TÉCNICO LISBOA

Multi-Higgs Models, Flavour and CP Violation

João Aparício Pereira Magalhães Alves

Thesis to obtain the Master of Science Degree in

Engineering Physics

Supervisor: Prof. Gustavo da Fonseca Castelo Branco

Examination Committee

Chairperson:	Prof. Jorge Manuel Rodrigues Crispim Romão
Supervisor:	Prof. Gustavo da Fonseca Castelo Branco
Members of the Committee:	Prof. Filipe Rafael Joaquim
	Prof. Maria Margarida Nesbitt Rebelo da Silva

September 2018

Acknowledgments

I would like to start by thanking my supervisor Gustavo C. Branco for his help, guidance and support during the last two years in which we have worked together, for introducing me to the world of research in particle physics and for all of the effort placed in my development as a physicist.

Next I would also like to thank João P. Silva and Jorge C. Romão for the important role played on my education on particle physics and of course to Miguel Nebot for many helpful discussions on the topics of this MSc thesis. I have also to thank CFTP for financial support and for the many opportunities to attend Summer Schools and Conferences that it provided to me.

I could not end the acknowledgments without refering by name the many friends that have helped me through out the last five years of war and their fallen pens. As promised, I thank Ana Sovelas, Diogo Carvalho, Francisco Horta, Gonçalo Castro, Inês Roça, João Oliveira, José Senart, Miguel Gonçalves, Miguel Pardal, Patrícia Estrela, Pedro Pereira, André Moreira, Bernardo Malaca, Carlos Garrido, Cristina Melício, Nélson Costa and Tiago Silva. Many apologies to anyone with missing pens that I might have missed.

Finally, I thank my family, friends and selfish collegues who did not lend me a pen for their friendship and support.

Resumo

Revemos Modelos Multi-Higgs (MHDMs) e discutimos a proliferação de parâmetros livres no sector de Yukawa do seu Lagrangiano. Demonstramos que um princípio inspirado nos modelos BGL pode ser implementado de modo a diminuir significantemente o número destes parâmetros. Realizamos uma análise sistemática de todos os Modelos com dois-Higgs (2HDM) que satisfazem esse princípio, resultando na introdução de duas novas classes de 2HDM, os models gBGL e jBGL. Usámos resultados experimentais de modo a aplicar limites às Interacções Neutras com Violação de Sabor (FCNC) mediados por escalares a nível árvore de MHDMs. A condição de Violação de Sabor Unitaría (UFV) é introduzida para controlarmos a intensidade das FCNC. Dentro dos modelos com UFV, duas subclasses emergiram como as mais plausíveis a serem extensões do Modelo Padrão, os modelos destros e esquerdinos. Demonstramos que a estrutura das FCNC a nível árvore desses models é tal que os contrangimentos experimentais que aplicamos a estes são relaxados significamentemente em comparação com o MHDM geral.

Palavras-chave: Modelos Multi-Higgs, Interacções Neutras com Violação de Sabor, Simetrias de Sabor Abelianas, Violação de CP.

Abstract

We review Multi-Higgs Doublet Models (MHDMs) and discuss the proliferation of free parameters in their Yukawa sector. It is shown that a BGL inspired principle can be implemented in order to significantly reduce such number. A systematic analysis of all Two-Higgs Doublet Models (2HDM) that satisfy it is done, resulting in the discovery of two previously unknown classes of 2HDM, gBGL and jBGL models. We use experimental results to apply bounds on the tree-level scalar mediated Flavour Changing Neutral Couplings (FCNC) of MHDMs. The Unitary Flavour Violation condition is introduced to control the intensity of FCNC. Amongst such models, two subclasses emerge as the most plausible extensions of the Standard Model, left and right models. It is shown that the structure of the tree-level FCNC of these models is such that the experimental bounds applied to them are significantly softened in comparison to the constraints applied to the general MHDM.

Keywords: Multi-Higgs Doublet Models, Flavour Changing Neutral Couplings, Abelian Flavour Symmetry, CP Violation.

Contents

List of Tables

List of Figures

Glossary

2HDM	Two-Higgs Doublet Models
AFS	Abelian Flavour Symmetry
BAU	Baryon Asymmetry of the Universe
dof	degrees of freedom
FCC	Flavour Changing Couplings
FCNC	Flavour Changing Neutral Couplings
h.c.	hermitian conjugate
HME	Hadronic Matrix Element
LH	left-handed
MFV	Minimal Flavour Violation
MHDM	Multi-Higgs Doublet Model
NFC	Natural Flavour Conservation
QFT	Quantum Field Theory
RH	right-handed
SM	Standard Model
SSB	Spontaneous Symmetry Breaking
vev	vacuum expectation value
VIA	Vacuum Insertion Approximation
WB	Weak Basis
WBI	Weak Basis Invariant
WBT	Weak Basis Transformation

Chapter 1

Introduction

In the beginning of the XXth century, the classical view of the Universe was shattered by the discoveries of Special Relativity and Quantum Mechanics. Later, both were combined in the consistent, predictive and renormalizable framework of Quantum Field Theory (QFT) which can make use of the Path Integral Formalism [1–3] based on fields, which are continuous degrees of freedom (dof). The generating functional $\mathcal{Z}$ is introduced in it as a fundamental quantity constructed from the classical action $\mathcal{S}_{cl}$. Thus, symmetries are implemented in QFT as a set of transformations under which $\mathcal{S}_{cl}$ is invariant. Therefore, the Lagrangian of a theory must be a Lorentz invariant in order for it to be in agreement with Special Relativity.

As of today, the Standard Model of Particle Physics (SM) is the best available description of the Universe. It is a QFT invariant under the gauge group $G_{SM} = SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ that is spontaneously broken through the Higgs mechanism into $SU(3)_C \otimes U(1)_Q$. Its dof are shown in the table below. Given that three copies of each fermion have been found in nature [4], we define $e \equiv (e, \mu, \tau)$ as a three-dimensional vector in flavour space (similar definitions are made for the other fermionic fields).

Field	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	$O(3,1)$	Field	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	$O(3,1)$
q_L	3	2	$\frac{1}{6}$	Spinor	G^a_μ	8	1	0	Vectorial
u_R	3	1	$\frac{2}{3}$	Spinor	W^a_μ	1	3	0	Vectorial
d_R	3	1	$-\frac{1}{3}$	Spinor	B_μ	1	1	0	Vectorial
l_L	1	2	$-\frac{1}{2}$	Spinor	ϕ	1	2	$\frac{1}{2}$	Trivial
e_R	1	1	-1	Spinor					

Table 1.1: Field content of the SM. We do not show the antiparticles to the scalar and fermionic fields.

Given that all of the symmetries of the SM are longitudinal (they do not distinguish the fermionic generations), no relations between the different flavour parameters of the theory arises. Thus, the number of free parameters in the SM is increased significantly. As such, the implementation of such symmetries is crucial to enhance the predictive power of the SM and of any extensions of it.

A CP transformation [5, 6] is composed of a parity transformation and a charge conjugation. As such, it transforms a right-handed (RH) fermion into a left-handed (LH) antifermion, and vice-versa. This transformation is crucial to our understanding of Cosmology given that the observed Baryon Asymmetry of the Universe (BAU) [7, 8] may only be generated from an initial situation where matter and antimatter were equally distributed if CP is violated. Nowadays, it is clear that the SM does not predict enough CP violation to explain the observed BAU [9, 10]. Thus, extensions of it with new sources of CP violation are required to satisfy the current cosmological bounds.

Recently, there has been a special interest in the scalar sector of the SM and some of its extensions due to the discovery by both the ATLAS [11] and CMS [12] collaborations of a particle that can be interpreted as the Higgs boson of the SM. A central question that remains is whether the couplings of such particle to the quarks and leptons are those of the SM or whether nature chose a more complex scalar sector. The simplest extension of the scalar sector of the SM consists of adding more scalars [13, 14]. The first Multi-Higgs Doublet Model (MHDM) was introduced by Lee [15] in order to generate spontaneous CP violation. Given that CP may not be conserved by their scalar potential, MHDMs become interesting candidates to solve the problem of Baryogenesis [16, 17]. There are also dark matter candidates in the framework of MHDMs [18, 19] and a possible solution to the strong CP problem in the form of the Peccei-Quinn mechanism [20]. However, if no extra symmetries are introduced, the general MHDM has Flavour Changing Neutral Couplings (FCNC) at tree-level that must be suppressed in order to avoid violation of stringent experimental bounds. Glashow and Weinberg [21] and Paschos [22] proposed Natural Flavour Conservation (NFC) in which the quarks of a given charge receive their mass from only one scalar. This solution can be obtained from a Z_2 symmetry, and it completely eliminates FCNC at tree-level. A more interesting approach to control FCNC is provided by BGL models [23], where there are tree-level FCNC in one sector of the theory but their flavour structure is completely defined by the CKM matrix. Minimal Flavour Violation (MFV) [24] has been proposed as a generalization of these models in which tree-level FCNC exist in both sectors of the theory while maintaining a structure that is completely defined by the CKM matrix.

