo numérico de vigas de secção soldada de aço de alta resistência com reforços transversais, baseado numa análise de elementos finitos não linear, em que as dimensões de reforços retangulares simples e duplos são variadas. Este estudo avalia o comportamento dos reforços transversais para diferentes configurações das imperfeições iniciais e compara as forças de compressão obtidas no reforço com a compressão de dimensionamento do EC 3-1-5, que se verifica ser muito conservativa para o dimensionamento dos reforços.

Para avaliar a contribuição para o momento fletor no reforço causado pelo carregamento lateral provocado pela encurvadura da alma, o reforço é substituído por apoios flexíveis. Desta forma obtem-se configuração e intensidade do carregamento lateral quando o reforço funciona como apoio efetivo da alma. Esta configuração também é importante para avaliar a importância relativa da resistência e da rigidez de flexão no dimensionamento do reforço.

É proposto um método elástico simples para o dimensionamento dos reforços transversais de vigas de secção soldada de aço de alta resistência com base no seu comportamento e nas forças obtidas através da análise de elementos finitos.

Palavras-chave
Aço de alta resistência; Vigas de secção soldada; Reforços transversais; Resistência pós-critica; Análise não-linear
ABSTRACT

Steel and steel-concrete composite plate girder bridge decks are commonly designed with intermediate transverse stiffeners. They divide the web in small panels increasing their buckling resistance, although their most important role is the contribution to the post-buckling resistance. After the plate buckles, the compressive stresses remain constant, while the tensile stresses develop rapidly due to the vertical anchorage provided by the transverse stiffener.

A numerical study is developed for high strength steel plate girders with transverse stiffeners, based on a non-linear finite element analysis, where the dimensions of single and double-sided rectangular stiffeners are varied. This study evaluates the behaviour of the intermediate transverse stiffeners for different initial imperfections of the plate girder and compares the compression forces obtained in the stiffener with the recognized very conservative EC 3-1-5 design method.

To investigate the contribution to the bending moment in the stiffener caused by the lateral loading due to the buckled web, the stiffeners are replaced by a continuous lateral elastic support. With this set-up it is possible to analyse the shape and magnitude of the lateral loading to which the stiffener is subject in order to effectively sub-divide the web panels. This set-up is also helpful to evaluate which of strength or flexural rigidity is the most important characteristic for transverse stiffener design.

A new elastic method for the design of the intermediate transverse stiffeners is developed, less conservative than the actual code, based on their behaviour and loads obtained from the finite element analysis.

Keywords

High strength steel; Plate girders; Transverse stiffeners; Post-buckling resistance; Non-linear analysis
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NOTATIONS

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<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>amp</td>
<td>Maximum amplitude of the initial geometric imperfection</td>
</tr>
<tr>
<td>FE</td>
<td>Finite element</td>
</tr>
<tr>
<td>HSS</td>
<td>High strength steel</td>
</tr>
<tr>
<td>SA</td>
<td>Stiffener A</td>
</tr>
<tr>
<td>SB</td>
<td>Stiffener B</td>
</tr>
<tr>
<td>S355</td>
<td>Steel with yield strength of 355 MPa</td>
</tr>
<tr>
<td>S690</td>
<td>Steel with yield strength of 690 MPa</td>
</tr>
</tbody>
</table>

Capital Latin letters

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{f1}$</td>
<td>Cross-sectional area of the compression flange</td>
</tr>
<tr>
<td>$A_{f2}$</td>
<td>Cross-sectional area of the tension flange</td>
</tr>
<tr>
<td>$A_{st}$</td>
<td>Effective cross-sectional area of the transverse stiffener</td>
</tr>
<tr>
<td>$C_w$</td>
<td>Equivalent second moment of area for a “T” type stiffener</td>
</tr>
<tr>
<td>$E$</td>
<td>Modulus of elasticity of structural steel, taken as 210 GPa</td>
</tr>
<tr>
<td>$F$</td>
<td>Applied load</td>
</tr>
<tr>
<td>$G$</td>
<td>Shear modulus of structural steel</td>
</tr>
<tr>
<td>$I_{st}$</td>
<td>Moment inertia of the effective cross section of the stiffener</td>
</tr>
<tr>
<td>$M_{CG}$</td>
<td>Moment in the stiffener caused by the asymmetry of the stiffener</td>
</tr>
<tr>
<td>$M_{Ed}$</td>
<td>Design bending moment</td>
</tr>
<tr>
<td>$M_{f,k}$</td>
<td>Characteristic moment resistance of the flanges</td>
</tr>
<tr>
<td>$M_{f,Rd}$</td>
<td>Design moment resistance of the flanges</td>
</tr>
<tr>
<td>$M_{LL}$</td>
<td>Moment in the stiffener caused by the lateral loading</td>
</tr>
<tr>
<td>$M_{st}$</td>
<td>Moment in the stiffener</td>
</tr>
<tr>
<td>$M_{δ}$</td>
<td>Moment caused by second order effects</td>
</tr>
<tr>
<td>$N_{Ed}$</td>
<td>Design axial force</td>
</tr>
<tr>
<td>$V_{k,Rd}$</td>
<td>Design resistance for shear</td>
</tr>
<tr>
<td>$V_{bf,Rd}$</td>
<td>Contribution of the flanges for the design resistance for shear</td>
</tr>
<tr>
<td>$V_{bw,Rd}$</td>
<td>Contribution of the web for the design resistance for shear</td>
</tr>
<tr>
<td>$V_{cr}$</td>
<td>Shear buckling resistance</td>
</tr>
<tr>
<td>$V_{Ed}$</td>
<td>Design shear force</td>
</tr>
<tr>
<td>$V_{plw,Rd}$</td>
<td>Plastic design shear resistance</td>
</tr>
</tbody>
</table>
### Small Latin letters

- $a$: Length of the stiffened web plate
- $a_1$: Length of the stiffened web plate $A_1$
- $a_2$: Length of the stiffened web plate $A_2$
- $b_1$: Length of the stiffened web plate $B_1$
- $b_2$: Length of the stiffened web plate $B_2$
- $b_{eff}$: Effective width
- $b_f$: Flange width
- $b_{ft}$: Tension flange width
- $b_{fc}$: Compression flange width
- $b_{st}$: Width of the one outstand of the stiffener
- $c$: Distance between the plastic hinges in the flanges
- $e_0$: Initial eccentricity of the stiffener
- $e_{CG}$: Eccentricity of the centre of gravity of the stiffener in relation to the web plate
- $f_y$: Yield strength
- $f_{yf}$: Yield strength of the flange
- $f_{yw}$: Yield strength of the web
- $h_w$: Clear web depth between flanges
- $k_t$: Plate buckling coefficient for shear stresses
- $q_{eq}$: Equivalent uniformly distributed lateral loading
- $t_f$: Thickness of the flange
- $t_{st}$: Thickness of the outstand of the stiffener
- $t_w$: Thickness of the web plate

### Small Greek letters

- $\alpha$: Ratio of compression force in the stiffener to shear in the web panel
- $\delta$: Lateral deflection of the stiffener
- $\delta_{max}$: Maximum admissible lateral deflection
- $\gamma_{M0}$: Partial safety factor for the resistance of cross sections
- $\gamma_{M1}$: Partial safety factor for structural member subject to instability
- $\varepsilon$: Factor $\varepsilon = \sqrt{\frac{235}{f_y}}$
- $\sigma_{cr}$: Elastic critical buckling stress
- $\sigma_y$: Yield strength
- $\lambda_w$: Reduced slenderness web plate
- $\chi_w$: Reduction factor for shear
- $\tau_{CR}$: Elastic critical shear buckling stress
- $\omega_0$: Initial eccentricity of the stiffener assumed for design
1 INTRODUCTION

1.1 Motivation

Webs of steel and steel-concrete composite bridge plate girder decks contribute mainly to the shear resistance. When a web panel is loaded with a shear force below its buckling resistance, the principal tensile and compressive stresses are of equal magnitude, as seen in Figure 1.1 a). It has been shown that after the web panel buckles the response is non-linear: the compressive stresses do not increase significantly, while the tensile stresses develop rapidly due to the membrane effect shown by the plates (Figure 1.1 b)). This vertical component of the diagonal tensile stresses is anchored in the edges of the web panel by the flanges and by the transverse stiffeners, while the horizontal component is easily supported by the web and flanges. The ultimate resistance is reached when the diagonal web tensile stresses reach the material yielding strength.

The plate girders with slender webs ($\lambda_w \geq 1.5$) exhibit a significant post-buckling resistance and in order to take advantage of this resistance capacity, the transverse stiffeners need to constitute a rigid support to the panels. When high strength steel (HSS) is adopted the plate girders’ webs are slenderer. Therefore, the post-buckling resistance up to plastic failure is higher and the transverse stiffeners are submitted to higher compressive forces. It is commonly accepted that the present form to design stiffeners much overestimates their design forces. So, at least when HSS is adopted, less conservative rules are welcomed. The motivation for this work is therefore to understand the complex stiffened web behaviour up to failure and to develop a variant design proposal for the intermediate transverse stiffeners, less conservative than the actual design criteria present in EC 3-1-5 1 [1], by evaluating the axial and lateral loading caused the membrane stress field during the post-buckling phase. This subject has been studied first by Basler [2], in the sixties of last century and since then by many researchers during the last thirty years [3-20]. Still, no studies have been conducted specifically for the case of stiffeners of high strength steel plate girders, where recent works have found the rules of EC 3-1-5 very conservative [21-26].

![Figure 1.1 – Simply supported plate loaded in shear](image)
1.2 Main objective

The main objective of this dissertation is to perform a numerical study, based on a geometrically and materially non-linear FE analysis, to evaluate the behaviour and internal forces in the intermediate transverse stiffeners up to failure. The internal forces in the stiffeners will be compared to the EC 3-1-5 design loads, and the design method will be discussed. In the following, a new methodology for the design of the intermediate transverse stiffeners is investigated, based on their behaviour and real internal forces obtained from the FE analysis, less conservative than the actual code design procedure.

1.3 Structure of the dissertation

Following this Introduction chapter, the dissertation is organized as follow:

- Chapter 2 summarizes the design of the web panels to shear and the design of the intermediate transverse stiffeners according to EC 3-1-5.
- In Chapter 3, the FE modelling is calibrated with laboratory tests that involved the two failure mechanisms related to shear involving the intermediate transverse stiffeners. Different initial imperfections of the web and transverse stiffeners are considered to investigate the sensibility of the plate girder to this issue.
- In Chapter 4 a numerical study is performed, based on a non-linear finite element analysis, where the dimensions of the intermediate stiffeners are varied. The behaviour and loads in the transverse stiffeners are evaluated for different initial imperfections of the plate girder and the results are compared with the current design loads according to EC 3-1-5.
- In Chapter 5, the intermediate transverse stiffeners are replaced by a continuous lateral elastic support in order to investigate the effect of the bending moment in the stiffener, due to the lateral loading created after the web panel buckles.
- In Chapter 6 a new method to design the intermediate transverse stiffeners is developed, based on a verification of strength and rigidity, where the bending moment in the stiffener is simulated by an equivalent uniformly distributed lateral load.
- Chapter 7 closes this work with some final thoughts and conclusions.

Two annexes are also included:

- Annex A presents some tables summarizing the numerical study developed in Chapter 4 and it also includes the calculation of the forces in the stiffener according to EC 3-1-5.
- Annex B includes the axial and bending moment diagrams, as well as the deflection of the stiffener along its height to assist the analysis of the results presented in Annex A.
2 DESIGN OF INTERMEDIATE TRANSVERSE STIFFENERS ACCORDING TO EC 3-1-5

2.1 Resistance to shear

According to EC 3-1-5 [1], the design resistance for shear of stiffened or unstiffened webs $V_{b,\text{rd}}$, is taken as the sum of the contribution of the web $V_{bw,\text{rd}}$ and the contribution of the flanges $V_{bf,\text{rd}}$, but is limited to the plastic resistance of the web, $V_{plw,\text{rd}}$ (Eq. (2.1)) [30, 31],

$$V_{b,\text{rd}} = V_{bt,\text{rd}} + V_{bf,\text{rd}} \leq V_{plw,\text{rd}}$$

where the plastic resistance of the web is defined by:

$$V_{plw,\text{rd}} = \eta \frac{f_{yw} h_w t_w}{\sqrt{3} \gamma_{M1}}$$

$V_{plw,\text{rd}}$ depends on the steel grade adopted in the web, because the EC 3-1-5 allows to take into account the hardening of the steel indirectly by using the coefficient $\eta$, as follows

$$\eta = 1.0 \text{ to } 1.2 \text{ if } f_y \leq 460 \text{ MPa}$$

$$\eta = 1.0 \text{ if } f_y > 460 \text{ MPa}$$

- Contribution from the web

Based on the model proposed by Hoglund [20], the contribution from the web considers both the elastic and post-buckling resistance and it is given by Eq. (2.5),

$$V_{bw,\text{rd}} = \chi_w f_{yw} h_w t_w \sqrt{3} \gamma_{M1}$$

where the shear buckling factor $\chi_w$ depends on the web slenderness $\tilde{\lambda}_w$ and on the type of end support being used. Table 2.1 shows the expressions used to calculate $\chi_w$ and their plot is shown in Figure 2.1. For $\tilde{\lambda}_w < 1.08$ the curves have the same reduction factor. Only for $\tilde{\lambda}_w \geq 1.08$ is the reduction factor different, moreover it depends on the type of end support. The webs with a rigid end support have a greater shear capacity (higher $\chi_w$) in comparison to the webs with a non-rigid end post.

<table>
<thead>
<tr>
<th>$\tilde{\lambda}_w$</th>
<th>Rigid end post</th>
<th>Non-rigid end post</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0.76 \frac{f_{yw}}{\tau_{cr}}$</td>
<td>$\eta$</td>
<td>$\eta$</td>
</tr>
<tr>
<td>$\tilde{\lambda}_w &lt; 0.83/\eta$</td>
<td>$\eta$</td>
<td>$\eta$</td>
</tr>
<tr>
<td>$0.83/\eta \leq \tilde{\lambda}_w &lt; 1.08$</td>
<td>$0.83/\tilde{\lambda}_w$</td>
<td>$0.83/\tilde{\lambda}_w$</td>
</tr>
<tr>
<td>$\tilde{\lambda}_w \geq 1.08$</td>
<td>$1.37/(0.7 + \tilde{\lambda}_w)$</td>
<td>$0.83/\tilde{\lambda}_w$</td>
</tr>
</tbody>
</table>
• **Contribution from the flanges**

The contribution from the flanges is usually small when compared to the resistance provided by the web. This contribution is associated to the formation of plastic hinges in the flanges, at the section where the diagonal tensile stress field is anchored, as shown in Figure 2.2.

![Figure 2.2 - Failure mechanism involving the plastic hinges in the flanges](image)

Still based on the model proposed by Hoglund [20], the contribution from the flanges $V_{bf,rd}$ is given by:

$$V_{bf,rd} = \frac{b_f \cdot t_f^2}{c} \cdot \frac{f_{yf}}{f_{M1}} \left[ 1 - \left( \frac{M_{Ed}}{M_{f,rd}} \right)^2 \right]$$  \hspace{1cm} (2.6)

where $c$ is the distance represented in Figure 2.2 and is calculated by Eq. (2.7)

$$c = a \cdot \left( 0.25 + \frac{1.6 \cdot b_f \cdot t_f^2 \cdot f_{yf}}{t_w \cdot h_w^2 \cdot f_{yw}} \right)$$  \hspace{1cm} (2.7)

and $M_{f,rd}$ is the moment resisted by the effective cross-section of the flanges; in the presence of a longitudinal axial force $N_{Ed}$ its value should be reduced, and it is given by Eq. (2.8).
\[ M_{f,rd} = \frac{M_{f,k}}{\gamma_M} \left( 1 - \frac{N_{Ed}}{(A_{f1} + A_{f2}) \cdot \frac{f_{fy}}{\gamma_M}} \right) \]  

(2.8)

## 2.2 Buckling resistance

The critical shear force of a web panel can be calculated by Eq. (2.9).

\[ V_{cr} = \tau_{cr} \cdot h_w \cdot t_w \]  

(2.9)

The elastic critical plate buckling stress, \( \tau_{cr} \), is obtained by:

\[ \tau_{cr} = k_\tau \cdot \sigma_E \]  

(2.10)

where \( \sigma_E \) is given by Eq. (2.11)

\[ \sigma_E = \frac{\pi^2 E t_w^2}{12(1 - v^2) h_w^2} = 190 \, 000 \left( \frac{t_w}{h_w} \right)^2 [\text{MPa}] \]  

(2.11)

and \( k_\tau \) is the buckling coefficient that depends on the aspect ratio of the web panel and is evaluated assuming the edges are simply supported (Eq. (2.12) and (2.13)).

\[ k_\tau = 4.00 + 5.34 \left( \frac{h_w}{a} \right)^2 \text{ if } a/h_w < 1 \]  

(2.12)

\[ k_\tau = 5.34 + 4.00 \left( \frac{h_w}{a} \right)^2 \text{ if } a/h_w \geq 1 \]  

(2.13)

This assumption is conservative because the flanges and transverse stiffeners provide some rigidity, not allowing the free rotation of the web panels’ edges. In Figure 2.3 is represented the variation of \( k_\tau \) with the aspect ratio of the web panel. Only for \( a/h_w \) below 3 does the increase in the buckling coefficient start to be noticeable in relation to a web panel with infinite length \( a \). This means that significant increases in \( V_{cr} \) are only possible by designing web plates with a small aspect ratio or thicker panels. However, a higher \( V_{cr} \) is not a key feature since the EC 3-1-5 allows to take advantage of the post-buckling capacity of the web panel, which is a considerable part of the resistance to shear, especially for slender webs made of HSS, as seen in Figure 2.1.

