Multi-Higgs Models, Flavour and CP Violation

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September 2018

Abstract

We review Multi-Higgs Doublet Models (MHDMs) and discuss the proliferation of free parameters in their Yukawa sector. We show that a BGL inspired principle can be implemented in order to significantly reduce such number and subsequently performed a systematic analysis of all Two-Higgs Doublet Models (2HDMs) that satisfy it. This resulted in the discovery of two previously undiscovered classes of 2HDM. We also used experimental results to bound the tree-level scalar mediated Flavour Changing Neutral Couplings (FCNC) of MHDMs. The Unitary Flavour Violation (UFV) condition is introduced to control the intensity of FCNC. Amongst such models, two subclasses emerge as the most plausible extensions of the SM, left and right models. It is shown that the structure of the tree-level FCNC of these models is such that experimental bounds applied to them are significantly softened in comparison to the constraints applied to the general MDHM.

Keywords: MHDMs, FCNC, Abelian Flavour Symmetry, CP Violation

1. Introduction

Recently, there has been a special interest in the scalar sector of the SM and some of its extensions due to the discovery by both the ATLAS [1] and CMS [11] collaborations of a particle that can be interpreted as the Higgs boson of the SM. A central question that remains to be answered is whether the couplings of such particle to the quarks and leptons are those of the SM or whether nature chose a more complex scalar sector. The simplest extension of the scalar sector of the SM consists of adding more scalars [7, 17]. The first MHDM was introduced by Lee [19] in order to generate spontaneous CP violation. Given that CP may not be conserved in the scalar potential of a MHDM, these become interesting candidates to solve the problem of Baryogenesis [12, 14]. There are also dark matter candidates in the framework of MHDMs [4, 13] and a possible solution to the strong CP problem in the form of the Peccei-Quinn mechanism [22].

If no additional symmetries are introduced, the general MHDM has tree-level FCNC that must be suppressed in order to avoid violation of stringent experimental bounds. Glashow and Weinberg [16] and Paschos [20] proposed Natural Flavour Conservation in which the quarks of a given charge receive their mass from only one scalar. This solution can be implemented from a Z_2 symmetry, and it completely eliminates FCNC at tree-level. A more interesting approach to control FCNC is provided by BGL models [8], where there are tree-level FCNC in one sector of the theory but their flavour structure is completely defined by the CKM matrix. Minimal Flavour Violation (MFV) [5] has been proposed as a generalization of these models in which tree-level FCNC exist in both sectors of the theory while mantaining a structure that is completely defined by the CKM matrix.

Motivated by the success of BGL models in the reduction of the number of free parameters in their Yukawa sector, we propose a principle that significantly reduces the free parameters in the Yukawa sector of any MHDM that satisfies it. Then, we look for the full set of symmetry protected 2HDM that satisfy it as an alternative to the perturbative expansion of MFV.

Guided by NFC and BGL models, we introduce a condition that, if satisfied by a MHDM, ensures that its tree-level FCNC are controlled. As such, we develop a methodical approach to the suppresion of the tree-level FCNC of a MHDM that its larger in its scope than any of the current alternatives.

This paper is organized as follows. In Sec. 2, we settle the notation by reviewing the Yukawa sector of a MHDM, paying special attention to the number of free parameters in it as well as to the experimental constraints that it must satisfy. We start Sec. 3 by stating the principle which reduces the number of free parameters in the Yukawa sector of a MHDM. Then, we go over the proprieties of all 2HDMs that satisfy such principle. In Sec. 4, we introduce the condition that controls the tree-level FCNC of a MHDM. Then, we study the proprieties that arise from such condition and compare the experimental bounds appliable to such models to those that the general MHDM must satisfy. We also define two subclasses within such condition, comparing their proprieties to that of the initial condition. Finally, we offer our conclusions in Sec. 5.

2. Yukawa Sector of MHDM

In this paper, we will neglect the leptonic sector of MHDMs. As such, the Yukawa Lagrangian of the most general MHDM is given by

$$-\mathcal{L}_Y = \bar{q}_L^0 \phi_a \Gamma_a d_R^0 + \bar{q}_L^0 \tilde{\phi}_a \Delta_a u_R^0 + h.c., \quad (1)$$

where $\tilde{\phi}_a = i\sigma_2 \phi_a^*$. After SSB, the scalar fields ϕ_a may be decomposed as

$$\phi_a = e^{i\alpha_a} \begin{bmatrix} \varphi_a^+ \\ \frac{1}{\sqrt{2}}(v_a + \rho_a + i\eta_a) \end{bmatrix}. \quad (2)$$