Motivated by the success of BGL models in the reduction of the number of free parameters in their Yukawa sector, we seek to determine a principle from which significant reductions are naturally obtained. We then look for the full set of symmetry protected Two-Higgs Doublet Models (2HDM) that satisfy such principle as an alternative to the perturbative expansion of MFV.

After completing such task, we seek a condition which, if satisfied by any MHDM, ensures that its tree-level FCNC are controlled. This provides a methodical approach to suppress FCNC in the context of MHDM that is larger in its scope than any of the current alternatives. We will present the full set of 2HDMs protected by an Abelian Flavour Symmetry (AFS) with controlled FCNC.

This MSc thesis is organised as follows. In Chap. 2, we start by reviewing the properties of a general gauge theory, paying particular attention to G_{SM}. Then, we provide an overview of the Higgs mechanism applied to a general non-Abelian gauge theory and to the SM. Following such work, we settle the notation of this MSc thesis by working out the structure and properties of the SM and the general MHDM. We finish this chapter with an analysis of CP violation in the context of the SM and MHDMs. We start Chap. 3 by reviewing the success of BGL models in the reduction of the number of free parameters in the Yukawa sector of a 2HDM. Then, we propose the principle that guided our search for further 2HDMs with a reduced number of free parameters in their Yukawa sector. In the rest of Chap. 3, we analyse the scalar potential, the intensity of the tree-level FCNC and the contributions to the BAU of all 2HDMs that satisfy such principle. In Chap. 4, we propose a condition to ensure that the tree-level FCNC of a MHDM are controlled. After giving an overview of the common properties of those models, such as the expected intensity of their tree-level FCNC, we present the list of all 2HDMs that satisfy such condition. Finally, we offer our conclusions in Chap. 5.

Chapter 2

State of the Art

2.1 Gauge Theories

A QFT [1–3, 25] is only in agreement with Special Relativity provided its Lagrangian is a Lorentz invariant. A theory is said to be free if its equations of motion are linear, i.e., if there are only quadratic terms in the number of fields in its Lagrangian. Thus, the most general Lagrangians describing a free complex singlet or spinor of the Lorentz-Poincaré group [25] are

$$\mathcal{L}_{CS} = (\partial^\mu \varphi)^\dagger (\partial_\mu \varphi) - m^2 (\varphi^\dagger \varphi), \quad \mathcal{L}_f = \bar{\psi}\,(i\gamma^\mu \partial_\mu - m)\,\psi. \tag{2.1}$$

Considering that both fields have N internal dof results in an accidental global $U(N)$ symmetry of the free theories. Such invariance does not hold for local transformations

$$\psi(x) \to \psi'(x) = U(x)\psi(x) \Rightarrow \partial_\mu \psi \to \partial_\mu \psi' = U(\partial_\mu \psi) + (\partial_\mu U)\psi \neq U(\partial_\mu \psi). \tag{2.2}$$

In order for a theory to be invariant under the global and local transformations of a symmetry group, it is clear that $\partial_\mu \psi$ must be transformed as ψ. Thus, we must introduce the covariant derivative in order to make the theories in (2.1) locally (or gauge) invariant

$$\partial_\mu \to D_\mu = \partial_\mu + igA^a_\mu t^a = \partial_\mu + ig\widetilde{A_\mu}, \tag{2.3}$$

where t^a are the generators of the group written in the representation (rep) according to which the field in which D_μ is applied transforms. Since we only consider gauge theories based on Lie groups [26, 27], the generators satisfy the relations

$$[t^a, t^b] = if^{abc}t^c, \quad \mathrm{tr}(t^a t^b) = T(R)\delta^{ab}. \tag{2.4}$$

As argued previously, the covariant derivative of a gauge invariant theory is transformed to $UD_\mu\psi$. Thus, the transformation law of the gauge fields $\widetilde{A_\mu}$ is given by

$$\widetilde{A_\mu} \to \widetilde{A'_\mu} = U\widetilde{A_\mu}U^{-1} + \frac{i}{g}(\partial_\mu U)U^{-1}. \tag{2.5}$$

Any transformation of a Lie group can be written using the generators as $U = e^{it^a\alpha^a}$. Thus, infinitesimal transformations are given by $U \simeq 1 + i\tilde{\alpha}$, under which the gauge fields are transformed to

$$\delta A^a_\mu = -f^{abc}\alpha^b A^c_\mu - \frac{1}{g}\partial_\mu \alpha^a. \tag{2.6}$$

We have seen how the prescription $\partial_\mu\psi \to D_\mu\psi$ promotes a theory with a global symmetry to a (local) gauge theory. However, we must always write the most general Lagrangian compatible with the symmetries of the theory or risk obtaining a non-renormalizable QFT [1, 3]. As such, we seek further allowed terms, starting by defining the generalized Maxwell tensor

$$ig\widetilde{F_{\mu\nu}} \equiv [D_\mu, D_\nu] \Leftrightarrow \widetilde{F_{\mu\nu}} = \partial_\mu\widetilde{A_\nu} - \partial_\nu\widetilde{A_\mu} + ig[\widetilde{A_\mu}, \widetilde{A_\nu}] \Leftrightarrow F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - gf^{bca}A^b_\mu A^c_\nu, \tag{2.7}$$

which is transformed under (2.5) to

$$\widetilde{F_{\mu\nu}} \to \widetilde{F'_{\mu\nu}} = U\widetilde{F_{\mu\nu}}U^{-1}. \tag{2.8}$$

Thus, the following term is both a gauge and Lorentz invariant

$$\mathrm{tr}\left(\widetilde{F_{\mu\nu}}\widetilde{F^{\mu\nu}}\right) = T(R)F^a_{\mu\nu}F^{a\mu\nu}. \tag{2.9}$$

The most general gauge invariant Lagrangians[1] describing complex scalar or spinor fields are

$$\mathcal{L}_{CS} = (D^\mu\varphi)^\dagger(D_\mu\varphi) - m^2(\varphi^\dagger\varphi) - \frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu}, \;\; \mathcal{L}_f = \bar{\psi}\left(i\gamma^\mu D_\mu - m\right)\psi - \frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu}. \tag{2.10}$$

Notice how no mass terms for the gauge fields are allowed by gauge invariance. Thus, we conclude that the existence of massive vector fields is incompatible with the predictions of gauge theories.

If we had a gauge theory with multiple fields transforming according the irreps β_i of a Lie group, could each covariant derivative have its own coupling g_i? If so, we should write (2.3) as

$$D_\mu\psi_{\beta_i} = (\partial_\mu + igY_i t^a_\beta A^a_\mu)\psi_{\beta_i}, \;\; i = 1, ..., n_{fields}, \tag{2.11}$$

where t^a_β is a notation for the generator a written in the rep β. Given such expression, the transformation law of the gauge fields in (2.6) is modified to

$$\delta A^a_\mu = -f^{abc}\alpha^b_i A^c_\mu - \frac{1}{gY_i}\partial_\mu\alpha^a_i, \tag{2.12}$$

where each field may be transformed with its own α_i. Since we can only introduce one set of gauge fields to ensure gauge invariance for all matter fields, (2.12) cannot depend on the field index i, resulting in $\alpha^a_i = Y_i\alpha^a$ and $f^{abc} = 0$. As such, only in Abelian gauge theories (for which $f^{abc} = 0$) may each field

[1] The term $(\varphi^\dagger\varphi)^2$ is a renormalizable Lorentz and gauge invariant. Thus, it should be added to $\mathcal{L}_{CS}$.

have its own g. Thus, we say that the coupling strength of non-Abelian gauge theories is universal[2].

To conclude this section, we consider non-simple gauge groups. Since these are the direct product of simple groups, the covariant derivative is just written as

$$D_\mu = \partial_\mu + i \sum_G g_G t^a_G A^a_{G,\mu} + i \sum_H g_H Y_H A_{H.\mu}, \tag{2.13}$$

where we sum over all non-Abelian groups G and Abelian groups H that form the non-simple gauge group. To make the theory renormalizable we must include a term $-\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu}$ in the Lagrangian for each simple group under which the theory is invariant.

The current experimental results [4] have a structure which is compatible with a gauge theory based on the non-simple group $G_{SM} = SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$. Thus, we are tempted to consider the SM a QFT invariant under G_{SM}. However, making the SM gauge invariant renders all gauge bosons massless, which is in disagreement with several experimental results [4, 28, 29] that suggest massive mediators of the weak interactions. This implies that we must find a method to generate the W and Z-boson masses without breaking the gauge structure of G_{SM} to make the SM a valid theory.