![Figure 2.3 – Variation of the buckling coefficient with the aspect ratio of the web panel (adapted from [28, 30])](image-url)
2.3 Design of the intermediate transverse stiffeners

The design and safety verification of intermediate transverse stiffeners is based on the fulfilment of two requirements: a) rigidity and b) resistance.

2.3.1 Rigidity requirement

In order to provide a rigid lateral support for the web panel the transverse stiffeners should have a minimum flexural rigidity. The EC 3-1-5 assumes that there is an effective width of the web acting together with the outstand of the stiffener. The effective cross section of the stiffener is represented in Figure 2.4 and it considers that a width equal to \(15 \varepsilon t_w\), for each side of the outstand, is effectively acting with the stiffener. However, no more than the actual dimension available should be taken into consideration to avoid any overlap of contributing parts of adjacent stiffeners. This effective cross-section should be considered for the verifications of rigidity and strength [1, 30-32].

![Double-sided stiffener](image1)

Double-sided stiffener

![Single-sided stiffener](image2)

Single-sided stiffener

Figure 2.4 - Effective cross section of the stiffener according to EC 3-1-5

The minimum rigidity requirement is frequently easily checked with the stiffeners usually adopted in design practice. It depends on the aspect ratio of the panel and is given by Eq. (2.14) and (2.15).

\[
I_{st} \geq 1.5 \cdot h_w^3 \cdot \frac{t_w^3}{a^2} \quad \text{if} \quad a/h_w < \sqrt{2} \quad (2.14)
\]

\[
I_{st} \geq 0.75 \cdot h_w \cdot t_w^3 \quad \text{if} \quad a/h_w \geq \sqrt{2} \quad (2.15)
\]

Figure 2.5 represents the minimum moment of inertia requirement graphically. The inertia requirement increases very rapidly for an aspect ratio \(a/h_w\) of the web panel below \(\sqrt{2}\).

2.3.2 Strength requirement

The transverse stiffeners are usually designed assuming an initial sinusoidal imperfection, where the maximum amplitude \(w_0\) is defined in EC 3-1-5 by [1]:

\[
w_0 = \min \left( \frac{h_w}{300} ; \frac{a_1}{300} ; \frac{a_2}{300} \right) \quad (2.11)
\]

where \(a_1\) and \(a_2\) are the width of the web panels adjacent to the stiffener, and \(h_w\) its height.
The EC 3-1-5 assumes that the axial force in the stiffener is only present for the post-buckling phase, and that every increment in shear, after the shear buckling resistance, passes as compression through the stiffener (Figure 2.6 a)). The axial force is the stiffener is then calculated by Eq. (2.12).

\[ N_{Ed} = V_{Ed} - V_{cr} \]  

(2.12)

In case the shear force in the web panels adjacent to the stiffener is different, the check should be performed considering the shear force at the distance of 0.5 \( h_w \) from the edge of the panel with the largest shear force.

EC 3-1-5 allows the use of a buckling length of no less than 0.75 \( h_w \), if both ends are assumed to be fixed laterally and the buckling curve \( c \) should be assumed for the design. A larger buckling length should be used if there are conditions that provide less end restraint to the transverse stiffener.
When single-sided asymmetric stiffeners are used eccentricities should be accounted for, as shown in Figure 2.6 b). The length $e$ is the distance between the centre of the web and the centre of gravity of the effective cross section of the stiffener.

### 2.3.3 Torsional buckling requirement

The local torsional buckling of the stiffener presents a complex behaviour, because the buckling wavelengths of the simply supported plate panels and simply supported stiffeners will usually be different if analysed in separate. However, for compatibility reasons, the wavelengths of the actual stiffened plate must be the same.

To assure no torsional buckling of the transverse stiffener occurs EC 3-1-5 provides the criterion of Eq. (2.13) for stiffeners with open cross sections,

$$\frac{I_p}{I_T} \geq 5.3 \cdot \frac{f_y}{E}$$

where $I_p$ is the polar second moment of area of the stiffener around the edge fixed to the plate and $I_T$ is the Saint Venant torsional constant for the stiffener.

For the case of stiffeners with significant warping stiffness, only the least onerous of the criteria presented in Eq. (2.13) and Eq. (2.14) need to be met.

$$\sigma_{cr} \geq \theta \cdot f_y$$

In Eq. (2.14) $\sigma_{cr}$ is the elastic critical stress for buckling not considering rotational restraint from the plate, given by Eq. (2.15), and $\theta$ is a parameter to ensure class 3 behaviour (the value $\theta = 6$ is recommended in EC 3-1-5). Since the expression presented in Eq. (2.14) is seen as conservative, the replacement of $f_y$ by the real stress installed in the stiffener for the Ultimate Limit State, deserves to be investigated [27].

$$\sigma_{cr} = \frac{1}{I_p} \left( G I_p + \frac{\pi^2 E C_w}{L^2} \right)$$

It is important to stress that this is a conservative verification because both Eq. (2.13) and Eq. (2.14) neglect the rotational restraint provided by the web plate that can be significant if the span of the plate is small. Another important aspect to notice is that these criteria become increasingly more restrictive as the yield strength of the steel adopted increases. For a rectangular stiffener outstand the Eq. (2.13) can be simplified and written as in Eq. (2.16) bellow:

$$\frac{b_{st}}{t_{st}} \sqrt{\frac{f_y}{355}} \leq 10.56$$

Which leads to the maximum admissible ratio $b_{st}/t_{st}$ equal to 10.56 for steel S355, decreasing substantially for HSS S690 to 7.57 [27].
The calibration of the FE modelling is of great importance for the validation of the work presented in the following chapters. The goal of the present chapter is to demonstrate that plate girders can be modelled with a geometrically and materially non-linear FE analysis and yield results similar to the original girders tested in laboratory. Quadrilateral thin shell elements have been used to model the plate girders.

For this calibration it is important to choose tests representative of the stiffeners’ behaviour. With that in mind, three tests from [3] representative of the two types of failure involving the intermediate stiffeners have been chosen. In tests TGV4 and TGV7-2 the failure mechanism involves a web failure while the intermediate stiffeners remain effectively straight. In test TGV8-1 the failure mechanism involves the bowing of an intermediate stiffener. For all the tests chosen, load/deflection plots were included in the paper and these can be compared with the load/deflection plots obtained from the FE analysis to validate the modelling.

### 3.1 Plate girder geometry and material properties

The test girders are simply supported at each end and were designed to be identical, except for the dimensions of the intermediate stiffeners. All stiffeners are continuously welded to the web plate. Tests with double-sided intermediate stiffeners are not considered because their load/deflection plots are not shown in the paper. The plate girder was loaded at mid-span by a single jack with a control system that enables to load the girder to achieve and maintain a chosen mid-span deflection, so it was possible to record both the dynamic and static maximum load capacity. The dimensions of the stiffeners under the jack and at the supports, as well as the gap between the rigid end post stiffeners are unknown, so reasonable dimensions and yield stresses were attributed to ensure a rigid behavior of those structural elements. Information regarding geometry and material properties for the remaining components of the plate girder are well described in the paper and allow the modelling of the tests with reasonable accuracy. Figure 3.1 shows the geometry of the plate girders tested.
Initial geometric imperfections – Although it is mentioned in the paper that the initial deformations of the web plate and the intermediate stiffeners were recorded prior to testing, its shape and amplitude have not been reported in the paper. Therefore, different configurations for initial geometric imperfections were developed to investigate the sensibility of the FE models and are often necessary to help the convergence of the FE analysis by initiating buckling. Initial imperfections were created from trigonometric functions with the intention of giving the web panels and intermediate transverse stiffeners a bow shape, as in Figure 3.2.

Some imperfection shapes only account for web panels imperfection, and not of the intermediate stiffeners, allowing to study the sensibility of the plate girder to the stiffeners’ local imperfection. For the maximum amplitude of the lateral initial bow, the proposed value from EC 3-1-5 Annex C, of $h_w/200$ has been adopted (also proposed in the 2nd revision draft of EC 3-1-5 for the initial bow imperfection of the stiffeners) [1, 29]. All initial imperfections have only one bow along the vertical $y$ axis.

![Figure 3.2 - Initial geometric imperfections considered (magnified by a factor of 25)](image)
Material properties – Every FE model has been analyzed considering a non-linear material behavior. The EC 3-1-5 Annex C.6 recommendation of including strain hardening with a slope of \( E/100 \) for the after-yield stress-strain slope has been followed. The fracture of the material has been assumed to occur for a strain of 0.10. Figure 3.3 presents the stress-strain curve described above.

![Stress-strain curve](image)

Figure 3.3 – Stress-strain curve adopted for the material behaviour

3.2 FE calibration analysis on test TGV4

Both intermediate stiffeners in test TGV4 are single-sided and have different dimensions. Stiffener A possesses outstanding dimensions of 40.28 mm x 4.97 mm and has a yield stress of 422.3 MPa, while stiffener B possesses outstanding dimensions of 29.79 mm x 4.95 mm and yield stress of 283.4 MPa. The remaining characteristics are shown in Table 3.1.

<table>
<thead>
<tr>
<th>( a_1 ) (mm)</th>
<th>( a_2 ) (mm)</th>
<th>( b_1 ) (mm)</th>
<th>( b_2 ) (mm)</th>
<th>( h_w ) (mm)</th>
<th>( t_w ) (mm)</th>
<th>( b_{fl} ) (mm)</th>
<th>( b_{fc} ) (mm)</th>
<th>Web stress yield (MPa)</th>
<th>Flange stress yield (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>595</td>
<td>595</td>
<td>596.5</td>
<td>596.5</td>
<td>598.2</td>
<td>1.97</td>
<td>200.7</td>
<td>200.7</td>
<td>223.6</td>
<td>255.3</td>
</tr>
</tbody>
</table>

The test TGV4 is representative of a plate girder that fails in a classic shear mode, where the intermediate stiffeners can effectively sub-divide the web panel. When the peak load of 203.0 kN is reached the girder keeps increasing its mid-span deflection while the load remains approximately constant. The load/deflection plot from the experimental test girder is shown in Figure 3.4.
Table 3.2 summarises the FE analysis on the plate girder TGV4 for the nine initial imperfections considered. All peak loads were slightly higher than the load of the experimental test girder. The maximum peak load of 210.7 kN (+3.8%) was reached by the FE model with imperfection nº1 and the minimum peak load of 207.5 kN (+2.2%) was reached by the FE model with imperfection nº2. Regardless of the initial imperfection used, the FE model failed in a shear mode and the stiffeners remained effectively straight after the peak load was reached.

Table 3.2 - Failure load and comments on the FE analysis of test TGV4

<table>
<thead>
<tr>
<th>FE model of TGV4 with initial geometric imperfection</th>
<th>Failure Load (kN)</th>
<th>Comments on failure mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>210.7</td>
<td>SA and SB effectively straight, satisfactory post-peak load/deflection curve</td>
</tr>
<tr>
<td>2</td>
<td>207.5</td>
<td>SA and SB effectively straight, satisfactory post-peak load/deflection curve</td>
</tr>
<tr>
<td>3</td>
<td>208.4</td>
<td>SA and SB effectively straight, satisfactory post-peak load/deflection curve</td>
</tr>
<tr>
<td>4</td>
<td>208.4</td>
<td>SA and SB effectively straight, satisfactory post-peak load/deflection curve</td>
</tr>
<tr>
<td>5</td>
<td>208.7</td>
<td>SA and SB effectively straight, satisfactory post-peak load/deflection curve</td>
</tr>
<tr>
<td>6</td>
<td>208.5</td>
<td>SA and SB effectively straight, satisfactory post-peak load/deflection curve</td>
</tr>
<tr>
<td>7</td>
<td>208.6</td>
<td>SA and SB effectively straight, satisfactory post-peak load/deflection curve</td>
</tr>
<tr>
<td>8</td>
<td>208.4</td>
<td>SA and SB effectively straight, satisfactory post-peak load/deflection curve</td>
</tr>
<tr>
<td>9</td>
<td>208.6</td>
<td>SA and SB effectively straight, satisfactory post-peak load/deflection curve</td>
</tr>
</tbody>
</table>

As an example of the behaviour of the FE models of TGV4, Figure 3.5 shows the lateral displacement contours of the FE model with initial imperfection nº1, while under peak load. The transverse stiffeners were able to effectively sub-divide the web panel, allowing them to have an independent behaviour.
In Figure 3.6 the load/deflection curves of the original test girder and the FE models with initial imperfections n°1, 3 and 9 have been plotted for comparison. Until a load of about 175 kN the slopes of the FE models and the slope of the experimental test girder are very similar although for a certain load the test girder has a greater deflection, probably due to an initial slack in the experimental set-up. Near the peak load the experimental test girder is more rigid than the FE models, but these were also able to deflect under and approximately constant load.

3.3 FE calibration analysis on test TGV7-2

The laboratory test TGV7-2 was carried out after test girder TGV7-1, that reached a peak load of 188.0 kN and its failure was due to the bowing of stiffener A, which was then strengthened to allow for a
second test where the goal was to analyse the behaviour of stiffener B. This second test was named TGV7-2 and it is the test modelled in this calibration exercise. There is no information on how the stiffener A was strengthened, so it was modelled with the same dimensions and material characteristics of stiffener B (25.21 mm x 5.10 mm).

By using a FE model where the two intermediate stiffeners have the same dimensions the girder becomes almost symmetrical for the exception of the length of web panels A1-A2 and B1-B2 that are slightly different. The laboratory test TGV7-2 reached a peak load of 211.0 kN. The comments of failure presented in the paper reveal that stiffener B remained effectively straight and the post-peak load/deflection curve was satisfactory. The geometric and material characteristics used for the modelling of test girder TGV7-2 are shown in Table 3.3 (the only difference from the original test girder are the dimensions of stiffener A) [3].

<table>
<thead>
<tr>
<th></th>
<th>a₁ (mm)</th>
<th>a₂ (mm)</th>
<th>b₁ (mm)</th>
<th>b₂ (mm)</th>
<th>hₜ (mm)</th>
<th>tₜ (mm)</th>
<th>bₕt (mm)</th>
<th>bₕc (mm)</th>
<th>Web stress yield (MPa)</th>
<th>Flange stress yield (MPa)</th>
<th>Stiffener stress yield (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>596</td>
<td>596</td>
<td>590.5</td>
<td>590.5</td>
<td>599</td>
<td>1.98</td>
<td>200.7</td>
<td>200.6</td>
<td>221.2</td>
<td>250.3</td>
<td>283.4</td>
</tr>
</tbody>
</table>

The test girder TGV7-2 is representative of a plate girder where the intermediate stiffeners remain effective after the shear failure of the web. After achieving the peak load the plate girder deflects under an approximately constant load. The load/deflection and the residual contour plots of panels B1 and B2 are shown in Figure 3.7.

![Figure 3.7 - Load/deflection and residual contour plots of test TGV7-2 (Extracted from [3])](image)

Table 3.4 shows the peak loads achieved by the FE model TGV7-2 and the comments of failure for its initial imperfections. By comparing the peak loads from the FE analysis and laboratory test girder, it can be concluded that the models were very accurate. The maximum load of the models was reached with the initial geometric imperfection 5 (+0.67%) and the minimum peak load was registered with
imperfection nº3 (211.0 kN), the same peak load of the original test girder. All the FE models with other initial imperfections reached a peak load between 211.0 kN and 212.4 kN. The failure of the FE models was due to shear failure of the web panels, for the exception of the model with imperfection 5, where the stiffener A deflected slightly outwards after reaching the peak load.

Table 3.4 - Failure load and comments of the FE analysis of test TGV7-2

<table>
<thead>
<tr>
<th>FE model of TGV7-2 with initial geometric imperfection</th>
<th>Failure Load (kN)</th>
<th>Comments on the failure mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>212.1</td>
<td>SA and SB effectively straight, satisfactory post-peak load/deflection curve</td>
</tr>
<tr>
<td>2</td>
<td>211.5</td>
<td>SA and SB effectively straight, satisfactory post-peak load/deflection curve</td>
</tr>
<tr>
<td>3</td>
<td>211.0</td>
<td>SA and SB effectively straight, satisfactory post-peak load/deflection curve</td>
</tr>
<tr>
<td>4</td>
<td>212.3</td>
<td>SA and SB effectively straight, satisfactory post-peak load/deflection curve</td>
</tr>
<tr>
<td>5</td>
<td>212.4</td>
<td>SA deflecting, SB effectively straight, slight fall-off in post-peak load/deflection curve</td>
</tr>
<tr>
<td>6</td>
<td>211.1</td>
<td>SA and SB effectively straight, satisfactory post-peak load/deflection curve</td>
</tr>
<tr>
<td>7</td>
<td>212.2</td>
<td>SA and SB effectively straight, satisfactory post-peak load/deflection curve</td>
</tr>
<tr>
<td>8</td>
<td>211.1</td>
<td>SA and SB effectively straight, satisfactory post-peak load/deflection curve</td>
</tr>
<tr>
<td>9</td>
<td>211.6</td>
<td>SA and SB effectively straight, satisfactory post-peak load/deflection curve</td>
</tr>
</tbody>
</table>

Figure 3.8 shows the lateral displacement contours of the FE models with initial geometric imperfections 3 and 5. In Figure 3.8 a) all web panels are working independently because the intermediate stiffeners have remained effective. In Figure 3.8 b) the intermediate stiffeners have also been able to sub-divide the web panels and the model was able to reach a similar maximum load. However, stiffener A shows a larger lateral deflection when compared to the stiffeners in Figure 3.8 a), which led to an increase in bowing after the peak load was reached.