We can always make a Weak Basis Transformation (WBT) to the Higgs basis

$$H_a = O_{ab} e^{-i\alpha_b} \phi_b = \begin{bmatrix} C_a^+ \\ \frac{1}{\sqrt{2}}(v\delta_{a1} + R_a + iJ_a) \end{bmatrix}, \quad (3)$$

where O is an orthogonal matrix defined by $O_{1a} = \frac{v_a}{v}$ with $v^2 = \sum_{i=1}^N v_i^2$. In the Higgs basis, the neutral interactions in Eq. 1 are given by

$$-\mathcal{L}_Y^0 = \frac{R_a}{v} \bar{q}_L^0 N_{qa}^0 q_R^0 + i\epsilon_q \frac{J_a}{v} \bar{q}_L^0 N_{qa}^0 q_R^0 + h.c., \quad (4)$$

where there is an implicit sum over $q = u, d$ and we have introduced $\epsilon_d = 1$ and $\epsilon_u = -1$. We have also defined the couplings

$$N_{da}^0 = \frac{1}{\sqrt{2}} v O_{ab} e^{i\alpha_b} \Gamma_b, \quad N_{ua}^0 = \frac{1}{\sqrt{2}} v O_{ab} e^{-i\alpha_b} \Delta_b. \quad (5)$$

Since $N_{q1}^0 = D_q^0$ and the remaining N_{qa}^0 are 3×3 complex arbitrary matrices, $36(N-1)$ new parameters arise in the Yukawa sector of the general MDHM. Assuming CP-conservation in the scalar potential of a MHDM implies that its physical scalars are obtained as

$$h_a = R_{ab} R_b, \quad j_a = J_{ab} J_b, \quad (6)$$

where R and J are real orthogonal matrices satisfying $J_{a1} = J_{1a} = \delta_{a1}$. After performing the transformation $q_X^0 = U_{qX} q_X$ that diagonalizes the mass matrices $D_q^0 = N_{q1}^0$, we write the interactions between the physical neutral scalars and the physical fermions

$$-\mathcal{L}_Y^0 = \frac{h_a}{2v} \bar{q}(A_{qa}^+ + A_{qa}^- \gamma_5) q + i\epsilon_q \frac{j_a}{2v} \bar{q}(A_{qa}'^- + A_{qa}'^+ \gamma_5) q, \quad (7)$$

where we defined $A_{qa}^\pm = \tilde{N}_{qa} \pm \tilde{N}_{qa}^\dagger$ and $A_{qa}'^\pm = \tilde{N}_{qa}' \pm \tilde{N}_{qa}'^\dagger$ with

$$\tilde{N}_{qa} = R_{ab} N_{qb}, \quad \tilde{N}_{qa}' = J_{ab} N_{qb}. \quad (8)$$

The existence of tree-level FCNC in MHDMs can be probed experimentally in direct and indirect processes. Direct processes are defined by the presence of scalar particles in either the initial or final state of the event, while in indirect these can only appear in intermediate states. Experimental results that constrain MHDMs at tree-level include the decay width of the top quark [21], the width of the (125) scalar detected at the LHC [18] and meson-antimeson mixing [9, 23].

Out of all these processes, it turns out that the most stringent constraint arises from meson-antimeson mixing. At tree-level, these processes are controlled by the following Feynman diagrams.

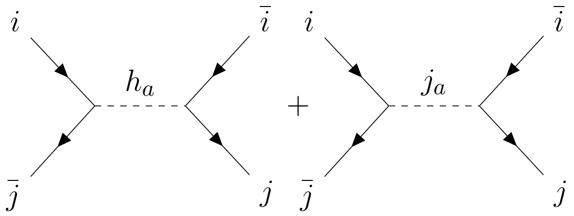

Figure 1: Tree-level Feynman diagrams that contribute to meson-antimeson mixing in a general MHDM.

From both diagrams, we compute the hadronic matrix element $M_{12}^P = \langle \bar{P}^0 | \mathcal{H}_{eff} | P^0 \rangle$ in the general MHDM to be

$$M_{12}^P = \sum_{a=1}^N \frac{m_P f_P^2}{48 v^2 m_{h_a}^2} \left[\chi^+ (A_{qa}^+)_{ji}^2 - \chi^- (A_{qa}^-)_{ji}^2 \right] - \sum_{a=2}^N \frac{m_P f_P^2}{48 v^2 m_{j_a}^2} \left[\chi^+ (A_{qa}'^-)_{ji}^2 - \chi^- (A_{qa}'^+)_{ji}^2 \right], \quad (9)$$

where we introduced

$$\chi^+ = \frac{m_P^2 - (m_i + m_j)^2}{(m_i + m_j)^2}, \qquad \chi^- = \frac{11m_P^2 - (m_i + m_j)^2}{(m_i + m_j)^2}. \tag{10}$$