The SM is an example of a chiral theory, i.e., the RH fermions are transformed differently under G_{SM} than the LH fermions (see Tab. 1.1). As such, direct fermionic mass terms ($\bar{\psi}\psi = \bar{\psi}_R\psi_L + \bar{\psi}_L\psi_R$) are not gauge invariant. Thus, we must also find a mechanism capable of generating the observed spectrum of fermionic masses [4]. To settle notation, we write the covariant derivative acting on a field that transforms according to the irrep (α, β, Y) of G_{SM}

$$D_\mu = \partial_\mu + i g_s G^a_\mu T^a_\alpha + i g W^a_\mu t^a_\beta + i g' Y B_\mu. \tag{2.14}$$

2.2 Higgs Mechanism

The Higgs mechanism [30–32] is the process of Spontaneous Symmetry Breaking (SSB) of a gauge theory by the achievement of a vacuum expectation value (vev) of one of its fields. A symmetry is said to be spontaneously broken if the vacuum is not invariant under it, but the Lagrangian is. For continuous symmetries, the Goldstone theorem [33] ensures the existence of one massless boson for each broken generator. In the case of gauge theories, the would-be Goldstone bosons are absorbed as the longitudinal polarization of the massive gauge bosons.

Given how Lorentz invariance stands as a good symmetry of nature [34, 35], only singlets of the Poincaré group may acquire a vev. Thus, only scalar fields[3] can be used in the Higgs mechanism.

[2] We have proven such result at tree-level. It can, however, be generalized to all orders of perturbation theory using the Ward Identities [1, 3].

[3] Combinations of non-scalar fields that transform like a singlet of the Poincaré group, such as $A_\mu A^\mu$ or $\bar{\psi}\psi$, may also acquire a vev without breaking Lorentz invariance.

2.2.1 Non-Abelian gauge theories

Consider a QFT invariant under a non-Abelian gauge group. Take its matter content to be a real scalar field with N internal dof[4] transforming according to the fundamental rep of the gauge group. The most general Lagrangian we can write for such theory is

$$\mathcal{L} = \frac{1}{2}(D^{\mu}\phi)^T(D_{\mu}\phi) - V(\phi) - \frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu}, \tag{2.15}$$

where the covariant derivative $D_{\mu}\psi$ and the generalized Maxwell tensor $F^a_{\mu\nu}$ are given by (2.3) and (2.7), respectively. The gauge invariance of $\mathcal{L}$ is written in terms of ψ and $\widetilde{A_{\mu}}$ as

$$\phi \to \phi' = U\phi, \;\; \widetilde{A_{\mu}} \to \widetilde{A'_{\mu}} = U\widetilde{A_{\mu}}U^{-1} + \frac{i}{g}(\partial U)U^{-1}. \tag{2.16}$$

Assume now that the minimization of $V(\psi)$ requires the field ψ to acquire a vev v

$$\phi(x) = v + \varphi(x), \tag{2.17}$$

such that $\langle 0\,|\,\varphi\,|\,0\rangle = 0$ and $V(v)$ is the global minimum of the scalar potential. Expanding $V(\phi)$ around v results in

$$V(\phi) = V(v) + \left.\frac{\partial V(\phi)}{\partial \phi_i}\right|_{\phi=v}(\phi_i - v_i) + \frac{1}{2}\left.\frac{\partial^2 V(\phi)}{\partial\phi_i\partial\phi_j}\right|_{\phi=v}(\phi_i - v_i)(\phi_j - v_j) + ... \equiv \frac{1}{2}\varphi^T M^2\varphi + \tilde{V}(\varphi), \tag{2.18}$$

where $\tilde{V}(\varphi)$ contains only interaction terms between the scalar fields and $M^2_{ij} = \left.\frac{\partial^2 V}{\partial\phi_i\partial\phi_j}\right|_{\phi=v}$. It is straightforward to check from (2.15) that $V(\psi)$ must be gauge invariant, which implies that the following relation must hold

$$\frac{\partial V(\phi)}{\partial\phi_i}\delta\phi_i = 0 \Rightarrow \frac{\partial^2 V(\phi)}{\partial\phi_i\partial\phi_j}(t^a)_{jk}\phi_k + \frac{\partial V(\phi)}{\partial\phi_j}(t^a)_{ji} = 0. \tag{2.19}$$

Noting that this relation holds for any field configuration, we take $\psi = v$ to obtain

$$(M^2)_{ij}(t^a v)_j = 0. \tag{2.20}$$

If the vacuum breaks B generators (implying the existence of $N - B$ generators for which $t^a v = 0$), it is trivial to use (2.20) to prove that M^2 must have B null eigenvalues corresponding to the B Goldstone modes[5]. We now introduce the unitary gauge as

$$\phi^T t^a v = 0, \tag{2.21}$$

in which we write the Lagrangian using the vevless fields φ as

[4] The analysis that follows apply to complex fields since they can be decomposed into two real fields.
[5] To prove the Goldstone boson we must only show that the symmetry was broken by v.

$$
\begin{aligned}
\mathcal{L} = &\frac{1}{2}(\partial^\mu\varphi)^T(\partial_\mu\varphi) - \frac{1}{2}\varphi^T M^2\varphi - \tilde{V}(\varphi) + \frac{1}{2}(\partial^\mu A^a_\mu)^2 - \frac{1}{2}(\partial_\mu A^a_\nu)^2 + \frac{1}{2}(M_A^2)^{ab}A^a_\mu A^b_\mu \\
&+ igA^a_\mu(\partial^\mu\varphi)^T t^a\varphi + g^2 A^a_\mu A^{b\mu}(\varphi^T t^a t^b v) + \frac{1}{2}g^2 A^a_\mu A^{b\mu}(\varphi^T t^a t^b \varphi) \\
&+ gf^{abc}A^b_\mu A^c_\nu(\partial^\mu A^{a\nu}) - \frac{1}{4}g^2 f^{abc}f^{ade}A^b_\mu A^c_\nu A^{d\mu}A^{e\nu},
\end{aligned}
\tag{2.22}
$$

where we defined the mass matrix M_A^2 for the gauge bosons as

$$
(M_A^2)_{ab} = g^2(v^T t^a t^b v). \tag{2.23}
$$

It is clear that M_A^2 has a null eigenvalue for each generator that is not broken by the vacuum, with the remaining positively defined. Notice that the massless bosons predicted by the Goldstone theorem are not present , having been aborbed as the longitudinal polarizations of the massive gauge bosons. Notice also that the Lagrangian in (2.2.1) is still gauge invariant, with (2.16) becoming

$$
\varphi \to U\varphi + (U-1)v, \quad \widetilde{A_\mu} \to \widetilde{A'_\mu} = U\widetilde{A_\mu}U^{-1} + \frac{i}{g}(\partial_\mu U)U^{-1}. \tag{2.24}
$$

In the Lagrangian of (2.15) we count N real scalar fields and N massless vector fields, totalling $3N$ independent dof. In the unitary gauge (see (2.2.1)), we observe $N-B$ real scalar fields, B massive and $N-B$ massless vector fields, resulting in the same number of dof. This was expected given that the Lagrangians of (2.15) and (2.2.1) are the same, only written in different variables.

In this subsection, we started by writing the Lagrangian of the theory in (2.15) using the dof that transform according to the fundamental rep of the gauge group . Then, we rewrote it in the Unitary gauge (see (2.2.1)) using the vevless fields of (2.17). Since physical particles are identified in a QFT as the poles of diagonal propagators of vevless fields [1], we must consider the physical modes of this theory to be the ones of (2.2.1) rather than those of (2.15). We have found the mechanism which generates mass terms for gauge bosons without breaking the structure of the theory. Such a feature is a direct consequence of the SSB of the gauge symmetry, which can be explicitly seen in the transformation law of (2.24). In it, we learn that the physical scalars φ do not transform according to any irrep of the gauge group if the vacuum is broken ($Uv \neq v$), which implies that the gauge group is no longer a symmetry of the theory[6].

2.2.2 $SU(2)_L \otimes U(1)_Y$ gauge group

Consider a gauge theory invariant under $SU(2)_L \otimes U(1)_Y$. Take its matter content to be a complex scalar transforming as the irrep $(2, Y_h)$. The most general Lagrangian we can write for such theory is

$$
\mathcal{L} = (D^\mu\phi)^\dagger(D_\mu\phi) - V(\phi) - \frac{1}{4}W^a_{\mu\nu}W^{a\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}, \quad D_\mu = \partial_\mu + ig\frac{\sigma^a}{2}W^a_\mu + ig'Y_h B_\mu, \tag{2.25}
$$

where $V(\phi) = \mu^2(\phi^\dagger\phi) + \lambda(\phi^\dagger\phi)^2$. We decompose the dof of the scalar ϕ in the unitary gauge as

[6] This concludes the demonstration of the Goldstone theorem.

$$\phi(x) = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & v + h(x) \end{bmatrix}^T , \tag{2.26}$$

implying that the potential acquires a vev

$$V(v) = \frac{1}{2}\mu^2 v^2 + \frac{1}{4}\lambda v^4. \tag{2.27}$$

Stability demands that we take v as the global minimum of the potential. Its existence requires $\lambda > 0$, with v being determined by

$$\frac{\partial V(v)}{\partial v} = 0 \Rightarrow v = 0 \ \wedge \ v = \sqrt{-\frac{\mu^2}{\lambda}}. \tag{2.28}$$

If $\mu^2 > 0$, the scalar potential is minimized by the trivial configuration $v = 0$. Given that such vacuum is invariant under all generators, there is no SSB for $\mu^2 > 0$.