In Figure 3.9 the load/deflection curves of the original test girder and the FE models with initial imperfections nº1, 3 and 5 have been plotted for comparison. It is possible to observe that the laboratory test girder seems to exhibit a higher rigidity and that the FE models started to yield for a lower load. After the peak load, the models with imperfections nº1 and 3 were able to deflect under an approximately constant load, while the model with imperfection nº5 had a faster decrease due to the lateral deflection of stiffener A.
Figure 3.8 - Lateral displacement contours (mm) of FE model TGV7-2.
Displacements in all directions are magnified by a factor of 5

Figure 3.9 - Load/deflection plots of laboratory test TGV7-2 and FE models
3.4 FE calibration analysis on test TGV8-1

Both intermediate stiffeners in test TGV8-1 are single-sided but have different dimensions. Stiffener A possesses outstanding dimensions of 20.50 mm x 3.22 mm and a yield stress of 247.6 MPa and stiffener B possesses outstanding dimensions of 15.95 mm x 5.71 mm and a yield stress of 212.4 MPa. The remaining geometric and material characteristics are presented in Table 3.5.

Table 3.5 - Geometric and material characteristics of TGV8-1 [3]

<table>
<thead>
<tr>
<th>a_1 (mm)</th>
<th>a_2 (mm)</th>
<th>b_1 (mm)</th>
<th>b_2 (mm)</th>
<th>h_w (mm)</th>
<th>t_w (mm)</th>
<th>b_f (mm)</th>
<th>b_r (mm)</th>
<th>web yield stress (MPa)</th>
<th>flange yield stress (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>596</td>
<td>596</td>
<td>590.5</td>
<td>590.5</td>
<td>599</td>
<td>1.98</td>
<td>200.7</td>
<td>200.6</td>
<td>221.2</td>
<td>250.3</td>
</tr>
</tbody>
</table>

The test TGV8-1 is representative of a girder that has a stiffener failure by bowing out of plane. After the peak load of 180.0 kN is reached the load/deflection has a negative slope, that indicates a large loss in rigidity. This loss in rigidity is due to the bowing of the intermediate transverse stiffener A and the panels A1 and A2 tended to function as a single web panel. The load/deflection and the residual contour plots of panels A1 and A2 are shown in Figure 3.10. Although the test TGV8-2 has not been modelled, it is relevant to mention it is the same girder of test TGV8-1 with a strengthened stiffener A, which was tested and reached a peak load of 188.0 kN and the failure was due to the bowing of stiffener B.

In Table 3.6 the failure load and comments of the FE models are shown for each initial geometric imperfection. The FE model with the initial imperfection n°1 was the only model able to find equilibrium beyond the peak load of the test girder, with 184.3 kN (+2.4%). The maximum load reached by the FE models with initial imperfections from n°2 to 9 is below the load from the laboratory test. The minimum peak load was obtained by the model with the initial geometric imperfection n°6, with a load of 172.9 kN (-3.9%). The paper section “Comments of failure” indicates the intermediate stiffener that bowed and if the bowing was towards the web plane (inwards) or towards the opposite side of the web plane.
It is interesting to notice that in some models it was the stiffener A that bowed and in other models it was the stiffener B. This is because the stiffeners have similar geometric properties in both area and inertia, so the stiffener that bows is very dependent of the initial geometric imperfection considered in the model. Another fact that leads to this conclusion is the laboratory tests TGV8-1 and TGV8-2 (failure by bowing of stiffener B) have roughly the same peak loads.

The lateral deflections of the web panels of the FE model TGV8-1 with imperfection n°7 and 9 are illustrated in Figure 3.11. In Figure 3.11 a) the girder has failed by the web plate and stiffener A bowing outwards which led panels A1 and A2 to function as a single web panel. It can also be seen that stiffener B and web panels B1 and B2 are bowing inwards although the lateral displacement magnitude in the web panel, at the section of stiffener B, is about half of the maximum lateral displacement in the web panel, at the section of stiffener A. In Figure 3.11 b) the failure was due to the inward bowing of stiffener B and web panels B1 and B2 tended to function as a single web panel after the peak load was reached.

<table>
<thead>
<tr>
<th>FE model of TGV8-1 with initial geometric imperfection</th>
<th>Failure load (kN)</th>
<th>Comments on the failure modes</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>184.3</td>
<td>SB bowing inwards, fall-off in post-peak load/deflection curve</td>
</tr>
<tr>
<td>2</td>
<td>176.8</td>
<td>SB bowing outwards, fall-off in post-peak load/deflection curve</td>
</tr>
<tr>
<td>3</td>
<td>175.2</td>
<td>SB bowing outwards, fall-off in post-peak load/deflection curve</td>
</tr>
<tr>
<td>4</td>
<td>175.1</td>
<td>SB bowing outwards, fall-off in post-peak load/deflection curve</td>
</tr>
<tr>
<td>5</td>
<td>178.5</td>
<td>SA bowing outwards, fall-off in post-peak load/deflection curve</td>
</tr>
<tr>
<td>6</td>
<td>172.9</td>
<td>SB bowing outwards, fall-off in post-peak load/deflection curve</td>
</tr>
<tr>
<td>7</td>
<td>173.6</td>
<td>SA bowing outwards, fall-off in post-peak load/deflection curve</td>
</tr>
<tr>
<td>8</td>
<td>172.6</td>
<td>SB bowing outwards, fall-off in post-peak load/deflection curve</td>
</tr>
<tr>
<td>9</td>
<td>180.0</td>
<td>SB bowing inwards, fall-off in post-peak load/deflection curve</td>
</tr>
</tbody>
</table>
To compare the laboratory test results of TGV8-1 with the FE models that include the initial geometric imperfections n°7 and 9, the load/deflection curves have been plotted in Figure 3.12. Both FE models show similar results, with the rigidity of the models being very accurate until a mid-span deflection of 2 mm. After the peak load is reached in the FE models, there is a sudden loss in rigidity associated to the bowing of one of the intermediate transverse stiffeners. The negative slope in the curve starts for an earlier deflection when compared to the laboratory test girder.
3.5 Conclusions from the FE model calibration

The use of the FE models has been proven to accurately predict the overall behaviour of the plate girders designed with intermediate transverse stiffeners used in this calibration. Some conclusions from the calibration of the test girders are presented below.

**FE models of TGV4 and TGV7-2:**
- ✓ The FE models exhibited less rigidity near the peak load when compared to the original test;
- ✓ The peak loads obtained from the FE models were very accurate;
- ✓ The initial geometric imperfection n°5 demonstrated a conservative post-peak behaviour;
- ✓ The FE models showed the same type of failure of the original test girder, for the exception of the model with imperfection n°5, that produced a conservative behaviour;
- ✓ The lateral displacement plots show the web panel is well subdivided by the intermediate transverse stiffeners.

**FE models of TGV8-1:**
- ✓ All initial imperfections considered in the FE models of test girder TGV8-1 had a failure mechanism that involved a large bowing shape in one of the intermediate transverse stiffeners, like the original test girder;
- ✓ The peak loads obtained from the FE models were, for most of the initial imperfections, slightly under the maximum load registered in the laboratory test;
- ✓ The FE models exhibited less rigidity after the peak was reached, and therefore conservative behaviour, by beginning the negative slope curve in the load/deflection plots before the laboratory test girder;
- ✓ The lateral displacement plots show large deflections at the intermediate stiffeners section and their ineffectiveness in sub-dividing the buckled web panels.
4 FE PARAMETRIC STUDY OF PLATE GIRDER WITH TRANSVERSE STIFFENERS

The aim of this chapter is to study the behaviour of single and double-sided intermediate transverse stiffeners in plate girders made of steel S355 and HSS S690. From the nine initial imperfections developed in Chapter 3, only imperfections n°1, 3 and 9 have been studied since they proved to be the governing ones. The distribution of the internal forces in the stiffeners and its magnitude, from when the peak load is acting on the girder, are compared with the EC 3-1-5 design loads. The aspect ratio of the web panel has been fixed to $a/h_w = 1$ and the web geometric slenderness to $h_w/200$. The only geometrical difference on the plate girders are the cross-sectional dimensions of the intermediate transverse stiffeners. In all cases, the width to thickness ratio is equal to 10 for stiffeners in steel S355 and 8 for stiffeners in HSS S690, to have always class 3 sections according to EC 3-1-1.

![Plate girder geometry](image)

Figure 4.1 - Plate girder geometry used in the parametric study (all dimensions in mm)

4.1 Criteria for obtaining internal forces

The intermediate transverse stiffener can be divided in two parts: the outstand part and the web part. The outstand part is the steel flange that is welded to the web panel and the web part is a certain width of the web that works together with the outstand due to the rigidity it provides. The EC 3-1-5 defines an
effective cross section of the stiffener $A_{st}$ which is composed by the stiffener’s outstand plus an effective width equal to $15 \varepsilon t_w$ for each side of the outstand, but not more than the actual dimension available. The definition of an effective cross section is of great importance for design proposes. However, because the goal is to study the behaviour of the stiffeners, it is necessary to develop some criteria to evaluate the real width of the web panel acting with the outstand in order to determine the axial force and bending moment diagrams. The definition of the effective cross section is determined by Eq. (4.1), assuming that the force resulting from the maximum stress acting in the effective cross section is equivalent to the force that results from the integration of the stresses in a wider section. Figure 4.2 presents this definition graphically.

$$\int \sigma(s) ds = \sigma_{max} \cdot b_{eff} \quad (4.1)$$

The stiffener is a structural element that has compression stresses due to the anchorage of the vertical component of the inclined tension field generated after the buckling of the web panel. It is expected that in the FE models part of this compression is being supported by the web part of the stiffener. The part of the web that contributes for resisting the internal forces has not been defined as a fixed width along the stiffener’s height, but rather the width with vertical compression stresses next to the outstand of the stiffener. It is important to note that this width may vary along the height of the stiffener and it is not exactly symmetrical for both sides of the outstand. Figure 4.3 illustrates the procedure for the calculation of the width considered to be acting with the stiffener. For an element of the web to be considered acting with the stiffener it must verify two criteria: it needs to be within a length $l$, equal to $h_w/2$, for each side of the stiffener’s outstand and it needs to have a compression vertical stress component.

With these criteria, a constant width is not directly defined; it is rather a consequence of which elements are used. Considering only the elements with compression stress leads to a maximization of the compression axial force installed in the web part and consequently it also maximizes the total compression force in the stiffener. For the evaluation of the axial force in the outstand part of the stiffener, elements with both compressive and tensile vertical stresses have been considered, since the installed
bending moment on the stiffener can overturn the installed uniform compression. The total axial force in the stiffener section at each level is the sum of the force in the web and in the outstand. Individualizing the two components of the axial force is important to understand how the stiffener behaves for the different initial imperfections of the plate girder.

![Diagram of stiffener section](image)

**Figure 4.3** - Definition of the web width acting with the stiffener (only the elements with vertical component of compression are shown in colour)

Some simplifications have also been considered for the calculation of the bending moment. The bending moment should be calculated in the centre of gravity of the stiffener, which is an unknown, since the cross section considered changes along its height. Due to the necessity of having a simple way to calculate the centre of gravity and since one of the goals is to compare the loads obtained in the FE models with the EC 3-1-5 design loads, the centre of gravity has been evaluated based on the effective cross section defined according to EC 3-1-5.

All web FE elements used in the calculation of the bending moment are assumed to have the same $x$ coordinate as the fibre of web in contact with the outstand. This means that every web element in a certain height ($y$ coordinate) contributes with the same lever arm (distance in the $x$ axis) for the bending moment in the stiffener, and thus all FE of the web are assumed to have the same out of plane displacement when the geometrical non-linear effect is used for evaluating the bending moment (Figure 4.4).
It is also important to define the orientation of the stiffener for the calculation of the bending moment diagram. The frame has been oriented in the upwards position, as seen in Figure 4.5, with the positives on the right-hand side and negatives on the left-hand side, meaning that the bending moment diagram is on the tension side of the stiffener.

Since the elements chosen for the calculation of the axial force are the same elements used in the calculation of the bending moment and were selected based on the maximization of the compression force in the stiffener, the consequences of also using those elements for the calculation of the bending moment should be analysed. By only using the web elements with vertical compression stress, the result is a maximization of the positive bending moment diagram for the single-sided stiffeners. It can also be concluded that this procedure for the calculation of the bending moment results in a minimization of the absolute value of the negative bending moment. These approximations will later be proven to be on the safe side, since in the most conditioning cases the stiffener has a positive bending moment diagram.

4.2 Plate girders made of steel S355

4.2.1 Plate girders modelled with double-sided stiffeners

The cross section of a double-sided stiffener is symmetrical in both axes, as opposed to a single-sided stiffener, that only has one axis of symmetry. The centre of gravity of the cross section of a single-sided stiffener does not coincide with the web plane, so when loaded with an axial force it is expected to increase the bending moment in the stiffener. With that in mind, this study starts by evaluating the behaviour of double-side stiffeners, which are not affected by sectional asymmetries.
All the outstands of the stiffeners studied have a rectangular shape and its proportions ensure class 3 sections according to EC 3-1-5. The ratio of width to thickness is equal to 10, which satisfies the minimum requirement for a class 3 section, made of steel S355, under pure compression (Eq. (4.2)).

\[
\frac{c}{t} = \frac{b_{st}}{t_{st}} = \frac{10}{1} < 14\varepsilon = 11.34
\]  

(4.2)

This parametric study consists in making small increments to the dimensions of the stiffener, which result in increasing both its area and moment of inertia, evaluating its behaviour and internal forces. The main variables that are going to be considered are: load carried by the plate girder; the axial force, bending moment and deflection of the stiffener for the increment of the peak load. The axial force and bending moment diagrams are also shown for the increment of the peak load. Another variable is the initial imperfection of the plate girder. For the parametric study only three initial imperfections are studied, which are imperfection n°1, imperfection n°3 and imperfection n°9. These imperfections were chosen based on the initial imperfection they give the stiffener, and therefore governing the design. In imperfection n°1, the stiffener is in a straight position, in imperfection n°3 the stiffener is bowing outwards (positive x direction) and in imperfection n°9 the stiffener is bowing inwards (negative x direction). Imperfections n°3 and 9 yield the same results for double-sided stiffeners, the only differences are in the signal of the bending moment diagram and deflection at mid-height. With that in mind, only imperfections n°1 and 3 are used for the parametric study of double-sided stiffeners.

Table 4.1 shows the outstand dimensions of the stiffeners and its area and inertia which are based on the effective cross-section defined by the EC 3-1-5. When the width of a double-sided stiffener is referred, it is the width of only one of the outstands of the stiffener, \(b_{st}\), as seen in Figure 4.1.

<table>
<thead>
<tr>
<th>(b_{st}) (mm)</th>
<th>(t_{st}) (mm)</th>
<th>(A_{st}) (cm(^2))</th>
<th>(I_{st}) (cm(^4))</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>2</td>
<td>3.1</td>
<td>1.3</td>
</tr>
<tr>
<td>30</td>
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<td>4.1</td>
<td>6.3</td>
</tr>
<tr>
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<td>5.5</td>
<td>19.1</td>
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<td>45.5</td>
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<td>60</td>
<td>6</td>
<td>9.6</td>
<td>93.1</td>
</tr>
<tr>
<td>70</td>
<td>7</td>
<td>12.2</td>
<td>170.6</td>
</tr>
<tr>
<td>80</td>
<td>8</td>
<td>15.2</td>
<td>288.7</td>
</tr>
</tbody>
</table>

For the range tested in the parametric study there are stiffeners that showed to be effective and others that did not show a satisfactory performance. A stiffener with a satisfactory behaviour is one provided with such strength and rigidity that allows for the maximum shear to be reached in the web panels. Initially the peak load in the girder rose significantly with small increments to the dimensions of the stiffener, but after a certain combination of both area and inertia, the maximum load has an increase that can be neglected. An effective stiffener is one that can provide support to the web panel allowing
the plate girder to reach the peak load and does not bow out of the web plane, allowing the girder to deflect under an approximately constant load [3].

The post-peak behaviour of the transverse stiffeners can be studied by plotting the load/deflection curve from the FE models. Figure 4.6 shows the load/deflection plot of the FE model with $b_{st} = 20 \text{ mm}$ for the initial imperfections nº1 and 3.