Assuming that the scalar potential of the theory is such that $m_h \ll m_{h_a}, m_{j_a}$ with h being the (125) particle found by the ATLAS and CMS collaborations, we simplify Eq. 9 to

$$M_{12}^P = \frac{m_P f_P^2}{48 v^2 m_h^2} \left[\chi^+ (A_{q1}^+)^2_{ji} - \chi^- (A_{q1}^-)^2_{ji}\right], \tag{11}$$

from which we obtain the following constraints that must be satisfied by a valid MHDM

$$\left|\chi^+ (A_{q1}^+)^2_{ji} - \chi^- (A_{q1}^-)^2_{ji}\right| < \frac{24 v^2 m_h^2 \Delta m_P}{m_P f_P^2}. \tag{12}$$

The right-hand side of these relations can be controlled by using the Cauchy-Schwarz and the triangle inequalities to be smaller than

$$\left|\chi^+ (A_{q1}^+)^2_{ij} - \chi^- (A_{q1}^-)^2_{ij}\right| \leq (\chi^+ + \chi^-) \sum_b \Big[|(N_{qb})_{ij}|^2 + |(N_{qb})_{ji}|^2\Big] + 2(\chi^+ - \chi^-) \sum_b \mathrm{Re}\left[(N_{qb})_{ij} (N_{qb})_{ji}\right]. \tag{13}$$

For Yukawa couplings of the order one, i.e., $(\Gamma_a)_{ij} \sim (\Delta_a)_{ij} \sim 1$, we conclude that

$$\sum_b \left[|(N_{qb})_{ij}|^2 + |(N_{qb})_{ji}|^2\right] \sim m_t^2 c_1, \qquad 2\sum_b \mathrm{Re}\left[(N_{qb})_{ij} (N_{qb})_{ji}\right] \sim m_t^2 c_2, \tag{14}$$

where c_i is a function of couplings of order one given that $\frac{1}{\sqrt{2}} v \approx m_t$. Using Eqs. 12-14, we determine that the general MHDMs are in agreement with experimental results provided the following relation holds

$$1.1 c_1^u - c_2^u < 3 \times 10^{-9}, \quad 1.2 c_1^d - c_2^d < 9 \times 10^{-10}. \tag{15}$$

This results were obtained from the most stringent constraints on the up and down sector of the theory, which turned out to come from $D^0 - \bar{D}^0$ and $K^0 - \bar{K}^0$ mixing, respectively. Eq. 15 implies that a cancellation of the order of 10^{-9} is required in order for the "natural" MHDM to be in agreement with experiment.

3. Controlling Parameters

In order to reduce the number of free parameters in the Yukawa sector of a MHDM, we propose the following principle:

"Each line of the mass matrix of a quark of a given charge should receive contributions from one and only one Higgs doublet."

If such propriety is to be stable under renormalization, it must be implemented through a symmetry of the full Lagrangian that can at most be broken softly by a mass term in the scalar potential.

We have found that only five 2HDM satisfy this principle: type I and type II 2HDMs [7,16,20], BGL models [8] and two previously undiscovered models [2,3] which we present in the rest of this section.

3.1. gBGL Models

gBGL models [2] are 2HDMs that are invariant under the Z_2 symmetry

$$(q_L^0)_3 \to -(q_L^0)_3, \quad \phi_2 \to -\phi_2. \tag{16}$$

As a result of the application of this symmetry, the Yukawa textures of gBGL models are written in this WB as

$$\Gamma_1 \sim \Delta_1 = \begin{bmatrix} x & x & x \\ x & x & x \\ 0 & 0 & 0 \end{bmatrix}, \quad \Gamma_2 \sim \Delta_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ x & x & x \end{bmatrix}. \tag{17}$$

Noting that we have yet to defined gBGL models in a Weak Basis Invariant (WBI) form (Eq. 16 and Eq. 17 are Weak Basis dependent), we introduce the WBI conditions

$$\Gamma_2^\dagger \Gamma_1 = \Delta_2^\dagger \Delta_1 = \Delta_2^\dagger \Gamma_1 = \Delta_1^\dagger \Gamma_2 = 0, \ \Gamma_i \neq 0, \tag{18}$$

which are sufficient and necessary to ensure that a given texture belongs to a gBGL model.