If $\mu^2 < 0$, the scalar potential is minimized by $v = \sqrt{-\frac{\mu^2}{\lambda}}$. Unlike before, $Yv \neq 0$ and $\sigma^a v \neq 0$, implying that SSB occured. We write the scalar potential in terms of the physical vevless field h as

$$V(h) = \frac{1}{2}m_h^2 h^2 \left(1 + \frac{h}{2v}\right)^2 . \tag{2.29}$$

We see from this expression that h as acquired a mass given by $m_h = \sqrt{-2\mu^2}$. Meanwhile, the gauge-kinetic terms are written as follows,

$$\begin{aligned}(D^\mu\phi)^\dagger(D_\mu\phi) = &\frac{1}{2}(\partial^\mu h)(\partial_\mu h) + \frac{1}{8}g^2v^2(W^1_\mu - iW^2_\mu)(W^{1\mu} + iW^{2\mu})\left(1 + \frac{h}{v}\right)^2 \\ &+ \frac{1}{8}(g^2 + 4g'^2Y_h^2)v^2\left(\frac{2g'Y}{\sqrt{g^2 + 4g'^2Y_h^2}}B_\mu - \frac{g}{\sqrt{g^2 + 4g'^2Y_h^2}}W^3_\mu\right)^2\left(1 + \frac{h}{v}\right)^2 .\end{aligned} \tag{2.30}$$

We now define the fields $W^\pm_\mu = \frac{1}{\sqrt{2}}(W^1_\mu \mp iW^2_\mu)$, $Z_\mu = c_W W^3_\mu - s_W B_\mu$ and $A_\mu = s_W W^3_\mu + c_W B_\mu$, where $c_W = \frac{g}{\sqrt{g^2+4g'^2Y_h^2}}$ and $s_W = \frac{2g'Y_h}{\sqrt{g^2+4g'^2Y_h^2}}$. Using them, we rewrite the Lagrangian as

$$\begin{aligned}\mathcal{L} = &\frac{1}{2}(\partial^\mu h)(\partial_\mu h) - \frac{1}{2}m_h^2h^2\left(1 + \frac{h}{2v}\right)^2 + m_W^2 W^+_\mu W^{-\mu}\left(1 + \frac{h}{v}\right)^2 \\ &+ \frac{1}{2}m_Z^2 Z_\mu Z^\mu\left(1 + \frac{h}{v}\right)^2 - \frac{1}{4}W^a_{\mu\nu}W^{a\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu},\end{aligned} \tag{2.31}$$

where $m_Z^2 = \frac{1}{4}(g^2 + 4g'^2Y_h^2)v^2$ and $m_W^2 = \frac{1}{4}g^2v^2 = m_Z^2c_W^2$. Notice that not only all of the fields in (2.31) are vevless, but also that the propagators that result from it are diagonal. Thus, there are three massive and one massless physical gauge bosons after SSB. Using the Goldstone theorem, we conclude that only three generators were broken, implying the existence of one linear combination of them conserved by the vacuum. In the unitary gauge, such combination is given by the charge $Q = t^3 + \frac{Y}{2Y_h}$[7]. Thus, we conclude that $SU(2)_L \otimes U(1)_Y$ was spontaneously broken into $U(1)_Q$.

We write the covariant derivative in terms of the physical gauge bosons as

[7] Note that Q is defined up to a normalization factor.

$$D_\mu = \partial_\mu + ig(t^+W_\mu^+ + t^-W_\mu^-) + i\left(gs_W t^3 + 2g'c_W Y_h \frac{Y}{2Y_h}\right) A_\mu + i\left(gc_W t^3 - 2g's_W Y_h \frac{Y}{2Y_h}\right) Z_\mu, \quad (2.32)$$

where we defined $t^\pm = \frac{t^1 \pm it^2}{\sqrt{2}}$. Introducing the electric charge $e \equiv gs_W = 2g'Y_h c_W$, we simplify this expression to the following form

$$D_\mu = \partial_\mu + ig(t^+W_\mu^+ + t^-W_\mu^-) + ieQA_\mu + \frac{ig}{c_W} Z_\mu(t^3 - s_W^2 Q). \quad (2.33)$$

2.3 Standard Model

As mentioned in Sec. 2.1, we must introduce a mechanism to the SM in order to generate the W and Z-boson masses. According to the previous section, the adition of a scalar transforming as the irrep $(1, 2, Y_h)$ achieves this. However, we must also generate the fermionic masses since the SM is a chiral theory. These will only be allowed if $Q_{fR} = Q_{fL}$, which fixes the hypercharge of the scalar $Y_h = \frac{1}{2}$. Having defined the matter content of the SM (see Tab. 1.1), we write its Lagrangian as

$$\mathcal{L} = \mathcal{L}_{kin} + \mathcal{L}_{NC} + \mathcal{L}_{CC} + \mathcal{L}_s + \mathcal{L}_{yuk} - \frac{1}{4}G_{\mu\nu}^a G^{a\mu\nu} - \frac{1}{4}W_{\mu\nu}^a W^{a\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}. \quad (2.34)$$

The different sectors of the SM Lagrangian are explicit below

$$\begin{aligned}
\mathcal{L}_s &= \frac{1}{2}(\partial^\mu h)(\partial_\mu h) - \frac{1}{2}m_h^2 h^2 \left(1 + \frac{h}{2v}\right)^2 + m_W^2 W_\mu^+ W^{-\mu}\left(1 + \frac{h}{v}\right)^2 + \frac{1}{2}m_Z^2 Z_\mu Z^\mu \left(1 + \frac{h}{v}\right)^2, \\
\mathcal{L}_{NC} &= -\frac{g_s}{2}\left(\sum_{f=u,d} \bar{f}^0 \gamma^\mu \lambda^a f^0\right) G_\mu^a - e\left(\sum_f Q_f \bar{f}^0 \gamma^\mu f^0\right) A_\mu - \frac{g}{c_W}\left[\sum_f \bar{f}^0 \gamma^\mu \left(g_R^f \gamma_R + g_L^f \gamma_L\right) f^0\right] Z_\mu, \\
\mathcal{L}_{kin} &= i\sum_f \bar{f}^0 \gamma^\mu \partial_\mu f^0, \quad \mathcal{L}_{CC} = -\frac{g}{\sqrt{2}}\left(\bar{u}_L^0 \gamma^\mu \gamma_L d^0 + \bar{\nu}^0 \gamma^\mu \gamma_L e^0\right) + h.c.,
\end{aligned} \quad (2.35)$$

where we introduced $g_R^f = -s_W^2 Q_f$ and $g_L^f = t_f^3 - s_W^2 Q_f$. Remember that we are using a notation in which f^0 is an n_g-dimensional vector in flavour space. Introducing $\tilde{\psi} = i\sigma_2 \psi^*$, the Yukawa Lagrangian of the SM is written in the unitary gauge as

$$-\mathcal{L}_{yuk} = \bar{q}_L^0 \psi \Gamma d_R^0 + \bar{q}_L^0 \tilde{\psi} \Delta u_R^0 + \bar{l}_L^0 \psi \Upsilon e_R^0 + h.c. = \left(\bar{d}_L^0 D_d^0 d_R^0 + \bar{u}_L^0 D_u^0 u_R^0 + \bar{e}_L^0 D_e^0 e_R^0\right)\left(1 + \frac{h}{v}\right) W_\mu^+ + h.c., \quad (2.36)$$

where we introduced $D_u^0 = \frac{1}{\sqrt{2}} v\Delta$, $D_d^0 = \frac{1}{\sqrt{2}} v\Gamma$ and $D_e^0 = \frac{1}{\sqrt{2}} v\Upsilon$. Note that these are arbitrary 3×3 complex matrices. The physical fermions are its eigenvectors, being written as $f_{L,R} = U_{fL,R}^\dagger f_{L,R}^0$. Notice that the diagonalization of the mass matrices is not a Weak Basis Transformation (WBT) given that the gauge sector of the Lagrangian is not invariant under it (a WBT requires $U_{dL} = U_{uL} \equiv U_L$). Nevertheless, only the Yukawa and the charged currents are not flavour invariant

$$\mathcal{L}_{yuk} = \left(\sum_f m_f \bar{f} f\right)\left(1 + \frac{h}{v}\right), \quad \mathcal{L}_{CC} = -\frac{g}{\sqrt{2}}\left(\bar{u}\gamma^\mu \gamma_L V_{CKM} d + \bar{\nu}\gamma^\mu \gamma_L e\right) W_\mu^+ + h.c., \quad (2.37)$$

where we defined $V_{CKM} = U_{uL}^\dagger U_{dL}$ and $\nu = U_{eL}^\dagger \nu^0$. The only Flavour Changing Couplings (FCC) at tree-level in the SM are mediated by the $W^\pm$ bosons (in the unitary gauge), resulting in the absence of tree-level FCNC in the SM. Thus, all flavour changing processes without transfer of charge are strongly suppressed in the SM. Notice also that there are no FCC in the leptonic sector of the SM. This is due to the massless nature of the neutrinos that allowed us to redefine them at will. As such, the tree-level FCC in the leptonic sector are recovered in extensions of the SM in which the neutrinos are given mass.