In Figure 4.6, the curve of imperfection nº1 has a negative slope after achieving the peak load, due to a sudden loss of lateral support provided by the stiffener to the buckled web. This lack of lateral support results in the bowing of the stiffener and the two adjacent web-panels are now tending to work as one single panel (Figure 4.7). In the curve of imperfection nº3, the loss of rigidity by the girder is not so accentuated because in imperfection nº3 the stiffener already has a significant initial bow. This also explains why for the same stiffener the girder with imperfection nº3 could not carry the same peak load. The initial bow increases steadily with the load, while in the FE model with imperfection nº1, the stiffener starts in a straight position and the bow increases rapidly near the peak load.

To make clear this effect, the absolute lateral deflection at mid-height of the stiffener has been plotted with the mid-span deflection of the girder in Figure 4.8. The initial imperfection is included in the lateral deflection of the stiffener and the absolute values were used because the stiffeners with imperfection nº1 bowed inwards and with imperfection nº3 bowed outwards. For a mid-span deflection between 0 mm and 4 mm the stiffeners have an identical curve, the only difference is that the curve from the plate girder with imperfection nº3 has started with an initial lateral deflection of 3 mm. For a mid-span deflection of 4 mm to 6 mm, which is where the girder reaches its peak load, both stiffeners experience a loss in rigidity that is more accentuated for the stiffeners of the plate girder with imperfection nº1, although the curve of imperfection nº1 is always above the curve of imperfection nº3. This explains why the girder with imperfection nº1 was able to reach a higher peak load and displayed a more sudden loss in rigidity in Figure 4.6. From a mid-span deflection of 6 mm onwards, the lateral displacement of the stiffeners with imperfections nº1 and 3 do not show any significant differences. These are two examples of girders with stiffeners that do not exhibit a satisfactory post-peak behaviour.

![Figure 4.6 - Load/deflection plot of the FE model with $b_{st} = 20 \text{ mm}$ for imperfections nº1 and 3](image-url)
Figure 4.7 - Lateral displacement (mm) contours in panels B1 and B2 for the FE model with $b_{st} = 20$ mm and imperfection nº1

Figure 4.8 - Absolute lateral deflection (mm) at mid-height of the stiffener with $b_{st} = 20$ mm for imperfections nº1 and 3 vs mid-span deflection of the plate girder (deflection includes the initial imperfection)

The FE models tested, in which the stiffeners have the biggest dimensions, are expected to allow the girder to achieve the maximum load and to exhibit a satisfactory post-peak behaviour, meaning the stiffeners have remained effective after the web panels fail in a classic shear mode. In Figure 4.9 are plotted the load/deflection curves of the FE models with imperfections nº1 and 3 for stiffeners with $b_{st} = 80$ mm. The two curves are very similar and show the girder was able to deflect under an approximately constant load. Figure 4.10 shows the lateral displacement at mid-height of the stiffeners for imperfections nº1 and 3, although this time the deflection does not include the initial imperfection. The lateral deflection of the stiffener keeps increasing with the mid-span deflection, but the values are so small that can be despised. The stiffener shows a very rigid behaviour, changing very little from its initial position and were able to effectively sub-divide the web panels as seen in Figure 4.11. When a heavy stiffener is used, the initial imperfection of the plate girder seems to have a very small influence in the overall behaviour. For a mid-span deflection of 14 mm, the stiffeners have a lateral deflection of
around 0.05 mm \((h_w/12000)\) and when the stiffeners were not effective the deflection was near 34 mm for the same mid-span deflection of the plate girder.

Figure 4.9 - Load/deflection plot of the FE model with \(b_{st} = 80\) mm for imperfections n°1 and 3

Figure 4.10 - Absolute lateral deflection (mm) at mid-height of the stiffener with \(b_{st} = 80\) mm for imperfections n°1 and 3 vs mid-span deflection of the plate girder (deflection does not include the initial imperfection)

Figure 4.11 - Lateral displacement (mm) contours in panels B1 and B2 for the FE model with \(b_{st} = 80\) mm and imperfection n°1
To evaluate the evolution of the loading in the stiffener, together with the shear force in the web panel, a curve with the maximum compression force acting in the stiffener for each load increment, and a curve with the shear force have been plotted with the mid-span deflection of the plate girder. Figure 4.12 a) shows the two curves for imperfection nº1 and Figure 4.12 b) shows the two curves for imperfection nº3. In both FE models the compression force in the stiffener is almost null until a mid-span deflection of about 1.5 mm and only from then on, the stiffeners start to be active. When the stiffener starts to be under compression, its axial force increases with a constant slope and plateaus when the maximum shear force is reached by the web panels. It should be noted that the slope of the axial force in the stiffener is lower than the slope of the shear force in the web panels. In the plate girder with imperfection nº3, the stiffener achieves a slightly larger load when compared to the plate girder with imperfection nº1, and in both cases the maximum axial force in the stiffener coincides, more or less, with the peak shear force.

![Graph of N_max and Shear vs Mid-span deflection](image)

**Figure 4.12 - Shear force and maximum compression force in the stiffener with mid-span deflection of the plate girder for $b_{st} = 60$ mm for the different initial imperfections**

As previously stated, one of the aims of the parametric study is to record the axial force and bending moment diagrams in the stiffener, according to the criteria developed in 4.1. In Figure 4.13 a) is shown the axial force diagram for the stiffeners in the plate girder with $b_{st} = 60$ mm and imperfection nº1. The normal force is sub-divided in the web ($N_{web}$) and outstand ($N_{st}$) components. As expected, the stiffener is under compression and most of that load is being carried by the outstand part. The total axial force in the stiffener has a parabolic shape and its maximum value occurs near mid-height of the stiffener, with the value of $-38.6$ kN. In Figure 4.13 b) the bending moment diagram is plotted. It has a negative signal and an “S” shape, peaking at $-0.37$ kNm.
In Figure 4.14 a) the axial force diagram is plotted for the plate girder with $b_{st} = 60$ mm and imperfection nº3. The outstand is also carrying most of the axial load in the stiffener. In this case the peak load in the stiffener occurs for a depth slightly under mid-height with a maximum compression value of $-46.4$ kN. In Figure 4.14 b) the bending moment diagram is plotted. This time the diagram has a parabolic shape and negative signal along its height, peaking at $0.48$ kNm. To help the interpretation of the behaviour of the stiffener, the evolution of the bending moment diagram with the mid-span deflection of the plate girder has been plotted in Figure 4.15 for both initial imperfections. When the imperfection nº1 is adopted, the diagrams maintained their values at the top and bottom, while it kept increasing at mid-height. When the imperfection nº3 is adopted, the diagrams increase their values more uniformly throughout their height while maintaining a parabolic shape.
A summary of the parametric study results for the double-sided stiffeners is shown in Annex A. For both imperfections the peak loads reached by the girder plateaus as the dimensions of the intermediate stiffeners get bigger. The FE models with imperfection n°1 reached this plateau faster than the models with imperfection n°3. It can be seen in Figure 4.16, where the curve of imperfection n°3 is below the curve of imperfection n°1. This demonstrates that the stiffeners in plate girders with initial imperfection n°3 need to be more robust to allow the girder to achieve the same carrying capacity. The absolute deflection at mid-height of the stiffeners also shows that stiffeners in plate girders with imperfection n°3 are more flexible than the ones with imperfection n°1. As it would be expected, the initial bow has a significant influence in the behaviour of the stiffener, but its influence gradually decreases as the rigidity of the stiffener increases.
4.2.2 Plate girders modelled with single-sided stiffeners

A single-sided stiffener only has one outstand and this induces a different behaviour to the stiffener due to a different loading as a consequence of not being symmetrical. In this parametric study, the single-sided stiffeners where also designed as a class 3 section under pure compression according to the EC 3-1-5 requirements, so the outstand maintains its rectangular shape with a ratio of width to thickness of 10. Because the stiffener only has one outstand, its centre of gravity is not centred with the web-plane. The axial compression force enters through the top of the stiffener by the flanges and increases until about mid-height, where the force peaks. This increase in axial force comes through the web panel which does not coincide with the centre of gravity of the stiffener (Figure 4.17) and increases the effects of the bending moment, which did not happen with the symmetrical stiffeners.

![Diagram of forces entering the eccentric single-sided stiffener](image)

Figure 4.17 - Forces entering the eccentric single-sided stiffener

For the parametric study of single-sided stiffeners the initial imperfections n°1, 3 and 9 were adopted. The objective of the parametric study is to increase the stiffener’s dimensions in small increments and evaluate its behaviour and the carrying capacity of the plate girder. The area and moment of inertia of the stiffener are also based on the effective cross section defined in EC 3-1-5. Table 4.2 summarises the properties of the single-sided stiffeners used in this parametric study. The variable $e_{CG}$ is the distance from the web to the centre of gravity and it increases rapidly as a larger stiffener is used.

<table>
<thead>
<tr>
<th>$b_{st}$ (mm)</th>
<th>$t_{st}$ (mm)</th>
<th>$A_{st}$ (cm$^2$)</th>
<th>$I_{st}$ (cm$^4$)</th>
<th>$e_{CG}$ (mm)</th>
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<td>884</td>
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</tr>
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</table>
In Figure 4.18 are plotted the load/deflection curves of the girders, modelled with $b_{st} = 25\,\text{mm}$ for initial imperfections nº1, 3 and 9. These are three examples of stiffeners that did not show an effective post-peak behaviour because the carrying capacity decreased significantly after reaching the peak load. All FE models reached the peak load for a mid-span deflection of about 4.5 mm. After that, the curves have a negative slope due to the bowing of the intermediate stiffeners. The plate girder with imperfection nº1 was able to reach a higher load when compared to the plate girders with the other imperfections, although it also showed a more sudden loss in rigidity. For a mid-span vertical deflection of about 6 mm, the load/deflection curve of the plate girder with imperfection nº1 is already below the other two curves. The load/deflection curves for imperfections nº3 and 9 do not show any significant difference between each other.

Figure 4.19 shows the absolute lateral deflection at mid-height of the stiffener (including the initial imperfection) with the mid-span deflection of the girder. The absolute values have been plotted, because in the FE models with imperfections nº1 and 9, the stiffener bowed inwards and for imperfection nº3 the stiffener bowed outwards. The curves have the same shape until a mid-span deflection of about 3.5 mm. After that, the stiffeners become less rigid and that loss in rigidity is more accentuated in the stiffeners of the plate girder with imperfection nº1. All stiffeners have a similar lateral deflection for a mid-span deflection of the plate girder of about 6 mm onwards.

Figure 4.18 - Load/deflection plot of the FE model with $b_{st} = 25\,\text{mm}$ for imperfections nº1, 3 and 9
The stiffeners with $b_{st} = 70$ mm have been chosen to describe the behaviour of an effective single-sided stiffener. The load/deflection curves have been plotted in Figure 4.20. The three curves have a very similar shape. The FE models with imperfections nº1 and 9, peaked at 519.6 kN and 519.5 kN respectively, while the girder with imperfection nº3 peaked slightly under at 506.6 kN. The plate girders reached the peak load for a mid-span deflection of about 6 mm and were able to deflect under an approximately constant load, demonstrating a satisfactory post-peak behaviour.

To evaluate the maximum axial load carried by the stiffener and the shear force in the web panel with the mid-span deflection of the plate girder, the two curves have been plotted in Figure 4.21 for the three different initial imperfections considered. The axial force in the stiffener is close to zero for a mid-span deflection below 1.5 mm. From then on, the maximum load in the stiffener starts increasing with a constant slope and reached its peak value for approximately the same mid-span deflection the girder registers its peak load. The curve of the load in the stiffener, for imperfection nº1, has a lower slope when compared to the slopes of imperfections nº3 and 9. The peak axial loads in the stiffener are also higher for imperfections nº3 and 9, but when the plate girder has the imperfection nº3, the stiffener unloads more rapidly when compared to imperfection nº9.
For the stiffeners of the FE models with $b_{st} = 70$ mm, which had a satisfactory behaviour, the evolution of the axial force and bending moment diagrams have been evaluated. The diagrams have been plotted in Figure 4.22 for different mid-span deflections of the plate girder and for the three initial imperfections considered.
For a mid-span deflection of the plate girder until 5 mm, the axial force diagrams have a well-defined parabolic shape and the peak force occurs at about mid-height of the stiffener. From then on, the shape of the diagram changes slightly but it does not deviate much from its initial shape. The axial force in the top of the stiffener increases, especially in the diagrams of imperfection nº1. In the diagrams of
imperfection n°3 and 9 the peak axial force occurs for a lower height, while it remains approximately at mid-height in the diagram of imperfection n°1. The diagrams of imperfections n°3 and 9 also show that those stiffeners carry a higher load, when compared with the diagrams of imperfection n°1.

The bending moment diagrams of the single sided stiffeners with $b_{st} = 70 \text{ mm}$ are plotted in Figure 4.23 for equally spaced mid-span deflections of the plate girder. All diagrams have a parabolic shape for a mid-span deflection up to 5 mm, starting at the top of the stiffener with a small value and slowly increasing until reaching the peak bending moment at about mid-height. The diagrams of imperfection n°1 reached a peak bending moment of 0.99 kNm for a mid-span deflection of 5 mm and the value of the bending moment in the stiffener started decreasing for a mid-span deflection of 7 mm. The diagrams of imperfection n°9 reached a peak bending moment of 0.73 kNm for a mid-span deflection of 9 mm. After a deflection of 5 mm the diagrams modified their initial parabolic shape to an “S” shape, and the peak bending moment shifted to the bottom of the stiffener. The diagrams of imperfection n°3 were the ones that reached a higher value for the bending moment. The diagrams not only maintained their parabolic shape but also maintained an approximately constant peak bending moment. The peak bending moment registered was 2.34 kNm for a mid-span deflection of 8 mm.
In Figure 4.24 the axial force and bending moment diagrams are plotted for when the plate girder is under peak load. The axial force is sub-divided in the web and outstand component to help better understand the behaviour of the single sided stiffeners. The total axial force diagram in the stiffener has a parabolic shape for all three imperfections. The web part of the stiffener is more loaded than the outstand for all imperfections, but especially in the stiffener of imperfection nº3. For the stiffener of imperfection nº3 the outstand has very little compression, so almost all the load of the stiffener is being carried by the web component. It is also for imperfection nº3 that the axial force and bending moment are the highest, with $-49.2 \, \text{kN}$ and $2.30 \, \text{kNm}$ respectively. The peak axial force and bending moment are $-34.6 \, \text{kN}$ and $0.84 \, \text{kNm}$ respectively, for imperfection nº1, while for Imperfection nº9 it is $-44.6 \, \text{kN}$ and $0.72 \, \text{kNm}$.

Figure 4.24 - Diagrams in the stiffener with $b_{st} = 70 \, \text{mm}$ while the plate girder with imperfection nº1 is under peak load
4.2.3 Discussion of results

There is a clear difference in the behaviour of plate girders modelled with effective or ineffective transverse stiffeners. Although a small stiffener contributes significantly to an increase in the ultimate load capacity of the girder, the post-peak behaviour is usually unsatisfactory. This happens because a small stiffener can divide the web into panels but starts bowing out of plane before they reach their maximum shear force. Once the stiffener bows, the web panels tend to work as a single web panel leading to a sudden loss in the rigidity and resistance of the plate girder.

In the plate girders modelled with small stiffeners, the peak load was always higher when the initial imperfection nº1 was adopted because the stiffener was in a straight position which made it less likely to bow in comparison with the stiffeners in girders with initial imperfections nº3 and 9. That is why the negative slope in the load/deflection curve is higher when the imperfection nº1 is used. The higher slope indicates a larger loss in rigidity by the plate girder due to the bowing of the stiffener.
When a rigid stiffener was used, the girder reached its peak load and was able to deflect under an approximately constant load, regardless of the initial imperfection, and both single and double-sided stiffeners exhibited an effective behaviour.

In the plate girders modelled with single-sided stiffeners, the peak load reached by the girder with imperfection n°3 was always below the loads reached by girders with imperfections n°1 and 9, even for very rigid stiffeners. An explanation for this is that the web part of the stiffener is responsible for carrying most of the compression load, which is significantly higher than the compression in the web for the other two imperfections, so it yields for a lower shear force.

The plate girders modelled with double-sided stiffeners with both imperfections n°1 and 3 were able to reach the same peak loads. This shows that the use of double-sided stiffeners makes the plate girder less susceptible to the initial imperfection, although the girder with imperfection n°3 required a larger stiffener to stabilise the peak load. In this case, the web part of the stiffener of the plate girders with imperfection n°3 is not as loaded in compression as when single-sided stiffeners are used, so the web panel is able to carry the same shear force.

The criteria developed in 4.1 needs to be validated to ensure that the simplifications considered for the calculation of the forces in the web are reliable and lead to a good estimate of the loading in the stiffener. Figure 4.27 shows the compression membrane forces in the web for girders with single and double-sided stiffeners. The distribution of the compression membrane forces are more symmetrical when single-sided stiffeners are used, in comparison with the compression forces in the double-sided stiffeners, that shift to the opposite side of the web panel that reached failure. This shift in the compressive forces only occurs right before the plate girder reaches its peak load.