We write the non-diagonal couplings of gBGL models in the Weak Basis (WB) of Eq. 17 as

$$(N_u)_{ij} = \left[t_\beta \delta_{ij} - (t_\beta + t_\beta^{-1}) n_i^* n_j\right] (m_u)_{jj}, \qquad (N_d)_{ij} = \left[t_\beta \delta_{ij} - (t_\beta + t_\beta^{-1}) V_{mi}^* V_{nj} n_m^* n_n\right] (m_d)_{jj}, \tag{19}$$

where $n_i = (U_{uL})_{3i}$ is an unitary vector. Since n only appears in Eq. 19 as the rephasing invariant combination $n_i^* n_j$, we can parametrize it as

$$n = (\cos\theta, e^{i\varphi_1}\sin\theta\cos\psi, e^{i\varphi_2}\sin\theta\sin\psi). \quad (20)$$

Thus, the Yukawa sector of gBGL models is parametrized by four parameters - two real angles and two complex phasis.

Notice that despite Eq. 17 suggesting that only two classes of BGL models are also gBGL models (tBGL and bBGL), this is not the case. In fact, all six classes of BGL models can be seen as special implementations of gBGL models defined by

$$\begin{aligned} n_d &= (V_{ud}^*, V_{cd}^*, V_{td}^*),\, n_u = (1,0,0), \\ n_s &= (V_{us}^*, V_{cs}^*, V_{ts}^*),\, n_c = (0,1,0), \\ n_b &= (V_{ub}^*, V_{cb}^*, V_{tb}^*),\, n_t = (0,0,1). \end{aligned} \quad (21)$$

The most general scalar potential of gBGL models is given by

$$\begin{aligned} V(\phi) &= \mu_{11}^2(\phi_1^\dagger\phi_1) + \mu_{22}^2(\phi_2^\dagger\phi_2) + \lambda_1(\phi_1^\dagger\phi_1)^2 \\ &+ \lambda_2(\phi_2^\dagger\phi_2)^2 + \lambda_3(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) \\ &+ \lambda_4(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1) + \lambda_5\left[e^{i\delta_5}(\phi_1^\dagger\phi_2)^2 + h.c.\right], \end{aligned} \quad (22)$$

where all parameters are real due to hermiticity. It is straightforward to prove that there is CP-conservation in this scalar potential, implying that the bound in Eq. 12 can be applied to gBGL models.

Using Eq. 19 on Eq. 12 results in the following constraint on gBGL models

$$c_{\beta\alpha}^2(t_\beta + t_\beta^{-1})^2|n_i n_j^*|^2 < \frac{12v^2 m_h^2 \Delta m_P}{5m_P^3 f_P^2}. \quad (23)$$

Out of all available meson states, the most stringent of these constraints arises from $D^0 - \bar{D}^0$ mixing. Given that $|n_1 n_2^*| \leq \frac{1}{2}$, we conclude that gBGL models will always be in agreement with experiment provided the following condition holds

$$|c_{\beta\alpha}(t_\beta + t_\beta^{-1})| < 0.02\,. \quad (24)$$

The existence of additional sources of CP and flavour violation in gBGL models has the potential to enhance the contributions to the BAU with respect to the SM expectations [15]. In particular, there is a WBI of dimension four in mass with a non-zero imaginary part [6, 10] given by

$$\mathrm{Im}\left[\mathrm{Tr}\left[N_u^0 D_u^{0\dagger} D_d^0 D_d^{0\dagger}\right]\right] = \mathrm{Im}\left[\mathrm{Tr}\left[N_u D_u V D_d^2 V^\dagger\right]\right]. \quad (25)$$

Using Eq. 19 on Eq. 25 and keeping only the leading contributions in the quark masses and powers of the Wolfenstein parameter λ [24] results in the following contribution to the BAU of gBGL models

$$BAU_{gBGL} \sim (t_\beta + t_\beta^{-1})\frac{m_t^2 m_b^2}{E^4}\mathrm{Im}\,[n_3^* n_2 V_{tb}^* V_{cb}], \quad (26)$$

where $E \sim 100\ GeV$ is an energy of the order of the electroweak scale. Since the prediction of the BAU in the SM is given by

$$BAU_{SM} \sim \frac{m_t^4 m_b^4 m_c^2 m_s^2}{E^{12}}\mathrm{Im}\,[Q_{ijkm}] \quad (27)$$

with $Q_{ijkm} = V_{ij}V_{km}V_{im}^*V_{kj}^*$, we conclude that gBGL models have room for an enhancement of the BAU of the order of

$$\begin{aligned} \frac{BAU_{gBGL}}{BAU_{SM}} \sim & 10^{16}(t_\beta + t_\beta^{-1})|n_2^* n_3| \times \\ & \times \sin[\arg(n_3^* n_2 V_{tb}^* V_{cb})]. \end{aligned} \quad (28)$$