2.4 Multi-Higgs Doublet Models

MHDMs [13–15] are one of the simplest extensions of the SM. They are QFT based on the SSB of G_{SM} to $SU(3)_C \otimes U(1)_Q$ which share the fermionic content of the SM. In fact, MHDMs only differ from the SM through its scalar sector, which is composed of N scalars transforming according to the irrep $(1, 2, \frac{1}{2})$ of G_{SM}.

The extension of the scalar sector of the SM was not arbitrary. To understand why, consider a MHDM with N scalars transforming as the irreps $(1, 2T_a + 1, Y_a)$ of G_{SM}. The mass terms generated for the W and Z-bosons in such theory are given by

$$\mathcal{L} \subset \frac{1}{2} g^2 W_\mu^+ W^{-\mu} v_a^T (T^2 - T_3^2) v_a + \frac{g^2}{2c_W^2} Z_\mu Z^\mu v_a^T T_3^2 v_a. \tag{2.38}$$

Thus, the tree-level prediction for the parameter ρ is given in these models by

$$\rho = \frac{m_W^2}{m_Z^2} = c_W^2 \frac{\sum_a v_a^2 \left[T_a(T_a+1) - Y_a^2\right]}{\sum_a 2 v_a^2 Y_a^2}, \tag{2.39}$$

where we used the conservation of $Q = t^3 + Y$ after SSB. The experimental value $\rho \approx c_W^2$ [4] justifies the expansion of the scalar sector of the SM - for $T_a = Y_a = \frac{1}{2}$ we obtain $\rho = c_W^2$ at tree-level.

2.4.1 Scalar Potential

In a generic MHDM, the most general scalar potential can be written as

$$V(\phi) = \mu_{ab}(\phi_a^\dagger \phi_b) + \lambda_{ab,cd}(\phi_a^\dagger \phi_b)(\phi_c^\dagger \phi_d). \tag{2.40}$$

Given that the Lagrangian of any QFT must be hermitian [1], the coefficients μ_{ab} and $\lambda_{ab,cd}$ are forced to satisfy the relations

$$\mu_{ab} = \mu_{ba}^*, \quad \lambda_{ab,cd} = \lambda_{cd,ab} = \lambda_{ba,dc}^*. \tag{2.41}$$

While a naive counting of the number of parameters in (2.40) is $2N^2(N^2+1)$, hermiticity reduces it down to $\frac{1}{2}N^2(N^2+3)$. Since such number scales with N^4, MHDM with high N become unattractive due to a proliferation of free parameters. As such, we will only analyse 2HDM, or rather NHDM with $N = 2$, in this MSc thesis.

	Modulus	Phasis	Total
μ_{ab}	$\frac{1}{2}N(N+1)$	$\frac{1}{2}N(N-1)$	N^2
$\lambda_{ab,cd}$	$\frac{1}{4}N^2(N^2+3)$	$\frac{1}{4}N^2(N^2-1)$	$\frac{1}{2}N^2(N^2+1)$
Total	$\frac{1}{4}N(N^3+5N+2)$	$\frac{1}{4}N(N^3+N-2)$	$\frac{1}{2}N^2(N^2+3)$

Table 2.1: Number of parameters in the scalar sector of an NHDM.

Since the scalar potential must be such that a charge is conserved after SSB[8], we can decompose the dof of each scalar field as follows,

$$\phi_a = e^{i\alpha_a}\begin{bmatrix}\varphi_a^+ \\ \frac{1}{\sqrt{2}}(v_a+\rho_a+i\eta_a)\end{bmatrix}. \tag{2.42}$$

The vevs v_a of the scalar fields are determined by minimizing the scalar potential, which results in

$$\begin{aligned}\frac{\partial\langle 0\,|\,V\,|\,0\rangle}{\partial v_a} &= 0 \Rightarrow v_b\mathrm{Re}\left[\tilde{\mu}_{ab}\right] + v_b v_c v_d \mathrm{Re}\left[\tilde{\lambda}_{ab,cd}\right] = 0,\\ \frac{\partial\langle 0\,|\,V\,|\,0\rangle}{\partial \alpha_a} &= 0 \Rightarrow v_a v_b\mathrm{Im}\left[\tilde{\mu}_{ab}\right] + v_a v_b v_c v_d \mathrm{Im}\left[\tilde{\lambda}_{ab,cd}\right] = 0,\end{aligned} \tag{2.43}$$

where a sum over all scalar indices with exception of a is implies and we introduced

$$\tilde{\mu}_{ab} = \mu_{ab}e^{i(\alpha_b-\alpha_a)} \ \wedge \ \tilde{\lambda}_{ab,cd} = \lambda_{ab,cd}e^{i(\alpha_d-\alpha_c+\alpha_b-\alpha_a)}. \tag{2.44}$$

Notice that the Weak Basis (WB) of (2.42) does not have physical meaning. We can perform a WBT into the Higgs basis. defined by only one scalar with a vev[9]. Such WBT is realized by a real orthogonal matrix O such that $O_{1a} = \frac{v_a}{v}$, where $v^2 = \sum_{i=1}^N v_i^2$. In the Higgs basis, we decompose the dof of the scalar fields as

$$H_a = O_{ab}e^{-i\alpha_b}\phi_b = \begin{bmatrix}O_{ab}\varphi_b^+ \\ \frac{1}{\sqrt{2}}(v\delta_{a1}+O_{ab}\rho_b+iO_{ab}\eta_b)\end{bmatrix} \equiv \begin{bmatrix}C_a^+ \\ \frac{1}{\sqrt{2}}(v\delta_{a1}+R_a+iJ_a)\end{bmatrix}. \tag{2.45}$$

In this WB, the scalar potential is rewritten as

$$V(H) = \Upsilon_{ab}(H_a^\dagger H_b) + \Upsilon_{ab,cd}(H_a^\dagger H_b)(H_c^\dagger H_d), \tag{2.46}$$

where we have introduced

$$\Upsilon_{ab} = O_{ac}\tilde{\mu}_{cd}(O^T)_{db} \ \wedge \ \Upsilon_{ab,cd} = O_{ae}O_{cg}\tilde{\lambda}_{ef,gh}(O^T)_{fb}(O^T)_{hd}. \tag{2.47}$$

Using (2.43), it is straightforward to prove the following relations

$$\mathrm{Re}\left[\tilde{\Upsilon}_{a1}\right] = \mathrm{Im}\left[\tilde{\Upsilon}_{a1}\right] = 0 \Leftrightarrow \tilde{\Upsilon}_{a1} = 0, \tag{2.48}$$

[8]Unlike in the SM, conservation of electric charge after SSB is not a given in the general MHDM. Therefore we must determine the conditions in which the potential conserves Q in all models.

[9]For MHDM with $N > 2$, the Higgs basis is not uniquely defined since we can rotate the vevless scalars at will.

where we assumed $v_a \neq 0$ and defined

$$\tilde{\Upsilon}_{ab} = \Upsilon_{ab} + \Upsilon_{ab,11} v^2. \tag{2.49}$$

The mass terms of the scalar fields of a MHDM can be easily obtained by keeping only the quadratic terms in (2.46) and applying the conditions in (2.48). Using such method, we obtain

$$V_M = \tilde{\Upsilon}_{ab} C_b^+ C_a^- + \frac{1}{2} \begin{bmatrix} R_a & J_a \end{bmatrix} \begin{bmatrix} \mathrm{Re}\left[T_{ab}^+\right] & -\mathrm{Im}\left[T_{ab}^-\right] \\ -\mathrm{Im}\left[T_{ba}^-\right] & \mathrm{Re}\left[T_{ab}^-\right] \end{bmatrix} \begin{bmatrix} R_b \\ J_b \end{bmatrix}, \tag{2.50}$$

where we introduced

$$T_{ab}^{\pm} = \tilde{\Upsilon}_{ab} + \Upsilon_{a1,1b} v^2 \pm \Upsilon_{a1,b1} v^2. \tag{2.51}$$

Using (2.48), it is straightforward to prove $T_{a1}^- = 0$. Thus, we conclude from (2.50) that C_1^+ and J_1^+ are zero mass eigenstates of the scalar potential, i.e., they are the would-be Goldstone bosons (G^+ and G^0, respectively) that are absorbed in the unitary gauge by the gauge bosons in the general MHDM.