The single-sided stiffeners might have a more centred compression force in the web because it is responsible for carrying more load in comparison with the double-sided stiffeners, where most of the compression force is carried by the outstand part. When the girder is in the post-buckling phase, but not near the peak load, the compression forces in the web part of the stiffener is more centred. The membrane forces in the web are within the width of \( h_w/2 \) for each side of the stiffener and the decision

![Figure 4.27 - Compression membrane forces in the web panels B1 and B2 while the plate girder made of steel S355 is under peak load](image)

| a) Single-sided stiffener | b) Double-sided stiffener |

40
of only considering the elements with compressive membrane forces in detriment of a fixed width was accurate, since it would minimize the compression force in the stiffener.

When double-sided stiffeners are used, the compression force is well distributed by the web and outstand parts of the stiffener, although the outstand part carries more load. The shape of the diagrams in both cases is parabolic, but the diagrams do not start with a null axial force. This shows that the stiffener is axially loaded at the top and bottom, but that value is far from the peak that occurs at about mid-height of the stiffener. The compression force is higher for the stiffeners in plate girders with initial imperfection n°3 when compared to the stiffeners in girders with initial imperfection n°1. In the examples presented in Figure 4.13 and Figure 4.14 the maximum axial force was −38.6 kN and −46.4 kN for stiffeners in plate girders with imperfections 1 and 3 respectively.

The bending moment diagram in the stiffener of the plate girder with initial imperfection n°1 has an “S” shape with the highest value at mid-height, whereas the diagram of the stiffener with imperfection n°3 as a parabolic shape. Although the absolute values of the bending moment diagrams are of similar magnitude, the deflection of the stiffener is higher when imperfection 3 is used, as seen in Figure B.13 of Annex B. This is because the bending moment in the stiffener, when the imperfection n°1 is used, is low at the extremities and only high at mid-height. While the absolute values of bending moment diagram of the stiffener with imperfection n°3 are higher at the extremities in comparison with the diagram of imperfection n°1. In Figure B.13 it is also possible to observe that the shape of the stiffener’s deflection throughout its height is very similar to the shape of the bending moment diagram.

In the single-sided stiffeners the compression force is well distributed between the web and outstand when the initial imperfections n°1 and 9 were used, but the web part carried slightly more load than the outstand. For single-sided stiffeners in the plate girder with imperfection n°3, the compression force is almost entirely carried by the web part while the outstand has very little compression. Figures B.5 and B.6, from Annex B, show that when small stiffeners were used the outstand part had tensile forces at mid-height, which indicates the stiffener is mainly resisting the bending moment. The stiffeners with imperfection n°3 and 9 were more axially loaded in compression than the stiffeners with imperfection n°1.

The bending moment diagram has positive signal and a parabolic shape in the stiffener with imperfections n°1 and 3, but in the stiffener of imperfection n°3 the bending moment at mid-height is about twice as high than the stiffener with imperfection n°1. When the imperfection n°1 is used, the stiffener is initially in a straight position and the bending moment is positive, because the compression entering the stiffener through the web causes a positive moment due to the centre of gravity being in the outstand part. The same happens for stiffeners with initial imperfection n°3, but in this case the bending moment is amplified because the stiffener already has a bow shape from its initial position, which increases the moment caused by the compression forces. Another reason is that the compression force is higher for this imperfection and since the bending moment depends on the axial force because
of the eccentric centre of gravity it also contributed to the increase of the bending moment. Due to this high bending moment there are more fibres in the exterior of the outstand with tensile stress, which led to a very small total compression in the outstand for the case shown in Figure 4.25. In stiffeners with a smaller width and imperfection n°3, the outstand even reached tensile axial forces at mid-height, which increased the force in the web part because the total compression force in the stiffener did not change much.

The single-sided stiffeners have shown that they can be as effective as the double-sided stiffeners because both types allow the girder to reach similar peak loads and show a satisfactory behaviour in providing an adequate support to the web panels. When double-sided stiffeners are fitted, the axial compression load is mostly carried by the outstand part of the stiffener, while in the single-sided stiffeners the web part is the most axially loaded because the outstand has, in most cases, tensile stresses in the exterior fibres due to the positive bending moment. Regarding the magnitude of the axial force, the imperfection n°3 is the most critical of imperfections for both types of stiffeners and it also leads to significantly higher bending moment when single-sided stiffeners are used. When double-sided stiffeners are fitted, the bending moment is slightly greater for imperfection n°1 than for imperfection n°3, but the plate girders with imperfection n°3 carried less load than the ones with imperfection n°1, for the same stiffeners, as seen in Figure 4.16. So, it can be concluded that imperfection n°3 is the most demanding imperfection for the two types of stiffeners.

Because imperfection n°1 leads to a negative bending moment diagram when double-sided stiffeners are used and a positive bending moment diagram when single-sided stiffeners are used (due to the asymmetry of the stiffener), a more demanding imperfection than imperfection n°1, for the plate girders with single-sided stiffeners, would have been to consider an initial imperfection where the web panels bow inwards (negative amplitude in the x axis), as seen in Figure 4.28. This imperfection would lead to a greater bending moment because whatever is causing it in the case of double-sided stiffeners would be causing a bending moment with the same signal as of that caused by the eccentricity of the stiffener.

Now that we know which is the most critical imperfection, we can compare the results obtained from the FE models with the design loads from EC 3-1-5. In Figure 4.29 the axial force in the stiffener obtained from the FE model of a plate girder with initial imperfection n°3 and the force to design the stiffener according to EC 3-1-5 are shown. The maximum compression force in the stiffener while the plate girder was under the peak load was −49.2 kN, far from the EC 3-1-5 design load of −124 kN. When imperfections n°3 and 9 were used, the axial force in the stiffener was about 40% of the EC 3-1-5 force, while when imperfection n°1 was adopted, the force was only about 30%. It is now important to discuss...
the evolution of the force with the shear in the web panels of the plate girder, so we can better understand the behaviour of the stiffener during the process of loading.

![Diagram](image)

Figure 4.29 – Axial force diagram of single-sided stiffener with $b_{st} = 70$ mm fitted in the plate girder with imperfection nº3 and the EC 3-1-5 design load

In Figure 4.30 the FE model shear and stiffener forces, as well as the force in the stiffener according to EC 3-1-5 expression (Eq. (2.12)) are plotted. According to EC 3-1-5, the axial force in the stiffener is zero until the buckling resistance is reached and is equal to $V_{Ed} - V_{cr}$ when the shear force is higher than $V_{cr}$. The force in the stiffener is almost null until a mid-span deflection of the plate girder of about 1.5 mm, which indicates that the stiffeners only start to be axially loaded after the web panels reach the buckling shear force. Once the stiffener starts to be loaded, the force increases proportionally to the shear force, although its slope is significantly inferior to the shear force slope. This means that after the buckling of the web panels, not all shear force passes through the stiffener and that the compression force can be described by the expression presented below (Eq. (4.1) and Eq. (4.2)),

\[
\begin{align*}
N_{Ed} &= 0 & \text{if } V_{Ed} < V_{cr} \\
N_{Ed} &= \alpha \cdot (V_{Ed} - V_{cr}) & \text{if } V_{Ed} \geq V_{cr}
\end{align*}
\]

where $\alpha$ is the ratio between the slope of the curve of the maximum compression force in the stiffener and the slope of the shear force curve in the web panel in the post buckling phase. To evaluate the value of $\alpha$, for a mid-span deflection of the girder between 2 mm and 4 mm, the shear force and stiffener force plots have been curve fitted into a linear equation. The constant $\alpha$ is the ratio between the stiffener force and shear force slopes, taking for this case the value of 0.29. The factor $\alpha$ can be understood as the measure of the compression force that passes through the stiffener for an increment of 1 kN in shear force of the web panels during the post-buckling phase.
Figure 4.30 – Evolution of the maximum compression force in the stiffener with the EC 3-1-5 expression

The maximum value of the bending moment plateaued once the stiffener used showed an effective behaviour, like what happened with the compression force. In the case of single-sided stiffeners that did not happen, because the distance between the centre of gravity and the web gets wider as a more robust stiffener is used. This indicates that the bending moment has some correlation with the dimensions of the stiffener. When comparing the moment of the single and double-sided stiffeners with the same cross-section area, it can be concluded that the single-sided stiffeners are subjected to a greater bending moment. The origin of the bending moment is not so easy to understand in the case of double-sided stiffeners because the centre of gravity coincides with the line of the web. It indicates that the bending moment might also be caused by a different action, like lateral forces due to the support of the buckled web panel. If these forces exist and are significant, they are present in the two types of stiffeners because both provide lateral support to the web panel.

Table 4.3 compares the peak bending moment obtained from the FE analysis with the EC 3-1-5 design bending moment. For the initial imperfection nº3, when small single-sided stiffeners were used the bending moment from the FE model was greater than the bending moment from the Eurocode, but as the dimensions of the stiffener increased, the bending moment of the Eurocode surpassed the bending moment acting in the stiffener. For the remaining initial imperfections, the bending moment in the stiffener was always below the bending moment from the Eurocode.

Figure 4.31 shows the peak shear force reached by the web panels for the different initial imperfections considered and for the different moment of inertia of the stiffeners used in the parametric study. The minimum inertia required by the EC 3-1-5 has also been plotted for comparison. The curves of the plate girders modelled with imperfection nº3 are the ones that have the lower loads. The minimum inertia required by EC 3-1-5 is not enough to ensure the web panels reach their potential maximum peak load and that the stiffeners have a satisfactory behaviour.
Table 4.3 - Comparison of the bending moment from the FE analysis and EC 3-1-5

<table>
<thead>
<tr>
<th>Initial Imperfection nº</th>
<th>$b_{st}$ (mm)</th>
<th>$M_{FEM , (\text{max})}$ (kNm)</th>
<th>$M_{EC3-1-5}$ (kNm)</th>
<th>$M_{FEM} / M_{EC3-1-5}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Imp nº 1</td>
<td>25</td>
<td>0.28</td>
<td>0.37</td>
<td>74%</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>0.25</td>
<td>0.58</td>
<td>43%</td>
</tr>
<tr>
<td></td>
<td>35</td>
<td>0.34</td>
<td>0.82</td>
<td>41%</td>
</tr>
<tr>
<td></td>
<td>40</td>
<td>0.43</td>
<td>1.09</td>
<td>39%</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>0.56</td>
<td>1.69</td>
<td>33%</td>
</tr>
<tr>
<td></td>
<td>60</td>
<td>0.70</td>
<td>2.35</td>
<td>30%</td>
</tr>
<tr>
<td></td>
<td>70</td>
<td>0.84</td>
<td>3.04</td>
<td>28%</td>
</tr>
<tr>
<td></td>
<td>80</td>
<td>1.02</td>
<td>3.73</td>
<td>27%</td>
</tr>
<tr>
<td>Imp nº 3</td>
<td>25</td>
<td>0.50</td>
<td>0.37</td>
<td>133%</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>0.64</td>
<td>0.58</td>
<td>111%</td>
</tr>
<tr>
<td></td>
<td>35</td>
<td>1.00</td>
<td>0.82</td>
<td>123%</td>
</tr>
<tr>
<td></td>
<td>40</td>
<td>1.69</td>
<td>1.09</td>
<td>155%</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>1.83</td>
<td>1.69</td>
<td>108%</td>
</tr>
<tr>
<td></td>
<td>60</td>
<td>2.06</td>
<td>2.35</td>
<td>88%</td>
</tr>
<tr>
<td></td>
<td>70</td>
<td>2.30</td>
<td>3.04</td>
<td>76%</td>
</tr>
<tr>
<td></td>
<td>80</td>
<td>1.77</td>
<td>3.73</td>
<td>48%</td>
</tr>
<tr>
<td>Imp nº 9</td>
<td>25</td>
<td>0.24</td>
<td>0.37</td>
<td>65%</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>0.17</td>
<td>0.58</td>
<td>30%</td>
</tr>
<tr>
<td></td>
<td>35</td>
<td>0.26</td>
<td>0.82</td>
<td>32%</td>
</tr>
<tr>
<td></td>
<td>40</td>
<td>0.26</td>
<td>1.09</td>
<td>24%</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>0.30</td>
<td>1.69</td>
<td>18%</td>
</tr>
<tr>
<td></td>
<td>60</td>
<td>0.49</td>
<td>2.35</td>
<td>21%</td>
</tr>
<tr>
<td></td>
<td>70</td>
<td>0.72</td>
<td>3.04</td>
<td>24%</td>
</tr>
<tr>
<td></td>
<td>80</td>
<td>0.97</td>
<td>3.73</td>
<td>26%</td>
</tr>
</tbody>
</table>

Variation of shear with moment inertia of the stiffener

Figure 4.31 – Shear force reached by the web panels with the moment of inertia of the stiffeners
4.3 Plate girders made of HSS S690

For the FE models in the parametric study of plate girders made of HSS S690, only the initial imperfection nº3 was used, because it was concluded in 4.2.3 that it is the most demanding from the three initial imperfections considered in the parametric study of plate girders made of steel S355. Both single and double-sided stiffeners used have the outstand ratio of width to thickness equal to 8 to ensure a class 3 behaviour according to EC 3-1-5.

4.3.1 Plate girders modelled with double-sided stiffeners

Table 4.4 shows the characteristics of the double-sided stiffeners made of HSS S690 used in this parametric study.

<table>
<thead>
<tr>
<th>$b_{st}$ (mm)</th>
<th>$t_{st}$ (mm)</th>
<th>$A_{st}$ (cm$^2$)</th>
<th>$I_{st}$ (cm$^4$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>2.5</td>
<td>2.7</td>
<td>1.7</td>
</tr>
<tr>
<td>30</td>
<td>3.75</td>
<td>3.9</td>
<td>7.8</td>
</tr>
<tr>
<td>40</td>
<td>5</td>
<td>5.7</td>
<td>23.8</td>
</tr>
<tr>
<td>50</td>
<td>6.25</td>
<td>8.0</td>
<td>56.9</td>
</tr>
<tr>
<td>60</td>
<td>7.5</td>
<td>10.8</td>
<td>116.3</td>
</tr>
<tr>
<td>70</td>
<td>8.75</td>
<td>14.1</td>
<td>213.2</td>
</tr>
<tr>
<td>80</td>
<td>10</td>
<td>17.9</td>
<td>360.9</td>
</tr>
</tbody>
</table>

Similar to what was done to study the transverse stiffeners in plate girders modelled with steel S355, it was also necessary to evaluate the load/deflection curves of the plate girders modelled with HSS S690, because they help to understand the influence of the transverse stiffeners in the overall behaviour of the plate girder.

The load/deflection curve of the plate girders with $b_{st} = 20$ mm and $b_{st} = 80$ mm have been plotted in Figure 4.32. When stiffeners with $b_{st} = 20$ mm were fitted, the load/deflection curve shows the plate girder started to lose some rigidity after a mid-span deflection of 4 mm, in comparison with the plate girder fitted with effective stiffeners ($b_{st} = 80$ mm). The peak load reached by the two plate girders is also very different. When stiffeners with $b_{st} = 20$ mm were used, the peak load reached 775.7 kN, while when stiffeners with $b_{st} = 80$ mm were used, the curve peaked at 928.6 kN.

To evaluate the force carried by the stiffener, the shear force in the web panel and the maximum compression force acting in the stiffener have been plotted in Figure 4.33. The force in the stiffener is almost null until a mid-span deflection of about 1.5 mm. From then on, the force in the stiffener increases with an approximately linear slope that is inferior to the slope of the shear force. After a mid-span deflection of 8 mm the axial force plateaus while the shear force increases, which might be explained by the additional carrying capacity provided by the flanges. When the shear force plateaus, the maximum...
The compression force also remains approximately constant. This shows that when the girder was deflecting under an approximately constant load, the force in the stiffener changed very little.

Figure 4.34 shows the evolution of the axial force diagram with the mid-span deflection of the plate girder. The compression force is low at the top of the stiffener, and then starts increasing until it reaches its peak at about mid-height and it starts decreasing again, reaching the bottom of the stiffener with a force like the one at the top. This gives the diagrams a parabolic shape that is very well defined until the girder reaches its peak load, for a mid-span deflection of about 15 mm. From then on, the peak force in the stiffener decreases and the shape of the diagram is not so well defined.

Figure 4.33 - Shear force and maximum compression force in the stiffener with mid-span deflection of the plate girder for $b_{st} = 80$ mm and initial imperfection nº3
Figure 4.34 - Evolution of the axial force diagram in the stiffener \( (b_{st} = 80 \text{ mm}) \) with the mid-span deflection of the plate girder

Figure 4.35 shows the evolution of the bending moment diagram with the mid-span deflection of the plate girder. The peak value of the diagrams occurs at about one third from the top of the stiffener and the diagram has an “S” shape due to the negative values of bending moment in the bottom third of the stiffener. After a mid-span deflection of 12 mm, the negative values of the diagram do not increase, while the positive ones keep rising. This odd shape is difficult to interpret, so the lateral forces caused by the buckled web panels to the stiffeners should be better investigated.