3.2. jBGL Models

jBGL models [3] are 2HDMs that are invariant under the Abelian Flavour Symmetry (AFS)

$$\phi_2 \to \Upsilon\phi_2, \quad q_{L3}^0 \to \Upsilon^{-1} q_{L3}^0,\ d_R^0 \to \Upsilon^{-1} d_R^0, \quad (29)$$

where $|\Upsilon| = 1$. After applying this symmetry, it is straightforward to write the Yukawa textures of jBGL models in this WB as

$$\Gamma_1 \sim \Delta_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ x & x & x \end{bmatrix}, \quad \Gamma_2 \sim \Delta_1 = \begin{bmatrix} x & x & x \\ x & x & x \\ 0 & 0 & 0 \end{bmatrix}. \quad (30)$$

Since both Eq. 29 and Eq. 30 are WB dependent, we introduce the WBI conditions

$$\Gamma_2^\dagger\Gamma_1 = \Delta_2^\dagger\Delta_2 = \Delta_2^\dagger\Gamma_2 = \Delta_1^\dagger\Gamma_1 = 0, \quad \Gamma_i \neq 0 \quad (31)$$

that are necessary and sufficient to ensure that a given texture belongs to a jBGL model.

We write the non-diagonal couplings of jBGL models in the WB of Eq. 30 as

$$\begin{aligned} (N_u)_{ij} &= \left[t_\beta\delta_{ij} - (t_\beta + t_\beta^{-1})n_i^* n_j\right](m_u)_{jj}, \\ (N_d)_{ij} &= \left[-t_\beta^{-1}\delta_{ij} + (t_\beta + t_\beta^{-1})V_{mi}^* V_{nj} n_m^* n_n\right](m_u)_{jj}, \end{aligned} \quad (32)$$

where $n_i = (U_{uL})_{3i}$ can be parametrized as in Eq. 20. Thus, jBGL models have four additional parameters in their Yukawa sector. Notice also from Eq. 32 that no BGL models is a particular implementation of jBGL models.

The most general scalar potential of jBGL models is given by

$$V(\phi) = \mu_{11}^2(\phi_1^\dagger\phi_1) + \mu_{22}^2(\phi_2^\dagger\phi_2) + \mu_{12}^2\Big[e^{i\dagger}(\phi_1^\dagger\phi_2) + h.c.\Big] + \lambda_1(\phi_1^\dagger\phi_1)^2 + \lambda_2(\phi_2^\dagger\phi_2)^2 + \lambda_3(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) + \lambda_4(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1), \tag{33}$$

where all parameters are real due to Hermiticity. We have softly broken the symmetry of Eq. 29 through the addition of the μ_{12}^2 term in order to avoid a physical Goldstone boson after SSB. Despite that, CP is still conserved by this potential, implying that the bound in Eq. 12 can be applied to jBGL models.

Comparing Eq. 19 and Eq. 32, we conclude that gBGL models and jBGL models are indistinguishable when analysing only one sector of the theory. As such, all results derived for gBGL models that involved FCNC in just one of the sectors of the theory translate directly to jBGL models. Thus, jBGL models are always in agreement with experiment provided

$$|c_{\beta\alpha}(t_\beta + t_\beta^{-1})| < 0.02\ . \tag{34}$$

Furthermore, there is room in jBGL models for an enhancement of the BAU in comparison to the SM expectations of the order of

$$\frac{BAU_{jBGL}}{BAU_{SM}} \sim 10^{16}(t_\beta + t_\beta^{-1})|n_3 n_2^*| \times \sin\left[\arg(n_3^* n_2 V_{tb}^* V_{cb})\right]. \tag{35}$$

Due to this propriety, the distinction between gBGL and jBGL models is not straightforward, being clear that it must involve observables related to both sectors of the theory. Nevertheless, we can use the diagonal couplings between the (125) scalar detected at the LHC and the physical fermions to perform such distinction. We compute the following ratio at tree-level for gBGL models to be

$$\left.\frac{BR(h\to\bar{c}c)}{BR(h\to\bar{s}s)}\right|_{gBGL} \approx \left(\frac{m_c}{m_s}\right)^2. \tag{36}$$

Meanwhile, the same ratio is given at tree-level for jBGL models by

$$\left.\frac{BR(h\to\bar{c}c)}{BR(h\to\bar{s}s)}\right|_{jBGL} \approx \left(\frac{m_c}{m_s}\right)^2 \times \left\{\frac{s_{\beta\alpha} - c_{\beta\alpha}\left[t_\beta - 2s_{2\beta}^{-1}|n_2|^2\right]}{s_{\beta\alpha} + c_{\beta\alpha}\left[t_\beta^{-1} - 2s_{2\beta}^{-1}|n_2|^2\right]}\right\}^2 \tag{37}$$

which allows for the distinction between gBGL and jBGL models. Note also that the same ratio is given at tree-level in the SM by Eq. 36, which implies that this combination of observables cannot distinguish between gBGL models and the SM.