Given that $\tilde{\Upsilon}_{ab} = \tilde{\Upsilon}_{ba}^*$, the physical charged scalars are obtained from the C_a^+ states as

$$h_a^+ = C_{ab} C_b^+, \tag{2.52}$$

where $C_{1a} = C_{a1} = \delta_{a1}$ and $CC^\dagger = C^\dagger C = 1$. Note that in this notation $h_1^+ = G^+$. In the general MHDM, the physical neutral bosons K_{a^*} (where a^* runs from 1 to $2N$) are a combination of the R_a and J_a states. They are obtained from the diagonalization of (2.50) to be

$$K_{a^*} = K_{a^*,a} J_a + K_{a^*,N+b} R_b, \tag{2.53}$$

where $K_{a^*,1} = K_{1,a^*} = \delta_{a^*1}$ and $KK^T = K^T K = 1$. Note that in this notation $K_1 = G^0$. However, it can be shown (see Sec. 2.5.2) that the R_a and J_a states decouple in a CP-conserving potential. As such, the neutral sattes can be written for such potential as

$$h_a = R_{ab} R_b \ \wedge \ j_a = J_{ab} J_b, \tag{2.54}$$

where $J_{a1} = J_{1a} = \delta_{a1}$ and $RR^T = R^T R = JJ^T = J^T J = 1$. In this notation, $j_1 = G^0$.

2.4.2 Yukawa Sector

The quark sector of the Yukawa Lagrangian of the most general MHDM is given by

$$-\mathcal{L}_Y = \bar{q}_L^0 \phi_a \Gamma'_a d_R^0 + \bar{q}_L^0 \tilde{\phi}_a \Delta'_a u_R^0 + h.c. \tag{2.55}$$

Using the decomposition of the scalars ϕ_a given in (2.42), we obtain

$$
\begin{aligned}
-\mathcal{L}_Y = & \bar{d}_L^0\left(\frac{1}{\sqrt{2}}v_a\Gamma_a\right)d_R^0 + \bar{u}_L^0\left(\frac{1}{\sqrt{2}}v_a\Delta_a\right)u_R^0 + \rho_a\left[\bar{d}_L^0\left(\frac{1}{\sqrt{2}}\Gamma_a\right)d_R^0 + \bar{u}_L^0\left(\frac{1}{\sqrt{2}}\Delta_a\right)u_R^0\right] \\
& + i\eta_a\left[\bar{d}_L^0\left(\frac{1}{\sqrt{2}}\Gamma_a\right)d_R^0 - \bar{u}_L^0\left(\frac{1}{\sqrt{2}}\Delta_a\right)u_R^0\right] + \varphi_a^+\left(\bar{u}_L^0\Gamma_a d_R^0 - \bar{u}_R^0\Delta_a^\dagger d_L^0\right) + h.c.,
\end{aligned} \tag{2.56}
$$

where $\Gamma_a = e^{i\alpha_a}\Gamma'_a$ and $\Delta_a = e^{-i\alpha_a}\Delta'_a$. From (2.56), we identify the mass matrices to be given by $D_d^0 = \frac{1}{\sqrt{2}}v_a\Gamma_a$ and $D_u^0 = \frac{1}{\sqrt{2}}v_a\Delta_a$. The interaction terms in (2.56) are written in the $\{\Gamma_a, \Delta_a\}$ basis. However, such basis is inadequate given that a combination of its elements possesses physical meaning. As such, we rewrite these couplings on a basis which has those combinations as elements. Such a goal is achieved through the previously introduced O matrix,

$$
N_{da}^0 = \frac{1}{\sqrt{2}}vO_{ab}\Gamma_b \;\wedge\; N_{ua}^0 = \frac{1}{\sqrt{2}}vO_{ab}\Delta_b. \tag{2.57}
$$

In such basis, (2.56) is written as

$$
-\mathcal{L}_Y = \frac{R_a}{v}\left(\bar{q}_L^0 N_{qa}^0 q_R^0\right) + i\epsilon_q\frac{J_a}{v}\left(\bar{q}_L^0 N_{qa}^0 q_R^0\right) + \frac{\sqrt{2}}{v}C_a^+\left(\bar{u}_L^0 N_{da}^0 d_R^0 - \bar{u}_R^0 N_{ua}^{0\dagger} d_L^0\right) + h.c., \tag{2.58}
$$

where there is an implicit sum over $q = d, u$, $\epsilon_d = 1$, $\epsilon_u = 1$ and we have made a WBT to the Higgs basis in order to remove the dependence in O from the Lagrangian. Finally, we diagonalize the mass matrices through the transformations $f_X^0 = U_{fX}f_X$ in order to obtain the Yukawa interactions of the physical fermions

$$
-\mathcal{L}_Y = \frac{R_a}{v}\bar{q}(N_{qa}\gamma_R + N_{qa}^\dagger\gamma_L)q + i\epsilon_q\frac{J_a}{v}\bar{q}(N_{qa}\gamma_R - N_{qa}^\dagger\gamma_L)q + \left[\frac{\sqrt{2}}{v}C_a^+\bar{u}\left(VN_{da}\gamma_R - N_{ua}^\dagger V\gamma_L\right)d + h.c.\right], \tag{2.59}
$$

where $V = U_{uL}^\dagger U_{dL}$ is the CKM matrix and we defined $N_{xa} = U_{xL}^\dagger N_{xa}^0 U_{xR}$. Since for $a \neq 1$ the N_{qa} couplings are 3×3 arbitrary complex matrices, $36(N-1)$ new parameters arise in the quark sector of the Yukawa Lagrangian of the general MDHM, including tree-level FCNC. We must now point out that the couplings in (2.59) are not written in terms of the physical bosons. Using (2.52), we write the interaction between the physical charged scalars and the fermions as

$$
-\mathcal{L}_Y^\pm = \frac{\sqrt{2}}{v}C_{ab}^*\,h_a^+\bar{u}(u_b + i\gamma_5 v_b)d + h.c., \tag{2.60}
$$

where we introduced the quantities

$$
u_a = \frac{1}{2}(VN_{da} - N_{ua}^\dagger V), \;\; v_a = \frac{1}{2i}(VN_{da} + N_{ua}^\dagger V). \tag{2.61}
$$

As for the neutral scalars, we can use (2.53) in (2.59) to obtain the interactions between the physical fermions and the physical neutral scalars in the general MHDM,

$$
-\mathcal{L}_Y^0 = \frac{K_{a^*,N+a}}{v}K_{a^*}\;\bar{q}\left(N_{qa}\gamma_R + N_{qa}^\dagger\gamma_L\right)q + i\epsilon_q\frac{K_{a^*,a}}{v}K_{a^*}\;\bar{q}\left(N_{qa}\gamma_R - N_{qa}^\dagger\gamma_L\right)q. \tag{2.62}
$$

If CP is conserved in the scalar potential of a MHDM, (2.62) is simplified to

$$-\mathcal{L}_Y^0 = \frac{h_a}{2v}\bar{q}(A_{qa}^+ + A_{qa}^-\gamma_5)q + i\epsilon_q\frac{j_a}{2v}\bar{q}(A_{qa}'^- + A_{qa}'^+\gamma_5)q, \tag{2.63}$$

where we introduced $A_{qa}^{\pm} = \tilde{N}_{qa} \pm \tilde{N}_{qa}^\dagger$ and $A_{qa}'^{\pm} = \tilde{N}_{qa}' \pm \tilde{N}_{qa}'^\dagger$ where

$$\tilde{N}_{qa} = R_{ab}N_{qb} \ \wedge \ \tilde{N}_{qa}' = J_{ab}N_{qb}. \tag{2.64}$$

2.4.3 Abelian Flavour Symmetries

In the previous subsections, we have studied the scalar potential and the Yukawa Lagrangian of the general MHDM. In this section, we require the theory to be invariant under a global symmetry S,

$$\phi_a \to \phi_a^S = S_{ab}\phi_b, \ \ q_L^0 \to (q_L^0)^S = S_L q_L^0, \ \ d_R^0 \to (d_R^0)^S = S_{dR}d_R^0, \ \ u_R^0 \to (u_R^0)^S = S_{uR}u_R^0, \tag{2.65}$$

where S_{ab}, S_L, S_{dR} and S_{uR} are unitary matrices in order to leave the kinetic terms invariant. Requiring (2.40) and (2.55) to be invariant under such symmetry forces the Yukawa couplings and the parameters of the potential to satisfy

$$\Gamma_a' = S_L^\dagger\Gamma_b' S_{dR}S_{ba}, \ \ \Delta_a' = (S^\dagger)_{ab}S_L^\dagger\Delta_b' S_{uR}, \ \ \mu_{ab} = S_{ac}\mu_{cd}(S^\dagger)_{db}, \ \ \lambda_{ab,cd} = S_{ae}S_{cg}\lambda_{ef,gh}(S^\dagger)_{fb}(S^\dagger)_{hd}. \tag{2.66}$$