Figure 4.35 - Evolution of the bending moment diagram in the stiffener \( (b_{st} = 80 \text{ mm}) \) with the mid-span deflection of the plate girder

In Figure 4.36 the axial force diagram and bending moment diagram in the stiffener are shown, while the plate girder is under peak load. The axial force diagram is divided in the web and outstand components. At the top of the stiffener the compression force is being resisted almost entirely by the web but then its force decreases along the height of the stiffener. The force in the outstand has a parabolic shape similarly to the total compression force acting in the stiffener, which peaks approximately at mid-height with the value of \(-111.7 \text{ kN}\). The component of the stiffener responsible for carrying most of the load is the outstand.
The bending moment diagram has an “S” shape has explained earlier, due to the negative value of the moment at the bottom third of the stiffener. The peak value of the bending moment is positive and occurs at about one third from the top of the stiffener with the value of 1.42 kNm.

![Bending moment diagram](image)

Figure 4.36 - Diagrams in the stiffener with \( b_{st} = 80 \) mm while the plate girder with imperfection \( n^33 \) is under peak load

4.3.2 Plate girders modelled single-sided stiffeners

Table 4.5 shows the characteristics of the single-sided stiffeners made of HSS S690 used in this parametric study.

<table>
<thead>
<tr>
<th>( b_{st} ) (mm)</th>
<th>( t_{st} ) (mm)</th>
<th>( A_{st} ) (cm²)</th>
<th>( I_{st} ) (cm⁴)</th>
<th>( e_{cg} ) (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>3.75</td>
<td>2.8</td>
<td>2.7</td>
<td>8.1</td>
</tr>
<tr>
<td>40</td>
<td>5</td>
<td>3.7</td>
<td>7.0</td>
<td>13.0</td>
</tr>
<tr>
<td>50</td>
<td>6.25</td>
<td>4.9</td>
<td>14.4</td>
<td>18.4</td>
</tr>
<tr>
<td>60</td>
<td>7.5</td>
<td>6.3</td>
<td>26.3</td>
<td>24.0</td>
</tr>
<tr>
<td>70</td>
<td>8.75</td>
<td>8.0</td>
<td>43.9</td>
<td>29.6</td>
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<tr>
<td>80</td>
<td>10</td>
<td>9.9</td>
<td>68.8</td>
<td>35.1</td>
</tr>
<tr>
<td>90</td>
<td>11.25</td>
<td>12.0</td>
<td>103.2</td>
<td>40.6</td>
</tr>
</tbody>
</table>

The load/deflection curve of the plate girders fitted with single-sided stiffeners with \( b_{st} = 30 \) mm and \( b_{st} = 60 \) mm have been plotted in Figure 4.37. Although the curve of \( b_{st} = 30 \) mm is less rigid than the curve of \( b_{st} = 60 \) mm after a mid-span deflection of 6mm and they reached different peak loads, the curves have a very similar shape. The plate girder fitted with stiffeners with \( b_{st} = 30 \) mm was able to carry a load of 813.7 kN and the one fitted with stiffeners with \( b_{st} = 60 \) mm carried a load of 894.9 kN.
Figure 4.37 - Load/deflection plot of the FE models with $b_{st} = 30 \text{ mm}$ and $b_{st} = 60 \text{ mm}$ for imperfection nº3

The plot in Figure 4.38 shows the evolution of the maximum compression force in the stiffener and shear force in the web panel with the mid-span deflection of the plate girder. The force in the stiffener is null until a mid-span deflection of about 1.5 mm. From then on, the compression force in the stiffener increased steadily and proportional to the post-buckling shear in the web panel, although its slope was inferior to the slope of the shear force curve.

Figure 4.38 - Shear force and maximum compression force in the stiffener with the mid-span deflection of the plate girder for $b_{st} = 60 \text{ mm}$ and initial imperfection nº3

Figure 4.39 shows the evolution of the axial force diagram with the mid-span deflection of the plate girder. The diagrams show that the stiffener has low values of compression at the top and bottom, but the compression force increases until it reaches its peaks at about mid-height. After a mid-span
deflection of 15 mm, the compression force at the top of the stiffener started to increase while the peak force at mid-height of the stiffener started to decrease.

Figure 4.39 - Evolution of the axial force diagram in the single-sided stiffener ($b_{st} = 60$ mm) with the mid-span deflection of the plate girder

Figure 4.40 shows the evolution of the bending moment diagram with the mid-span deflection of the plate girder. The diagrams have a parabolic shape and its values are always positive throughout the height of the stiffener. The peak values of the bending moment diagram occur at mid-height. For a mid-span deflection of the plate girder from 15 mm to 24 mm, the bending moment diagram changed very little. This shows the moment remained constant while the girder was deflecting under an approximately constant load.

Figure 4.40 - Evolution of the bending moment diagram in the single-sided stiffener ($b_{st} = 60$ mm) with the mid-span deflection of the plate girder

In Figure 4.41 the axial force and bending moment diagrams in the stiffener are shown, while the plate girder is under peak load. The axial force in the stiffener is almost entirely being resisted by the web component. The absolute value of axial force in the outstand is very little and it even reaches tensile forces at about mid-height. This distribution of forces in the stiffener is caused due to the exterior fibres of the outstand having tensile stresses to resist the high positive bending moment. A similar distribution
of forces was also registered for the single-sided stiffeners in plate girder modelled with steel S355 and initial imperfection nº3. Because the outstand has a tensile force at mid-height and the total force in the stiffener is the sum of the forces in the web and outstand, the web force is slightly higher than the peak total force in the stiffener, which was $-97.9$ kN. The axial force diagram also shows that the origin of the increase in the total compression force at the top of the stiffener is due to an increase in the force of the web.

The bending moment diagram is always positive, but low at the top and bottom of the stiffener and peaks at mid-height with the value of 5.23 kNm. At the top of the stiffener the diagram has and odd shape, caused by the high compression forces at the top of the web that cause an increase in the positive bending moment.

![Figure 4.41 - Diagrams in the single-sided stiffener with $b_{st} = 60$ mm while the plate girder with imperfection nº3 is under peak load](image)

4.3.3 Discussion of results

The load/deflection curve of the plate girders modelled with ineffective stiffeners did not show a loss in rigidity as sudden as the plate girders modelled with steel S355. If initial imperfection nº1 would have been used, the loss in rigidity by the plate girder might have been greater due to a more sudden bowing of the stiffener.

The post-peak behaviour of the plate girders is better when the HSS S690 is used, compared to the post-peak behaviour of the plate girders with ineffective stiffeners modelled with steel S355. This is because the girders made of HSS S690 did not lose as much carrying capacity after the peak load was achieved and the girder kept deflecting. However, the peak load is substantially higher when heavy stiffeners are used.
Similar to what was done in 4.2.3, the criteria developed in 4.1 needs to be validated to make sure it leads to reliable results for the analysis of the internal forces in the transverse stiffeners. The results were like the ones obtained from plate girders using steel S355. The compression forces in the web were almost symmetrical for each side of the stiffener when single-sided stiffeners were used. But when double-sided stiffeners were used, the compression force in the web shifted slightly for the opposite side of the web panel that failed, although this only happened near the failure load. Figure 4.42 shows an example of the compression membrane forces in the web for single and double-sided stiffeners while the plate girder is under peak load.

![Figure 4.42 - Compression membrane forces in the web panels B1 and B2 while the plate girder made of HSS S690 is under peak load](image)

The total axial compression force was slightly higher for the plate girders modelled with double-sided stiffeners because those girders were able to achieve a higher load compared to the plate girders modelled with single-sided stiffeners.

In the double-sided stiffeners the diagrams have a parabolic shape and the maximum force occurs at about mid-height. The outstand was the component of the stiffener that carried most of the axial load. From the plate girders tested, the higher compression force in the double-sided stiffener, while the girder was under peak load, was registered for the stiffener with $b_{st} = 80 \text{ mm}$ with $-111.7 \text{ kN}$.

In the single-sided stiffeners the axial force diagram also showed a parabolic shape, but in this case the distribution of the compression through the components of the stiffener was much different. As seen in Figure 4.41, the compression was mostly carried by the web, because the exterior elements of the outstand were under tension due to the positive bending moment the stiffener was resisting. From the plate girders tested, the higher compression force in the single-sided stiffener, while the girder was under peak load, was registered for the stiffener with $b_{st} = 40 \text{ mm}$ with $-106.3 \text{ kN}$. Figure 4.43 shows the vertical component of the membrane forces in the outstand of the stiffener with $b_{st} = 60 \text{ mm}$, while the girder was under peak load.

The bending moment diagram in the double-sided stiffeners is probably caused by lateral forces provoked by the buckled web. The compression forces enter the stiffener through its centre of gravity, so it is not caused by eccentricities. The diagram has mainly a positive signal, which was expected due
to the initial outwards bow of the stiffener. Although the negative values at the bottom are harder to interpret. The shape of the bending moment diagram in double-sided stiffeners with initial imperfection n°3 in plate girder modelled with steel S355 was similar.

The bending moment diagram in the single-sided stiffeners has a parabolic shape and peaks at mid-height of the stiffener. The bending moment diagram in a single-sided stiffener is higher in comparison with a double-sided stiffener with the same cross section area. This higher bending moment is mostly caused by the eccentricity of the single-sided stiffener and the initial imperfection n°3, which magnifies this effect.

Figure 4.44 shows the total axial force in the stiffener obtained from the FE model and the design load calculated as if the stiffener were to be designed according to EC 3-1-5. The double-sided stiffener with \( b_{st} = 80 \) mm has been chosen for comparison because it is the one with the highest compression force. The maximum compression in the stiffener while the plate girder was under peak load was \(-111.7\) kN, that compares with the design load of 247 kN. This means that the compression force acting in the stiffener was only 45% of the EC 3-1-5 design load. In 4.2.3, the comparison of the force in the stiffener from the FE model modelled with steel S355 showed that the stiffener carried 40% of force from EC 3-1-5. This leads to the impression that the stiffeners in plate girders modelled with HSS S690 are more axially loaded than the stiffeners in plate girders modelled with S355, but it might not be true, since the EC 3-1-5 is more conservative in the estimation of \( V_{rd} \) in plate girder with HSS S690 than with steel S355. To understand how much compression the stiffener is carrying, in relation to the shear force in the web panels, the ratio \( \alpha \) (from Eq. (4.1) and Eq. (4.2)) must be evaluated.

In Figure 4.45 it is shown the FE model shear force and the compression force in the stiffener as well as the force in the stiffener according to the EC 3-1-5 expression \( (N_{ed} = V_{ed} - V_{tr}) \).
Similar to what happened in the plate girders modelled with steel S355, the force in the stiffener is almost null until a mid-span deflection of the girder of about 1.5 mm. This indicates that the stiffener only starts to be axially loaded after the web panels reach their buckling shear resistance. Once the stiffener starts to be loaded the force rises proportionally to the shear force, but its slope is significantly inferior to the shear force slope. The values of the shear force and compression force in the stiffener, for a mid-span vertical deflection between 3 mm and 7 mm, have been curve fitted into a linear equation to obtain the slope values to calculate the ratio $\alpha$. In this case, the value $\alpha$ is 0.36, which means that in the post-buckling phase, an increase of 1 kN in shear leads to an increase in the stiffener’s compression force of 0.36 kN. The value of $\alpha$ calculated in 4.2.3 for the stiffener in the plate girder modelled with steel S355 was 0.29, lower than the value for girders modelled with HSS S690, leading us to conclude that the stiffeners are more axially loaded when the HSS S690 is adopted.

Figure 4.44 - Axial force diagram of double-sided stiffener with $b_{st} = 80$ mm fitted in the plate girder with imperfection n°3 and the EC 3-1-5 design load

Figure 4.45 - Shear force and maximum compressive force in the stiffener with the mid-span deflection of the plate girder
The maximum value of the bending moment diagram increased slightly as the dimensions of the stiffener became larger, like what happened for the stiffeners in plate girders made of steel S355. The bending moment of a single-sided stiffener is greater than the bending moment of a double-sided stiffener with the same outstand cross-sectional area. This means the eccentric stiffeners are more loaded in flexion than the double-sided stiffeners. The origin of the bending moment in the double-sided stiffeners might be provoked by the adjacent buckled web panels and the second order moment due to the installed compression. Both effects are also present in plate girders with single-sided stiffeners.

Figure 4.46 shows the evolution of the peak load reached by the plate girder, for the two types of stiffeners, with the inertia of their effective cross-section. In the figure it is also plotted the minimum inertia to design the stiffener according to EC 3-1-5 of 2.4 cm$^4$. This value is the same used to design the stiffeners in plate girders made of steel S355 because the equation to calculate the minimum inertia is independent of the yield strength of steel used.

The peak load of the plate girders modelled with single-sided stiffeners is always slightly lower than the load reached by plate girders with double-sided stiffener. This characteristic was also noticed in plate girders modelled with steel S355 and initial imperfection nº3. An explanation for this is that the web becomes weaker due to being more loaded in compression when single-sided stiffeners are used in girders with the initial imperfection nº3.

The minimum inertia requirement is far from the inertia necessary for the single and double-sided stiffeners to allow the girder to reach its maximum load capacity and for the stiffeners to exhibit an appropriate behaviour.

![Variation of shear force with moment inertia of the stiffener](image)

Figure 4.46 - Shear force reached by the web panels with the moment of inertia of the stiffeners in plate girders made of HSS S690
4.4 Conclusions for the design of transverse stiffeners

Below are presented the main conclusions from the parametric study using plate girders made of steel S355 and HSS S690 fitted with single and double-sided stiffeners.

✓ The simplifications considered to evaluate the forces acting in the stiffener proved to be satisfactory.
✓ From the initial imperfections considered, the initial imperfection nº3 is the most demanding for the stiffeners and consequently to the plate girder design.
✓ The axial force diagram has a parabolic shape and its maximum occurs at about mid-height of the stiffener, which indicates that the stiffener is being loaded in the top half and unloaded in the bottom half.
✓ The stiffener is not subjected to axial force until the web panel reaches its shear buckling resistance.
✓ In the post-buckling phase, the compression force in the stiffener increases steadily and proportionally to the increase of the shear force, but with less intensity.
✓ The ratio $\alpha$ is higher in plate girders modelled with HSS S690 than with steel S355, which indicates that the stiffeners with HSS S690 are more axially loaded.
✓ The bending moment increases as the dimensions of the stiffener become larger when single-sided stiffeners are adopted, due to the eccentricity between the web and the centre of gravity of the stiffener.
✓ For the same cross-sectional area, the single-sided stiffeners have a greater bending moment, also due to the eccentricity between the web and the centre of gravity of this type of stiffener.
✓ The maximum compression force acting in the stiffener from the FE models is much lower than the compression force used to design the stiffener according to EC 3-1-5.
✓ The minimum inertia requirement from EC 3-1-5 is not enough to ensure that the stiffeners have a satisfactory behaviour and that the plate girder is able to reach its maximum resistance.
✓ The forces caused by the buckled web panel to the stiffener should be better investigated since they are responsible for an important change of the stiffener mean compressive stress state. It should also help in the interpretation of the origin of the bending moment diagram shape of double-sided stiffeners.
5 EVALUATION OF THE LATERAL FORCES ACTING ON THE STIFFENER

It has been concluded from the previous chapter that the compression force acting in the stiffener is far less than the force adopted to design the stiffeners according to EC 3-1-5. The goal of this chapter is to evaluate the lateral loading acting on the stiffener, which is not directly taken into account on the standard design. For that purpose, the outstand of the stiffener has been replaced by lateral supports that only restrain the out of plane displacements of the web. These simplified FE model set-up also aims to understand which of the two aspects of the design — strength or flexural rigidity — is the most important characteristic, since the models simulate a plate girder with intermediate transverse stiffeners that have a very large moment of inertia and with an area, to resist the compression force, equal to the effective area of the web. Figure 5.1 shows the plate girder with lateral supports at the section of the intermediate transverse stiffeners.

5.1 Plate girders made of steel S355

The stiffeners studied in Chapter 4 were composed by two parts: the web and the outstand. In this chapter the outstand of the stiffener is replaced by lateral supports and their function is to allow the division of the web panel into smaller panels, simulating the function of a real stiffener by only providing elastic lateral support. The goal is to study the distribution and magnitude of the lateral forces caused by the buckled web panel along the height of the stiffener.

A challenge to this set-up is to see if the plate girder can reach the same peak loads it achieved when modelled with real transverse stiffeners since it is only able to withstand the compression force installed in the stiffener with the web part.

The lateral supports could have been modelled as rigid or as flexible supports. In this case, the lateral supports have been modelled with elastic springs with constant stiffness $k$. These supports must be stiff enough, so the section of stiffener deflects very little, allowing it to maintain a line of approximately zero displacement from its initial position. A stiffness of $k = 10 \text{kN/mm}$ has been adopted.
5.1.1 Behaviour of the plate girder

To analyse the behaviour of the plate girder the load/deflection curves must be plotted to determine if this FE modelling set-up simulates accurately the behaviour of a plate girder modelled with real stiffeners. Figure 5.2 shows the load/deflection curves of the FE models made of steel S355 with initial imperfections nº 1 and 3. The load seems to increase linearly until it reaches about 90% of its maximum capacity, for a mid-span deflection of about 4 mm. From then on, the plate girder loses some rigidity and the load peaks for a mid-span vertical deflection of about 7 mm and is able to deflect under an approximately constant load for both initial imperfections. The plate girder with imperfection nº 1 reached a peak load of 518.9 kN, while the girder with imperfection nº 3 reached a peak load of 510.6 kN. These loads are very similar to the ones obtained when the girders were modelled with real stiffeners, so it can be concluded that this FE modelling set-up with the elastic springs simulates well the behaviour of a plate girder designed with intermediate stiffeners.