4. Controlling FCNC

4.1. Unitary Flavour Violation

A model is said to have Unitary Flavour Violation if its couplings satisfy

$$N_{qa}^0 = L_q^a D_q^0 R_q^a, \tag{38}$$

where the matrices L_q^a must be determined by the scalar potential of the theory, but are otherwise arbitrary. Furthermore, the structure of Eq. 38 must be implemented by an AFS.

Since L_q^a and R_q^a must be determined by the scalar potential, they cannot be functions of the Yukawa couplings $(\Gamma_a)_{ij}$ and $(\Delta_a)_{ij}$. Thus, we conclude that they must be diagonal

$$(L_q^a)_{ij} = (l_q^a)_k (P_k)_{ij}, \quad (R_q^a)_{ij} = (r_q^a)_k (P_k)_{ij}, \tag{39}$$

where $(P_k)_{ij} = \delta_{ik}\delta_{jk}$ and the 3D-vectors $\vec{l}_q^a$ and $\vec{r}_q^a$ are determined by the system of equations

$$(\Gamma_b)_{ij} = 0 \lor (l_d^a)_i (r_d^a)_j = \frac{O_{ab}}{O_{1b}}. \tag{40}$$

Thus, we conclude that MHDMs with UFV are identified by $10(N-1)$ parameters distributed by $4(N-1)$ vectors. Furthermore, given that O is a real matrix, we can always choose $\vec{l}_q^a$ and $\vec{r}_q^a$ real as well.

Consider two couplings Γ_{b_1} and Γ_{b_2} of a MHDM with UFV satisfying $(\Gamma_{b_1})_{ij}(\Gamma_{b_2})_{ij} \neq 0$. Then, Eq. 40 requires that the following relation holds

$$\frac{O_{ab_1}}{O_{1b_1}} = \frac{O_{ab_2}}{O_{1b_2}} \Leftrightarrow \frac{O_{ab_1}}{O_{ab_2}} = \frac{O_{1b_1}}{O_{1b_2}}, \tag{41}$$

from which we read that the columns b_1 and b_2 of O must be proportional since this relation is valid for all a. Considering that no orthogonal matrix has that propriety, we conclude that the Yukawa couplings of any MHDM with UFV obey the relation

$$(\Gamma_a)_{ij}(\Gamma_b)_{ij} = 0,\ \forall_{a\neq b}. \quad (42)$$

Remembering that the Yukawa couplings of a MHDM with UFV are determined by an AFS, we conclude from this expression that all scalars that couple to fermions must transform differently under such symmetry.

The non-diagonal couplings of a MHDM with UFV are written after the diagonalization of the mass matrices as

$$(N_{qa})_{ij} =(l^a_q)_m(r^a_q)_n(m_q)_{kk}(U_{qL})^*_{mi}(U_{qL})_{mk}\times \times(U_{qR})^*_{nk}(U_{qR})_{nj}, \quad (43)$$

justifying the nomenclature UFV due to the intensity of the tree-level FCNC being controlled by unitary matrices in these models. Thus, the FCNC of a general MHDM with UFV cannot be arbitrary large, unlike was the case for the general MHDM.

In order to constrain MHDM with UFV, we begin by using the triangle and the Cauchy-Schwarz inequalities on Eq. 12 to obtain

$$\left|\chi^+(A^+_{q1})^2_{ji} - \chi^-(A^-_{q1})^2_{ji}\right| \le \chi^+c^2_{\beta\alpha}\sum_b\Big|(N_{qb})_{ji} + (N^\dagger_{qb})_{ji}\Big|^2 + \chi^-c^2_{\beta\alpha}\sum_b\Big|(N_{qb})_{ji} - (N^\dagger_{qb})_{ji}\Big|^2. \quad (44)$$