Notice that the texture of the symmetry is not Weak Basis Invariant (WBI). If we perform the WBT

$$\phi_a \to \phi_a' = W_{ab}\phi_b, \ \ q_R^0 \to q_L'^0 = W_L q_L^0, \ \ d_R^0 \to d_R'^0 = W_{dR}d_R^0, \ \ u_R^0 \to u_R'^0 = W_{uR}u_R^0, \tag{2.67}$$

the textures of the symmetry in (2.65) are transformed to

$$S_{ab}' = W_{ac}S_{cd}(W^\dagger)_{db}, \ \ S_L' = W_L S_L W_L^\dagger, \ \ S_{dR}' = W_{dR}S_{dR}W_{dR}^\dagger, \ \ S_{uR}' = W_{uR}S_{uR}W_{uR}^\dagger. \tag{2.68}$$

Since all matrices in (2.65) are unitary, we can use (2.67) to go into a WB in which the symmetry is diagonal,

$$S_{ab} = e^{i\theta_a}\delta_{ab}, \ \ S_L = \mathrm{diag}(e^{i\alpha_1}, e^{i\alpha_2}, e^{i\alpha_3}), \ \ S_{dR} = \mathrm{diag}(e^{i\beta_1}, e^{i\beta_2}, e^{i\beta_3}), \ \ S_{uR} = \mathrm{diag}(e^{i\gamma_1}, e^{i\gamma_2}, e^{i\gamma_3}). \tag{2.69}$$

It is common to call these symmetries AFS [36, 37] since not only do they distinguish the generations of fermions and scalars (hence flavour), but we can also find a WB in which they are diagonal (hence Abelian). Due to the accidental global symmetries of the MHDM Lagrangian[10] we can enforce $\theta_1 = \alpha_1 = 0$. In the WB in which the symmetry is diagonal, (2.66) is written as

$$\begin{gathered}(\Gamma_a')_{ij} = e^{i(\beta_j - \alpha_i + \theta_a)}(\Gamma_a')_{ij}, \ \ (\Delta_a')_{ij} = e^{i(\gamma_j - \alpha_i - \theta_a)}(\Delta_a')_{ij}, \\ \mu_{ab} = e^{i(\theta_a - \theta_b)}\mu_{ab}, \ \ \lambda_{ab,cd} = e^{i(\theta_a - \theta_b + \theta_c - \theta_d)}\lambda_{ab,cd}.\end{gathered} \tag{2.70}$$

These relations are structured in such a way that the parameters are forced to be null or left completely arbitrary. Notice that while the WB in which we wrote (2.40) and (2.55) was not physic, the WB

[10] $q_L^0 \to e^{i\alpha}q_L^0 \ \wedge \ u_R^0 \to e^{i\alpha}u_R^0 \ \wedge \ d_R^0 \to e^{i\alpha}d_R^0$ and $\psi_a \to e^{i\theta}\psi_a \ \wedge \ d_R^0 \to e^{-i\theta}d_R^0 \ \wedge \ u_R^0 \to e^{i\theta}u_R^0$.

of (2.70) is. (2.70) can be used to restrict the set of allowed AFSs. Since no massless quark has been detected [4], we conclude that the mass matrices $N^0_{d1} \propto v_a\Gamma_a$ and $N^0_{u1} \propto v_a\Delta_a$ cannot have a null determinant. Thus, there cannot be an entire column equal to zero for all couplings. This restricts the possible ways in which the RH fermions may transform to

$$\beta_j = -\theta_a, \alpha_2 - \theta_a, -\alpha_3 - \theta_a \;\wedge\; \gamma_j = \theta_a, \alpha_2 + \theta_a, \theta_3 + \theta_a. \tag{2.71}$$

A similar argument requires that no row can be entirely null for all Yukawa couplings. Thus, we obtain the possible ways in which LH fermions may be transformed

$$\alpha_i = \theta_a - \theta_b, \theta_a + \theta_b, \theta_a + \theta_b - \theta_c. \tag{2.72}$$

We construct the set of AFSs from which massless quarks may be absent[11] by combining both restrictions. As such, we find that all unbroken AFSs that are not ruled out by experimental results belong to the (at most) (N-1)-dimensional vectorial space

$$\mathcal{F} = \Big\{\alpha_i = a_a\theta_a, \beta_i = b_a\theta_a, \gamma_j = c_a\theta_a\Big\}, \tag{2.73}$$

where we imply a sum over a. The coefficients a_a, b_a and c_a are integers of modulus smaller than three. Using such fact, we majorate the total number of possible MHDMs arising from AFSs that are not ruled out by experiment[12] to $5^{3(N-1)}$. Since the total number of distinguishable AFSs was 2^{18N}, this realization has provided a reduction of the order of 10^9 possible models for $N = 2$ and 10^{12} for $N = 3$.

2.4.4 Experimental Constraints

The existence of tree-level FCNC (see (2.63)) can be probed experimentally in two types of processes, direct and indirect [6]. The first is defined by the presence of scalar particles in either the initial or final state of the event. The most stringent experimental constraints of this type arise from the decay width of the top quark [38]. In the general MHDM, such process is given at tree-level by the following Feynman diagrams

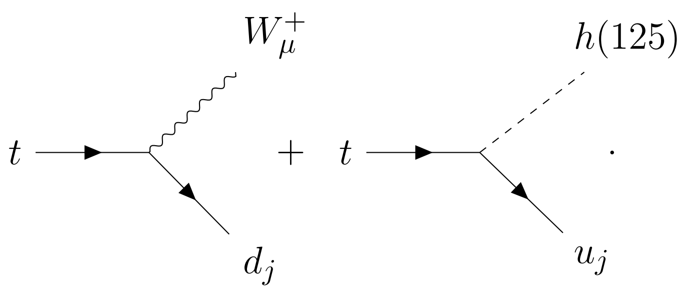

Figure 2.1: Feynman diagrams contributing at tree-level to the top quark width in the general MHDM.

[11] It is not assured that all models in this set are free from massless quarks, but rather that all odels outside of it have them.

[12] Such number can be further decreased by requiring the CKM matrix to not be Cabibbo-like and removing permutations of fermions and scalars.

We used the fact that only one scalar with a mass lower than the top has been found [4] to draw these diagrams. Furthermore, we have considered such boson to be a scalar since the possibility of the (125) particle detected at the LHC being a pure pseudoscalar has already been ruled out[13] [39, 40]. Neglecting the masses of all fermions but the top quark, we compute its decay width in the general MHDM with a CP-conserving potential to be

$$\Gamma = \frac{m_t^3}{16\pi v^2}(1 - 3x_W^4 + 2x_W^6)\sum_{j=d,s,b}|V_{tj}|^2 + \frac{m_t}{64\pi v^2}(1-x_h^2)^2\sum_{j=u,c}\left[|(A_{u1}^+)_{jt}|^2 + |(A_{u1}^-)_{jt}|^2\right], \quad (2.74)$$

where we defined $x_V = \frac{m_V}{m_t}$. Using the top quark pole mass $m_t = 174.2\ GeV$ [4], the scalar mass $m_h = 125.09\ GeV$ [4], the W-boson mass $m_W = 80.385\ GeV$ [4] and the upper bounds to the $t \to hq$ branching ratio of 0.19% from the ATLAS [41] and 0.56% from the CMS [42] collaborations , we obtain the following constraint on MHDM,

$$|(\tilde{N}_{u1})_{qt}|^2 + |\tilde{N}_{u1})_{tq}|^2 < 2m_t^2|V_{tb}|^2\frac{1-3x_W^4+2x_W^6}{(1-x_h^2)^2}\ \mathrm{BR}(t\to hq) \sim 440\ \mathrm{GeV}^2, \quad (2.75)$$

where we assumed $BR(t \to W^+b) \approx 1$. It is clear that this constraint only applied to MHDM with tree-level FCNC in the up sector. If no such couplings exist, we must use scalar decays of the type $h \to \bar{d}_i d_j$ to constrain MHDMs. While inexistent (at tree-level) in the SM [3], these decays are computed at tree-level in a general MHDM through the Feynman diagram

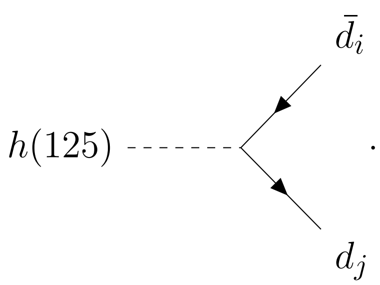

Figure 2.2: Tree-level Feynman diagram contributing to decay $h \to \bar{d}_i d_j$ in the general MHDM.