![Load/deflection curves of plate girders made of steel S355 with elastic lateral supports](image)

Figure 5.2 - Load/deflection curves of plate girders made of steel S355 with elastic lateral supports

Figure 5.3 shows the evolution of shear force and maximum compression force in the web, at the section of the stiffener, with the mid-span deflection. As seen on the girder modelled with real stiffeners, the compression force in the web is almost null until a mid-span deflection of about 1.5 mm. From then on, the force in the web increases with an approximately linear slope until the plate girder reaches its peak load. The maximum compression force in the web occurs approximately for the same mid-span deflection the girder reaches its peak load. The shape of the shear force and compression force curves are very similar for both initial imperfections.

The compression force at the section of the stiffener is being resisted by a certain width of web adjacent to the lateral supports. Figure 5.4 shows the evolution of the compression force diagrams with the mid-span deflection of the plate girder. The diagrams have a well-defined parabolic shape until a mid-span deflection of 6 mm and the maximum force occurs approximately at mid-height of the web.
After a mid-span deflection of 6 mm the diagram maximum force decreases, and its shape is also parabolic, although not so well defined. Another characteristic of the diagrams is that the compression force is significantly higher in the plate girder with imperfection nº3. While under peak load the maximum compression force in the web is \(-38.1\) kN for the plate girder with imperfection nº1 and \(-53.1\) kN for the plate girder with imperfection nº3. This trait of imperfection nº3 was also noticed when the plate girders were modelled with real stiffeners. Even though the plate girders with imperfection nº1 are able to carry more load, the compression force at the section of the stiffener is always higher for the plate girders with imperfection nº3.

5.1.2 Lateral loading

To simulate the flexural rigidity of the stiffener, its outstand has been replaced by lateral supports. These lateral supports could be modelled as rigid or semi-rigid. At first rigid supports were used since they were easier to model, and the section of the stiffener would maintain a line of zero displacement from its initial position. This approach was later abandoned because the use of rigid supports was influencing the values of the reaction forces. The supports were then modelled as semi-rigid by springs with a linear behaviour. Each node of the meshed web has a lateral support, at the section of the stiffener, as seen in Figure 5.1. With this modelling the distribution of the reaction forces in the supports are smoother along the height of the web. The web does not maintain a line of zero displacement from its initial position, but the springs were modelled with such rigidity that its displacements can be neglected.
The physical meaning of the reaction forces obtained from the FE models are the forces the transverse stiffener would do to counteract the action caused by the buckled web panel. It is more interesting to work with the symmetrical of the reaction forces, because their physical meaning is the loading caused by the web panel to the stiffener. An evaluation should be made, not only to the shape of the lateral loading, but also to the bending moment diagram consequent of that loading. The stiffener is treated as a simply supported beam with a span equal to the height of the web panel $h_w$. Because the loading in the stiffener is not self-balanced, the support reactions of the beam have been calculated to ensure the equilibrium and are shown in orange in the lateral force diagrams.

The lateral forces acting in the stiffener and the bending moment diagram they cause can be seen in Figure 5.5. The lateral loading has an “S” shape. At the top and bottom the lateral forces have a positive signal and at mid-height the forces have a negative signal. This characteristic is observed in the plate girders with both initial imperfections, although the forces at the middle of the stiffener have greater magnitude for the girder with imperfection nº1. The sum of the lateral forces is +6.0 kN for the girder with imperfection nº1 and +6.8 kN for the girder with imperfection nº3. The bending moment diagram of
the stiffener of the girder with imperfection nº1 has a similar shape to its lateral loading. It is positive at both the top and the bottom, but it is negative at mid-height. The maximum positive value is +0.29 kNm and the maximum negative value is −0.17 kNm. While the bending moment diagram in the stiffener of the girder with imperfection nº3 is always positive and its shape is almost parabolic, peaking at +0.53 kNm.

Figure 5.5 - Lateral loading and bending moment diagram while the plate girder made of steel S355 is under peak load

The evolution of the bending moment diagram resulting from the lateral forces acting on the stiffener for the plate girders with imperfections 1 and 3 are shown in Figure 5.6. The bending moment diagram of the stiffener in the girder with imperfection nº1 maintains its initial “S” shape. The maximum negative bending moment occurs for a mid-span deflection of the girder of 5 mm. After that the maximum negative value decreases, while the maximum positive value increases. The bending moment diagram for a mid-span deflection of between 7 mm and 9 mm is very similar. This shows the web has maintained its lateral loading in the stiffener while the plate girder is deflecting under an approximately constant load. The
bending moment diagram has, for all mid-span deflections, a positive signal in the stiffener of the girder with initial imperfection nº3. Until a mid-span deflection of the girder of 6mm, the value of the bending moment at mid-height decreases. This decrease is caused by the lateral loading with a negative signal that is acting at mid-height of the web. Although the plate girder is deflecting under an approximately constant load, the bending moment diagram keeps increasing and its curve starts to take a more parabolic shape. The magnitude of the diagram increases due to a decrease in the load magnitude at mid-height of the web that has a negative signal and also due to the increase of load magnitude that has a positive signal at the top and bottom the web.

![Figure 5.6](image)

Figure 5.6 - Evolution of the bending moment diagram caused by the lateral loading in the plate girder made of steel S355

5.2 Plate girders made of HSS S690

5.2.1 Behaviour of the plate girder

The same FE model set-up has been used to study the lateral forces acting in the intermediate stiffeners of plate girders made of HSS S690. The rigidity of the springs has been kept the same because it also leads to displacements that can be neglected.
The load/deflection curves of the plate girders with initial imperfections nº1 and 3 have been plotted in Figure 5.7. The curve of the girder with initial imperfection nº3 started to lose its rigidity before the girder with initial imperfection nº1. Apart from that, the two curves are very similar and both plateau after the girder reaches its peak load which was 896.8 kN for the girder with imperfection nº1 and 895.9 kN for the girder with imperfection nº3.

Figure 5.7 - Load/deflection curves of plate girders made of HSS S690 with lateral supports

In Figure 5.8 the shear force in the web panels and the maximum compression force registered in the web have been plotted with the mid-span deflection of the girder for the initial imperfections nº1 and 3. The maximum compression force in the web is almost zero until a mid-span deflection of about 1.5 mm for the plate girders with both initial imperfections. The force in the web then starts to increase steadily with a linear slope and reaches its peak for a mid-span deflection of the girder of about 12 mm.

Figure 5.8 - Shear force and maximum compression force in the web with mid-span deflection of the plate girder made of steel S355 with lateral supports

a) Imperfection nº1

b) Imperfection nº3
The maximum compression force in the web was $-106.6\,\text{kN}$ and $-112.3\,\text{kN}$ for the girders with imperfection nº1 and 3 respectively. For a mid-span deflection of 12 mm, the web panels have not yet achieved their maximum shear force, although when the mid-span deflection increases the shear force does not rise significantly and it can be said the curve has plateaued. The peak shear force in the web panels is reached for a mid-span deflection of about 20 mm, but while the shear force has plateaued from 12 mm to 24 mm, the maximum compression force in the web has been decreasing. The slope of the maximum compressive force in the web is higher for the plate girder with imperfection nº3.

The evolution of the compression force diagram in the web with the mid-span deflection of the girder has been plotted in Figure 5.9 for the girders with imperfections nº1 and 3. The shape of the diagrams is parabolic, and the compression force increases steadily until a mid-span deflection of 12 mm. For a mid-span deflection greater than 12 mm, the maximum compression force in the web decreases and its parabolic shape is not so well defined because the compression force in the bottom of the web has increased. The shape of the diagrams of the girders with imperfections nº1 and 3 is very similar, the only difference is that the maximum force in the web of the girder with imperfection nº3 decreases more rapidly, as seen in Figure 5.8. The maximum compressive force registered in the web while the girder was under peak load was $-91.1\,\text{kN}$ for the girder with imperfection nº1 and $-82.9\,\text{kN}$ for the girder with imperfection nº3.

Figure 5.9 - Evolution of the axial force diagram in the web for the plate girder made of HSS S690 with lateral supports
5.2.2 Lateral loading

The lateral loading caused by the buckled web panel and bending moment diagram consequent of the equilibrium of the lateral loading in a simply supported beam, are presented in Figure 5.10. The lateral loading also has an “S” shape, like the ones in the plate girders made of steel S355.

The major difference between the lateral loading of the two girders is that the lateral forces are negative in the middle of the web for the girder with imperfection nº1, while the forces are always positive in the girder with imperfection nº3 for the exception of the bottom part of the web, where there are some forces with negative signal. The bending moment diagram caused by the lateral loading in the girder with imperfection nº1 has a positive signal throughout its height, although it decreases at mid-height due to the negative loading in the middle section. The maximum value registered was 1.54 kN.m and occurred at the bottom third of the web.

Figure 5.10 - Lateral loading and bending moment diagram while the plate girder made of HSS S690 is under peak load
The bending moment diagram caused by the lateral loading of the girder with imperfection n°3 has a parabolic shape and registers higher values, because the lateral forces have a positive signal, for the exception of the bottom part where there are some forces with negative signal. The maximum value of the bending moment was 2.03 kNm and occurred approximately at mid-height of the web.

The evolution of the bending moment diagram, resulting from the lateral loading caused by the buckled web panel, with the mid-span deflection of the girder is shown in Figure 5.11. The bending moment of the stiffener with imperfection n°1 is negative at the middle of the stiffener until a mid-span deflection of 9 mm. After that the diagram is always on the positive side and the maximum values occur in the bottom third of the web.

The bending moment diagram resulting from the lateral loading caused by the buckled web panel in the girder with imperfection n°3 is always positive and the maximum value occurs approximately at mid-height of the web. The values are significantly higher when compared with the values from the girder with imperfection n°1, because the lateral forces at mid-height of the web, although small, have a positive signal in the girder with imperfection n°3.

Figure 5.11 - Evolution of the bending moment diagram caused by the lateral loading in the plate girder made of HSS S690
5.3 Discussion of results

The plate girders made of steel S355 modelled with lateral restraints at the section of the stiffener were able to carry similar loads to the ones obtained by the plate girders modelled with effective stiffeners and the peak load was slightly higher when the initial imperfection no1 was used. While the peak load reached by the plate girders made of HSS S690 was also similar to the ones obtained by plate girders with effective stiffeners, there was almost no difference in the peak load achieved, regardless of the initial imperfection used.

If we consider that the compression force is resisted by an effective web width of $2 \times 15\varepsilon t_{w}$, the maximum force the “stiffener” can carry is $-78.0$ kN and $-108.7$ kN for plate girders made of steel S355 and HSS S690 respectively. The maximum compression force registered in the web for plate girders made of steel S355 was $-44.7$ kN for the initial imperfection no1 and $-53.1$ kN for the initial imperfection no3, well below the compression force of $-78.0$ kN, that the effective area of the web can resist. While the maximum compression force registered in the plate girders made of HSS S690 with initial imperfection no1 and 3 was $-106.6$ kN and $-112.3$ kN respectively. This compression force is very near or over the maximum value according to the effective area admitted being resisting the compression force. This might explain why the plate girders made of HSS S690 with initial imperfection no1 and 3 resisted approximately the same load, which was expected to be higher when the initial imperfection no1 was used.

It can be concluded that, even with the small contribution of the web to resist the compression force, the plate girders are able to carry approximately the same load, which emphasises that the compression force in the stiffener is very small and that the main characteristic of the stiffener is its flexural rigidity in order to provide an effective support to the web panel.

The lateral displacement contours of the web panels B1 and B2 is plotted in . The lateral supports have effectively sub-divided the web panel by providing a rigid support. Since the lateral supports were modelled as elastic springs with a constant stiffness, the shape of the displacement of the web in the $x$ axis, at the section of the stiffener, is proportional to the shape of the lateral loading. As an example, we can see that for the plate girder with imperfection no1, made of steel S355, the green/red buckles of the web were responsible for the outwards deflection of the supports at the top and bottom of the web. And the blue buckles were responsible for the inwards deflection of supports at mid-height of the web.

Regardless of the steel grade adopted for the plate girder, the equilibrium of the lateral forces led to a higher bending moment when the initial imperfection no3 was used, proving that it is also the most demanding in terms of lateral forces provoked by the web panel. The values and shape of the bending moment diagrams resultant from the lateral forces are very similar to the diagrams obtained for double-sided stiffeners. The shape and magnitude of the bending moment values of the single-sided stiffeners are different which is explained by the eccentricity of its centre of gravity to the web.
The lateral loading, and consequently the bending moment, in the HSS S690 plate girders are higher because the web panel, for both steel grades, buckles for the same shear force and that leads to a long-lasting post-buckling phase for HSS S690 plate girders since they have a higher load carrying capacity.

5.4 Conclusions from the evaluation of the lateral loading acting on the stiffener

Below are presented the main conclusions from the evaluation of the lateral loading acting on the transverse stiffeners of plate girders made of steel S355 and HSS S690.

✓ The compression force acting at the section of the stiffener to balance the vertical component of the inclined tensile membrane field, can be carried by the web adjacent to the lateral supports.
and both its shape and magnitude of the axial force diagram are very similar compared to the diagrams of real stiffeners resulting from a FE analysis.

✓ Between area and moment of inertia, the most important geometric feature of the stiffener for the structural behaviour of the stiffened web panels is the moment of inertia, since the effective web width by itself is able to carry the compression force if laterally supported.

✓ The lateral loading acting on the stiffener explains the shape and magnitude of the bending moment diagrams of double-sided stiffeners. The single-sided stiffeners are also affected by this lateral loading, but its bending moment also has a large contribution resultant from its eccentricity to the web panel that has not been taken into consideration by using this FE model set-up.
6 NEW PROPOSAL FOR INTERMEDIATE TRANSVERSE STIFFENERS DESIGN

The goal of this chapter is to develop a simple procedure to design the intermediate stiffeners, less conservative than the actual design procedure according to EC 3-1-5 for steel grades S355 and HSS S690. This variant design method should be able to accurately simulate the internal forces and the behaviour of the single and double-sided stiffeners from the parametric study developed in Chapter 4.

6.1 Variant design method

From the FE analysis conducted in Chapters 4 and 5, it has been concluded that the intermediate transverse stiffeners are responsible for two main functions during the post-buckling phase:

- To provide lateral support to the buckled web panels;
- To resist a compression force resulting from the equilibrium of the vertical component of the diagonal tensile membrane field.

The bending moment in the stiffener is caused by the lateral forces resulting from the buckled web panel and, in case single-sided stiffeners are used, from the eccentric centre of gravity of the effective cross section.

The variant design method should be able to simulate the results obtained when the initial imperfection nº3 is adopted, since this imperfection leads to a larger axial force, bending moment and lateral mid-height deflection of the stiffener.

Since the bending moment diagrams obtained from the parametric study developed in Chapter 4 have a parabolic shape when single-sided stiffeners were used, for design purposes, it can be simulated by an equivalent uniformly distributed lateral load \( q_{eq} \) acting along the height of the stiffener, which produces the same bending moment and is capable of simulating the lateral deflection. The bending moment diagrams of the double-sided stiffeners, in plate girders with imperfection nº3, do not have a parabolic shape and their peak value, when compared to a single-sided stiffener with the same outstand area, is significantly lower. So, if an equivalent lateral load is used to simulate the bending moment it will not accurately predict the shape of the diagram, but it will lead to a more conservative design for the double-sided stiffeners case.

The effective cross section of the stiffener has been kept the same as defined in EC 3-1-5. This means the area of the stiffener is the sum of the area of the outstand plus the area within a width of web equal to \( 15 \varepsilon t_w \) for both sides, but not more than the actual dimension available.

Figure 6.1 shows the general calculation model used to design the stiffener that is admitted being simply supported at the flanges and subject to an equivalent uniformly distributed lateral load that simulates
the effects that produce the bending moment. The magnitude of the axial force, admitted constant along the height of the stiffener, can be evaluated by Eq. (6.1)

\[ N_{Ed} = \alpha \cdot (V_{Ed} - V_{CR}) \]  

(6.1)

where \( \alpha \) represents the percentage of post-buckling shear force that passes through the stiffener.