Using Eq. 43 and the Cauchy-Schwarz inequalities implies that

$$\left|(N_{qa})_{ji} \pm (N^\dagger_{qa})_{ji}\right|^2 \le 20m^2_{q3}\,|\vec{l}^a_q|^2|\vec{r}^a_q|^2, \quad (45)$$

where m_{q3} is the heaviest mass of the q sector. Thus, MHDMs with UFV are always in agreement with experiment provided

$$c^2_{\beta\alpha}J_LJ_R \le \frac{3v^2m^2_hm^2_j\Delta m_P}{5m^2_{q3}m_Pf^2_P(6m^2_P - m^2_j)}, \quad (46)$$

where $J_L = \sum_{a=2}^N|\vec{l}^a_q|^2$, $J_R = \sum_{a=2}^N|\vec{r}^a_q|^2$ and m_j is the mass of the heaviest quark in the meson state. As was the case for the "natural" MHDM, it turns out that the most stringent contraints of this type arise from the $D^0-\bar{D}^0$ and $K^0-\bar{K}^0$ mixings

$$c^2_{\beta\alpha}\left[\sum_{a=2}^N\sum_{m=1}^3(l^a_u)^2_m\right]\left[\sum_{a=2}^N\sum_{n=1}^3(r^a_u)^2_n\right] \le 10^{-10},$$
$$c^2_{\beta\alpha}\left[\sum_{a=2}^N\sum_{m=1}^3(l^a_d)^2_m\right]\left[\sum_{a=2}^N\sum_{n=1}^3(r^a_d)^2_n\right] \le 6\times10^{-8}. \quad (47)$$

It should be pointed out that we have considered the maximum value of $\sum_{k,m,n=1}^3|U^*_{mj}U_{mk}V^*_{nk}V_{ni} \pm U_{mi}U^*_{mk}V_{nk}V^*_{nj}|^2 \le 20$ despite the fact that such coefficient is of order one in most regions of the parameter space.

4.2. Left Models

A left model is a MHDM with UFV defined by $R^a_q = (1,1,1)$. Using a similar argument to the one used to prove Eq. 42, we conclude that the Yukawa couplings of a left model satisfy the relation

$$(\Gamma_a)_{ij}(\Gamma_b)_{ik} = 0,\ \ (\Delta_a)_{ij}(\Delta_b)_{ik} = 0,\ \forall_{j\neq k}. \quad (48)$$

Using these relations, it is straightforward to prove that the following set of WBI conditions are sufficient and necessary to ensure that a given texture belongs to a left model

$$\Gamma^\dagger_a\Gamma_b = \Delta^\dagger_a\Delta_b = \Gamma^\dagger_a\Delta_{p_a} = 0, \quad (49)$$

where the coefficients p_a identify the particular left model and are to be considered as all possible labels with exception of, at most, one. Note that these relations imply that each line of the mass matrix of a quark of a given charge should receive contributions from one and only one Higgs doublet, i.e., the models introduced in Sec. 3 are left models.

The non-diagonal couplings of left models are written as

$$(N_{qa})_{ij} = (l^a_q)_m(U_{qL})^*_{mi}(U_{qL})_{mj}(m_q)_{jj}. \quad (50)$$

Thus, left models are identified by $2(N-1)$ vectors $\vec{l}^a_q$. Given that the combination $U^*_{mi}U_{mj}$ is equal to $(\Lambda U)^*_{mi}(\Lambda U)_{mj}$ with Λ a diagonal and unitar matrix, there are only a maximum of six new parameters in the Yukawa sector of a left model (remember that $\vec{l}^a_q$ is a parameter of the scalar potential and $U_{dL} = U_{uL}V$) regardless of the number of scalars in the theory.

Replacing Eq. 50 on Eq. 44, using the Cauchy-Schwarz inequalities and neglecting the lightest of the quark masses (m_i) inside the meson state results in the following inequality

$$\left|(N_{qa})_{ji} \pm (N^\dagger_{qa})_{ji}\right|^2 \le m^2_j\left[\sum_{m=1}^3(l^a_q)^2_m\right]. \quad (51)$$

As such, we conclude that left models are always in agreement with the current experimental results provided the following relation holds

$$c^2_{\beta\alpha}\left[\sum_{a=2}^{N}\sum_{m=1}^{3}(l^a_q)^2_m\right] \leq \frac{12v^2 m_h^2 \Delta m_P}{m_P f_P^2 (6m_P^2 - m_j^2)}. \quad (52)$$

Unlike the previous cases, it turns out that the most stringent of these constraints in the down sector of the theory arises from $B^0_d - \bar{B}^0_d$ mixing. Regarding the up sector, $D^0 - \bar{D}^0$ mixing remains the most constraining result appliable to left models, which results in

$$\begin{aligned} c^2_{\beta\alpha}\left[\sum_{a=2}^{N}\sum_{m=1}^{3}(l^a_u)^2_m\right] &< 5\times10^{-5}, \\ c^2_{\beta\alpha}\left[\sum_{a=2}^{N}\sum_{m=1}^{3}(l^a_d)^2_m\right] &< 2\times10^{-4}. \end{aligned} \quad (53)$$