The total bending moment in the stiffener results from the effects of the lateral loading \( M_{LL}^{IMP 3} \), its centre of gravity \( M_{CG} \) and the second order effect \( M_\delta \). The maximum bending moment resulting from the equilibrium of the lateral loading of the buckled web when the plate girder has the initial imperfection no3 \( M_{LL}^{IMP 3} \) already includes the effect of the initial maximum amplitude \( e_0 \) of the stiffener. Equations (6.2) to (6.5) describe how to reach the equivalent lateral uniform load \( q_{eq} \):

\[ M_{eq} = M_{st} \]  

(6.2)

\[ \frac{q_{eq} \times h_w^2}{8} = M_{LL}^{IMP 3} + M_{CG} + M_\delta \]  

(6.3)

\[ \frac{q_{eq} \times h_w^2}{8} = M_{LL}^{IMP 3} + N_{Ed} \times e_{CG} + N_{Ed} \times \delta \]  

(6.4)

\[ q_{eq} = \frac{8 \times N_{Ed}}{h_w^2} \left( M_{LL}^{IMP 3} \frac{N_{Ed}}{N_{Ed}} + e_{CG} + \delta \right) \]  

(6.5)

The maximum deflection of the stiffener can be obtained by Eq. (6.6), assuming a simple supported beam loaded with the equivalent uniformly distributed load \( q_{eq} \):

\[ \delta = \frac{5 \times q_{eq} h_w^4}{384 \times E I_{st}} \]  

(6.6)

The intermediate transverse stiffener should be verified using a second order elastic analysis, verifying the following criteria of strength and rigidity:
The maximum stress in the stiffener $\sigma$, resulting from the axial force $N_{Ed}$ and the uniformly distributed lateral load $q_{eq}$, should not exceed the yield stress $\sigma_y$;

The deflection of the stiffener $\delta$, resulting from the uniformly distributed lateral load, must not exceed the maximum admissible deflection $\delta_{adm}$, which presently according with the EC 3-1-5 is stated to be $h_w/300$, but it is expected to be reduced to $h_w/500$ in the new version of the code.

The deflection of the stiffener can be calculated iteratively, or the value can be taken as the maximum admissible.

### 6.2 Discussion of results

A sensitivity analysis to the mid-height deflection of the stiffener should be made to establish a value that can be considered the maximum admissible deflection to be used for designing the stiffener. The variation of the peak shear force in the web panel with the deflection of the stiffener has been plotted in Figure 6.2 for the FE models with steel S355 and in Figure 6.3 for the FE models with HSS S690.

The figures only show the deflection values until $h_w/1800$, missing some values of the shear force of the FE models with double-sided stiffeners developed in Chapter 4, because their deflection rapidly tended to zero.

In Figure 6.2 the shear force values are more spread along the axis of the deflection of the stiffener when compared with Figure 6.3, because the deflection of the stiffeners in the girders with steel S355 tended more rapidly to zero. This is easily understood, since the dimensions of the stiffeners are similar and the stiffeners in the girders with HSS S690 are more loaded laterally and in compression, which leads to a greater deflection.

Because there were no plate girders made of steel S355 with double-sided stiffeners for a deflection between $h_w/150$ and $h_w/400$ in the parametric study developed in Chapter 4, an additional FE model with $b_{st} = 28$ m has been added. The plate girder reached a shear force of 250.8 kN and the mid-height lateral deflection of the stiffener was 2.25 mm ($h_w/267$).

For a deflection of the stiffener lower than $h_w/300$ there is no significant increase of shear force in the web panel for both single and double-sided stiffeners in plate girders modelled with steel S355 and HSS S690. For this reason, the value $h_w/300$ has been defined as the maximum admissible deflection for the stiffener.

The value of $\alpha$ is the percentage of post-buckling shear force that goes through the stiffener as compression. The values of $\alpha = 0.4$ for the plate girders made of steel S355 and $\alpha = 0.5$ for the plate girders made of HSS S690 have been adopted. This means that, for design purposes, an axial force of
−49.6 kN and −123.5 kN is acting in the transverse stiffener in plate girders made of steel S355 and HSS S690 respectively, which are slightly higher compression forces compared to the ones obtained from the FE models.

![Variation of shear with the deflection of the stiffener - S355](image)

Figure 6.2 - Variation of shear force in the web panel with the deflection of the stiffener for plate girder modelled with steel S355 and initial imperfection nº3

![Variation of shear with the deflection of the stiffener - HSS S690](image)

Figure 6.3 - Variation of shear force in the web panel with the mid-height deflection of the stiffener for plate girder modelled with HSS S690 and initial imperfection nº3

Table 6.1 presents the results from the design of the transverse intermediate stiffeners according to this simple variant method. By comparing the outstand area of the single and double-sided stiffeners it is possible to come up with some conclusions regarding the efficiency of each solution, because the outstand area is proportional to the volume of steel added to the plate girder to form the cross section of the stiffener (and if we ignore the fact that the double-sided stiffeners need twice the number of welds than a single-sided stiffener, we can also assume that the outstand area is proportional to the cost).
Table 6.1 - Results from the variant design method of the intermediate transverse stiffeners

<table>
<thead>
<tr>
<th></th>
<th>Steel S355</th>
<th>HSS S690</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Double-sided</td>
<td>Single-sided</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.4</td>
<td>0.4</td>
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<tr>
<td>$N_{Ed}$ (kN)</td>
<td>49.6</td>
<td>49.6</td>
</tr>
<tr>
<td>$b_{st}$ (mm)</td>
<td>31.8</td>
<td>46.2</td>
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<tr>
<td>$A_{outstand}$ (mm$^2$)</td>
<td>202</td>
<td>213</td>
</tr>
<tr>
<td>$I_{st}$ (cm$^4$)</td>
<td>7.8</td>
<td>10.5</td>
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<tr>
<td>$q_{eq}$ (kN/m)</td>
<td>12.5</td>
<td>26.3</td>
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<tr>
<td>$M_{eq}$ (kNm)</td>
<td>0.56</td>
<td>1.18</td>
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<tr>
<td>$M_{CC}$ contribution to $M_{eq}$</td>
<td>0 %</td>
<td>50 %</td>
</tr>
<tr>
<td>$M_{LL}$ contribution to $M_{eq}$</td>
<td>89 %</td>
<td>42 %</td>
</tr>
<tr>
<td>$M_{\delta}$ contribution to $M_{eq}$</td>
<td>11 %</td>
<td>8 %</td>
</tr>
<tr>
<td>$\sigma_{inside}$ (MPa)</td>
<td>-355</td>
<td>-259</td>
</tr>
<tr>
<td>$\sigma_{outside}$ (MPa)</td>
<td>125</td>
<td>292</td>
</tr>
<tr>
<td>$\delta$ (mm)</td>
<td>1.3</td>
<td><strong>2.0</strong></td>
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<tr>
<td>$\delta$ in relation to $h_w$</td>
<td>$h_w/462$</td>
<td>$h_w/300$</td>
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</tbody>
</table>

When steel grade S355 is used, the outstand area of the stiffeners is very similar, but the single-sided stiffener has about 5% more outstand area. Although in this case it is hard to compare which type is more efficient because the double-sided stiffener was conditioned by strength, while the single-sided stiffener was conditioned by rigidity. When HSS S690 is used, the outstand area of the single-sided stiffener is considerably higher (about 88%) than the area of the double-sided stiffener and both were conditioned by the present deflection limit.

When a more robust single-sided stiffener is used, its inertia increases, which is beneficial to control the deflection, but the problem is that part of that benefit is lost due to the increase of the distance between the web and the centre of gravity of the effective cross-section, which increases the bending moment. On the other hand, the centre of gravity of a double-sided stiffener always coincides with the web, so there is no aggravation of the bending moment when its dimensions increase.

A significant percentage of the bending moment in the single-sided stiffeners is caused by its eccentricity, while in the double-sided stiffeners the most important factor are the lateral loading caused by the buckled web, that also includes the effect of the initial eccentricity of the stiffener of $h_w/200$ (value used already in accordance with the new proposal of revision of the EC3-1-5). The second order $P-\Delta$ effect caused by the lateral displacement $\delta$ has a small contribution on the total bending moment, when compared to the other factors.

Figure 6.4 compares the bending moment diagram considered for the design of the single-sided stiffener ($b_{st} = 74.3$ mm) for the plate girder made of HSS S690 and the bending moment from the stiffeners
with $b_{st} = 70 \text{ mm}$ and $b_{st} = 80 \text{ mm}$, obtained from the FE analysis. The diagram used for design, resulting from the uniformly distributed lateral load, is a good approximation of the real diagram, but it has, for the most part, greater bending moment values, that might lead to an overestimation of the mid-height deflection. The single sided stiffener with $b_{st} = 70 \text{ mm}$ has a deflection of $h_w/340$ that is already lower than the present maximum acceptable and would therefore be a feasible stiffener. But because the bending moment diagram used for design is always slightly higher than the actual bending moment diagram, the deflection is overestimated. This is not an issue, because an overestimated lateral deflection is on the safe side and this method is still not so conservative as the standard one. The peak bending moment for the design of the stiffener is between the values of the $b_{st} = 70 \text{ mm}$ and $b_{st} = 80 \text{ mm}$ and show that this procedure leads to a good estimate of the bending moment in the stiffener.

The remaining results of the stiffeners designed by this simple variant method also led to a good estimate of the bending moment and lateral deflection when compared to the ones from the FE analysis and the results were still always a bit on the safe side.

![Diagram of a stiffener and bending moment diagram comparison](image)

**Figure 6.4** - Comparison of the bending moment diagram considered for the design of the single-sided stiffener with HSS S690 and the bending moments from the FE analysis
7 GENERAL CONCLUSIONS AND FURTHER DEVELOPMENTS

The compression forces in the stiffeners, obtained from the FE analysis, are much lower compared to the forces used to design the transverse stiffeners according to EC 3-1-5. During the post-buckling phase, only about 30% to 40% of the shear force in the web panel goes through the stiffener as compression, while the code assumes that all post-buckling shear produces compression in the stiffeners. Only web panels with an aspect ratio equal to one \( (a/h_w = 1) \) have been studied. The compression force in the stiffener should be evaluated for web panels with different geometries to confirm these percentages of post-buckling shear force in the stiffeners.

The ratio of post-buckling shear force that goes through the stiffener depends on the initial imperfection of the plate girder. From the three initial imperfections studied, imperfections nº3 and 9 yield the largest compression forces. The initial imperfection nº3 has been chosen as the most demanding, because it also led to a greater bending moment and higher lateral deflection of the stiffener. Since the axial force is bigger when the stiffener has an initial bow shape, the change in the compression force should be investigated for different amplitudes of this initial imperfection, to verify if larger stiffener compression forces can be found. For the development of future initial equivalent imperfections, a combination of the most relevant buckling modes should be used, and a torsional imperfection should be added to the stiffener to increase its susceptibility to the phenomenon of torsional buckling.

The bending moment in the stiffener is much more sensitive to the initial imperfection in comparison to the axial force. It is caused by the lateral loading due to web buckling, initial amplitude of the imperfection, second order effects caused by the lateral deflection of the stiffeners and finally also due to the asymmetric cross-section of the stiffener with respect to the web. This last effect only exists if a single-sided stiffener is adopted and, in that case, it accounts for the largest contribution to the bending moment. Although the single-sided stiffeners can be considered less efficient they are used more often in bridge design, because a plate girder with concealed stiffeners is considered to be more aesthetically pleasing.

It has been confirmed that the main function of the stiffener is to provide a rigid lateral support to the buckled web panels and that the compression force can be withstood by the web adjacent to the stiffeners. The stiffeners are, in most cases, conditioned by rigidity and not by strength. In that sense, to reach the maximum plate girder resistance, the rigidity criteria imposed to the stiffeners in the EC 3-1-5 should be increased. Moreover, the use of hybrid plate girders where the intermediate transverse stiffeners have a lower yield-stress than the web panel might be worth to be investigated, because it should lead to a more economical design.
The shape of the lateral loading acting on the stiffener is caused by the buckled web, so it is expected that those lateral forces might change their shape and magnitude, depending on the aspect ratio and slenderness of the web panel. A parametric study should be performed to develop a general expression for the calculation of those forces for a wide range of geometries to allow the complete definition of the equivalent uniformly distributed lateral load used for the proposed design method. The use of the most relevant buckling modes of the plate girder for the initial equivalent imperfection should be taken into consideration to investigate if they lead to larger lateral forces acting at the stiffener’s section.

The analysis was developed only for plate girders with transverse stiffeners without longitudinal stiffeners. On one hand, the longitudinal stiffeners will induce additional elastic support to the transverse stiffeners, but on the other hand they will change the compression force distribution transmitted to the stiffeners. Therefore, this solution, generally used on bridge webs higher than 3 m deserves to be investigated in a separate way.

Finally, numerical results presented on this research work could / should be confirmed with a new laboratory testing campaign that can contribute for the improvement of the present stiffener’s design rules.
REFERENCES


[33] ABAQUS/CAE - Version 6.13
## ANNEX A

Table A.1 - Parametric study results of single-sided stiffeners in plate girders made of steel S355

<table>
<thead>
<tr>
<th>Initial imperfection nº</th>
<th>$b_{st}$ (mm)</th>
<th>Peak load $= 2V$ (kN)</th>
<th>$N_{total}$ (kN)</th>
<th>$M^-$ (kNm)</th>
<th>$M^+$ (kNm)</th>
<th>$\delta$ (mm)</th>
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<td>nº1</td>
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Table A.2 - Calculation of $V_{cr}$ and $V_{bt,Rd}$

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<th>$h_w$ (mm)</th>
<th>$a$ (mm)</th>
<th>$t_w$ (mm)</th>
<th>$f_{yw}$ (MPa)</th>
<th>$\sigma_E$ (MPa)</th>
<th>$k_r$</th>
<th>$\tau_{cr}$</th>
<th>$\lambda_w$</th>
<th>$\eta$</th>
<th>$\chi_w$</th>
<th>$V_{cr}$ (kN)</th>
<th>$V_{bt,Rd}$ (kN)</th>
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<td>HSS S690</td>
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<td>600</td>
<td>3</td>
<td>690</td>
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Table A.3 - Calculation of $V_{bf,Rd}$ and $V_{b,Rd}$

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<thead>
<tr>
<th>$b_f$ (mm)</th>
<th>$t_f$ (mm)</th>
<th>$f_{yf}$ (MPa)</th>
<th>$M_{Ed}$ (KNm)</th>
<th>$M_{f,Rd}$ (kNmm)</th>
<th>$c$ (mm)</th>
<th>$V_{bf,Rd}$ (kN)</th>
<th>$V_{b,Rd}$ (kN)</th>
<th>$V_{pl,Rd}$ (kN)</th>
<th>$V_{Rd} - V_{cr}$ (kN)</th>
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Table A.4 - Parametric study results of double-sided stiffeners in plate girders made of steel S355

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<tr>
<th>$b_{st}$ (mm)</th>
<th>Peak load = 2V (kN)</th>
<th>$N_{total}$ (kN)</th>
<th>$M^-$ (kNm)</th>
<th>$M^+$ (kNm)</th>
<th>$\delta$ (mm)</th>
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<td>Initial imperfection nº1</td>
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<tr>
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Table A.5 - Parametric study results of single and double-sided stiffeners in plate girders made of HSS S690

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<th>$N_{total}$ (kN)</th>
<th>$M^-$ (kNm)</th>
<th>$M^+$ (kNm)</th>
<th>$\delta$ (mm)</th>
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ANNEX B
Figure B.1 – Diagrams of the double-sided stiffeners fitted in plate girders made of steel S355 with initial imperfection nº1 ($b_{st} = 20 \text{ mm to } 80 \text{ mm}$)
Figure B.2 – Diagrams of the double-sided stiffeners fitted in plate girders made of steel S355 with initial imperfection nº3 ($b_{st} = 20$ mm to 80 mm)
Figure B.3 – Diagrams of the single-sided stiffeners fitted in plate girders made of steel S355 with initial imperfection nº1 ($b_{st} = 25$ mm to $40$ mm)
Figure B.4 – Diagrams of the single-sided stiffeners fitted in plate girders made of steel S355 with initial imperfection nº1 ($b_{st} = 50$ mm to $80$ mm)
Figure B.5 – Diagrams of the single-sided stiffeners fitted in plate girders made of steel S355 with initial imperfection nº3 ($b_{st} = 25$ mm to $40$ mm)
Figure B.6 – Diagrams of the single-sided stiffeners fitted in plate girders made of steel S355 with initial imperfection nº3 ($b_{st} = 50 \text{ mm to } 80 \text{ mm}$)
Figure B.7 – Diagrams of the single-sided stiffeners fitted in plate girders made of steel S355 with initial Imperfection nº9 ($b_{st} = 25$ mm to $40$ mm)
Figure B.8 – Diagrams of the single-sided stiffeners fitted in plate girders made of steel S355 with initial Imperfection nº9 ($b_{st} = 50 \text{ mm to } 80 \text{ mm}$)
Figure B.9 – Diagrams of the double-sided stiffeners fitted in plate girders made of HSS S690 with initial imperfection nº3 ($b_{st} = 20 \text{ mm to } 50 \text{ mm}$)
Figure B.10 – Diagrams of the double-sided stiffeners fitted in plate girders made of HSS S690 with initial imperfection nº3

\( b_{st} = 60 \text{ mm to } 80 \text{ mm} \)
Figure B.11 – Diagrams of the single-sided stiffeners fitted in plate girders made of HSS S690 with initial imperfection nº3 ($b_{st} = 30\,\text{mm}$ to $60\,\text{mm}$).
Figure B.12 – Diagrams of the single-sided stiffeners fitted in plate girders made of HSS S690 with initial imperfection nº3 ($b_{st} = 70$ mm to 90 mm)
Figure B.13 – Deflection of the double-sided stiffeners fitted in plate girders made of steel S355
Figure B.14 – Deflection of the single-sided stiffeners fitted in plate girders made of steel S355
Figure B.15 – Deflection of the stiffeners fitted in plate girders made of HSS S690 with initial imperfection nº3