4.3. Right Models

A right model is a MHDM with UFV defined by $L^a_q = (1, 1, 1)$. Using a similar argument to the one used to prove Eq. 42, we conclude that the couplings of a right model satisfy the relation

$$(\Gamma_a)_{ji}(\Gamma_b)_{ki} = 0, \quad (\Delta_a)_{ji}(\Delta_b)_{ki} = 0, \ \forall_{j\neq k}. \quad (54)$$

Using these relations, it is straightforward to prove that the following set of WBI conditions are sufficient and necessary to ensure that a given texture belongs to a right model

$$\Gamma_a \Gamma_b^\dagger = \Delta_a \Delta_b^\dagger = 0. \quad (55)$$

This relations imply that a column of the mass matrix of a quark of a given charge should receive contributions from one and only one Higgs doublet.

The non-diagonal couplings of right models are written as

$$(N_{qa})_{ij} = (r^a_q)_n (U_{qL})^*_{ni}(U_{qL})_{nj}(m_q)_{ii}, \quad (56)$$

implying that right models are identified by $2(N-1)$ vectors $\vec{r}^a_q$. Since $U^*_{mi}U_{mj}$ is equal to $(\Lambda U)^*_{mi}(\Lambda U)_{mj}$ with Λ a diagonal and unitary matrix, there are only a maximum of twelve new parameters in the Yukawa sector of a right model regardless of the number of scalars in the theory. Replacing Eq. 56 on Eq. 44, neglecting the lightest of the quark masses inside the meson state and using the Cauchy-Schwarz inequalities results in

$$\left|(N_{qa})_{ji} \pm (N^\dagger_{qa})_{ji}\right|^2 \leq m_j^2 \left[\sum_{n=1}^{3}(r^a_q)^2_n\right]. \quad (57)$$

After comparing Eq. 57 to Eq. 51, we conclude that right models are always in agreement with experimental results provided

$$\begin{aligned} c^2_{\beta\alpha}\left[\sum_{a=2}^{N}\sum_{m=1}^{3}(l^a_u)^2_m\right] &< 5\times10^{-5}, \\ c^2_{\beta\alpha}\left[\sum_{a=2}^{N}\sum_{m=1}^{3}(l^a_d)^2_m\right] &< 2\times10^{-4}. \end{aligned} \quad (58)$$

5. Conclusion

In Sec. 3, we have shown the existence of a simple principle that reduces the 36 new free parameters in the Yukawa sector of the most general 2HDM to a maximum of four. Besides the already known type I, type II and BGL models, we have found two previously undiscovered classes of models which satisfy the principle. The first of them, gBGL, can be seen as a generalization of BGL models outside of the MFV framework. Contrary to them, gBGL models contain tree-level FCNC in both sectors that are parametrized by four additional flavour parameters beyond the CKM matrix. Nevertheless, we have shown that its FCNC are significantly suppressed, rendering gBGL models plausible extensions of the SM. We have also shown that such suppression has not came at the expense of the BAU generated by gBGL models. The second of the non-trivial 2HDMs that satisfies our principle is that of jBGL models. While they cannot be seen as a generalization of BGL models, extensive comparisons can be made to gBGL models. In fact, we found both to be indistinguishable when analysing only one sector of the theory. Thus, jBGL models have the controlled FCNC and enhanced BAU of gBGL models. However, we were still capable of making a clear distinction between the two models.

In Sec. 4, we have introduced the condition of UFV to obtain a large class of MHDMs with controlled tree-level FCNC. As a consequence of its application, the bounds in the down sector of the theory are relaxed by a factor of $\frac{m_t^2}{m_b^2}$ in comparison to the "natural" MHDM. After analysing their proprieties, we have encountered two important subclasses of UFV. The first, populated by left models, was found to be equivalent to the class of 2HDMs that satisfy the principle of Sec. 3. Both left and right models have an additional $\frac{m_j^2}{m_{q3}^2}$ factor of suppression in relation to the general MHDMs with UFV. As such, those models are the most plausible extensions of the SM in the context of MHDMs.

We leave a more detailed analysis of the phenomenology of the introduced models as future work, as well as a study of the scalar potential of the general MHDM with respect to its FCNC.

6. Acknowledgments

We would like CFTP for the support shown during the development of this work. We would also like to thank Gustavo C. Branco, Miguel Nebot and João P. Silva for useful suggestions and helpful discussions of our results.

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