Neglecting the fermionic masses with respect to the scalar mass, the decay width of this process is given by

$$\Gamma = \frac{m_h}{32\pi v^2}\left[|(A_{d1}^+)_{ji}|^2 + |(A_{d1}^-)_{ji}|^2\right] = \frac{m_h}{16\pi v^2}\left[|(\tilde{N}_{d1})_{ij}|^2 + |(\tilde{N}_{d1})_{ji}|^2\right]. \quad (2.76)$$

Using the upper bound on the width of the (125) scalar of $\Gamma_h < 26\ MeV$ [43], we constrain MHDM without tree-level FCNC in the up sector by requiring $\Gamma(h \to \bar{d}_i d_j)$ to be below such limit,

$$\left|(\tilde{N}_{d1})_{ij}\right|^2 + \left|(\tilde{N}_{d1})_{ji}\right|^2 < \frac{16\pi v^2}{m_h}\Gamma \sim 640\ \mathrm{GeV}^2 \quad (2.77)$$

Indirect processes [6] are defined as those in which scalar particles participate only as intermediate states. In the general MHDM, the most stringent of such processes [13, 38] is the neutral meson mixing $P^0 - \bar{P}^0$[14] that is given at tree-level by the Feynman diagrams

[13] The possibility of the (125) particle to have mixed scalar and pseudoscalar components has not yet been ruled out.

[14] We assume the quark content of the meson states to be $P^0 = i\bar{j}$ and $\bar{P}^0 = j\bar{i}$.

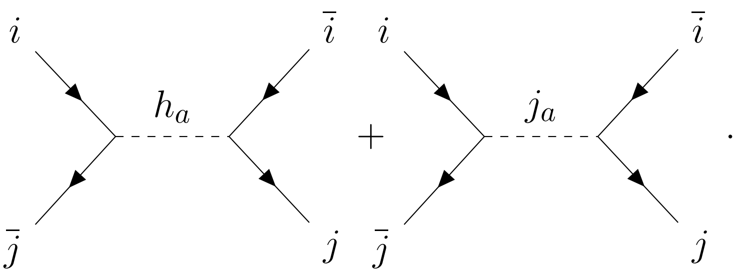

Figure 2.3: Feynman diagrams contributing at tree-level to $P^0 - \bar{P}^0$ mixing in a general MHDM.

As we did for the direct processes, we will assume that the scalar potential conserves CP. From both diagrams, we compute the effective Hamiltonian of meson mixing in a MHDM

$$\mathcal{H}_{eff} = \frac{1}{4v^2}\sum_{a=1}^{N}\frac{(\bar{j}\Gamma^a_{ji} i)^2}{m^2_{h_a}} - \frac{1}{4v^2}\sum_{a=2}^{N}\frac{(\bar{j}\Gamma'^a_{ji}\, i)^2}{m^2_{j_a}}, \tag{2.78}$$

where we introduced

$$\Gamma^a_{ji} = (A^+_{qa})_{ji} + (A^-_{qa})_{ji}\gamma_5, \;\; \Gamma'^a_{ji} = (A'^-_{qa})_{ji} + (A'^+_{qa})_{ji}\gamma_5. \tag{2.79}$$

Using (2.78), we compute M^P_{12} (see Sec. A) in a MHDM with a CP-conserving potential to be

$$M^P_{12} = \sum_{a=1}^{N}\frac{m_P f_P^2}{48v^2 m^2_{h_a}}\left[\chi^+(A^+_{qa})^2_{ji} - \chi^-(A^-_{qa})^2_{ji}\right] - \sum_{a=2}^{N}\frac{m_P f_P^2}{48v^2 m^2_{j_a}}\left[\chi^+(A'^-_{qa})^2_{ji} - \chi^-(A'^+_{qa})^2_{ji}\right], \tag{2.80}$$

where we defined

$$\chi^+ = \frac{m_P^2 - (m_i + m_j)^2}{(m_i + m_j)^2} \;\wedge\; \chi^- = \frac{11 m_P^2 - (m_i + m_j)^2}{(m_i + m_j)^2} = 11\chi^+ + 10. \tag{2.81}$$

Assuming that the scalar sector of the theory is such that $m_h \ll m_{h_a}, m_{j_a}$, where h is the (125) particle found by the ATLAS [11] and the CMS [12] collaborations, we can simplify (2.80) to

$$M^P_{12} = \frac{m_P f_P^2}{48v^2 m_h^2}\Big[\chi^+(A^+_{q1})^2_{ji} - \chi^-(A^-_{q1})^2_{ji}\Big]. \tag{2.82}$$

MHDMs are constrained by requiring that the contributions of the neutral scalars do not exceed the experimental value of Δm_P (see Sec. A). Thus, a valid MHDM must satisfy

$$\Big|\chi^+(A^+_{q1})^2_{ji} - \chi^-(A^-_{q1})^2_{ji}\Big| < \frac{24v^2 m_h^2 \Delta m_P}{m_P f_P^2}. \tag{2.83}$$

Out of the four meson systems, it turns out that the most stringent constraint arises from the $D^0 - \bar{D}^0$ system

$$\Big|\chi^+_D(A^+_{u1})^2_{cu} - \chi^-_D(A^-_{u1})^2_{cu}\Big| < 2\times 10^{-3}\ \mathrm{GeV}^2. \tag{2.84}$$

It is clear that this bound only applied if there are tree-level FCNC in the up sector of the theory. If that is not the case, the most stringent constraint related to indirect processes that may be applied to MHDMs arises from $K^0 - \bar{K}^0$ mixing

$$\left|\chi_K^+(A_{d1}^+)^2_{sd} - \chi_K^-(A_{d1}^-)^2_{sd}\right| < 7\times 10^{-3}\ \mathrm{GeV}^2. \tag{2.85}$$

2.5 CP Violation

A CP transformation [5] is defined as the product of a parity operation ($\vec{r} \to -\vec{r}$) and a charge conjugation ($Q_i \to -Q_i$ for all charges). The geometrical view of these operations is such that we classicaly expect both to commute with a time translation ($t \to t + \Delta t$). As such, the correspondence principle states that the quantum operators that represent CP should obey $[\mathcal{H}, P] = [\mathcal{H}, C] = 0$. Since these imply that C and P are symmetries of nature, we can only define a quantum operator for CP compatible with its classical interpretation in theories that conserve CP.

If CP may be violated, we apply the following procedure. First, we isolate the manifestly CP invariant sector of the Lagrangian, $\mathcal{L}_{inv}$. Then, we define the most generic CP transformation that leaves $\mathcal{L}_{inv}$ invariant. Such transformations are used as a probe to check whether the full Lagrangian is CP invariant. If none of them is a symmetry of $\mathcal{L} - \mathcal{L}_{inv}$, we say that the theory violates CP.

Since the kinetic terms in (2.1) are CP invariant, we can find the most generic CP transformations of singlet and spinorial fields

$$(CP)\phi_a(CP)^\dagger = U_{ab}\phi_b^{\dagger^T},\ \ (CP)f_a(CP)^\dagger = K_{ab}\gamma^0 C\bar{f}_b^T, \tag{2.86}$$

where U_{ab} and K_{ab} are unitary matrices that mix fields with similar quantum numbers and masses. Note that these transformations have the most general form for all QFTs.

2.5.1 Standard Model

Following the procedure devined above, we take $\mathcal{L}_{inv} = \mathcal{L} - \mathcal{L}_Y^q$. For such choice, the most generic CP transformations that we may define in the SM are

$$\begin{gathered}(CP)\phi(CP)^\dagger = \phi^{\dagger^T},\ \ (CP)q_L^0(CP)^\dagger = K_L\gamma^0 C\bar{q}_L^{0T},\\ (CP)u_R^0(CP)^\dagger = K_{uR}\gamma^0 C\bar{u}_R^{0T},\ \ (CP)d_R^0(CP)^\dagger = K_{dR}\gamma^0 C\bar{d}_R^{0T},\end{gathered} \tag{2.87}$$

under which the quark sector of the Yukawa Lagrangian is transformed to

$$-(CP)\mathcal{L}_Y^q(CP)^\dagger = \bar{q}_L^0\phi(K_L^\dagger\Gamma K_{dR})^* d_R^0 + \bar{q}_L^0\tilde{\phi}(K_L^\dagger\Delta K_{uR})^* u_R^0 + h.c. \tag{2.88}$$

Thus, the SM is CP invariant if and only if there is a choice of unitary matrices K_L, K_{dR} and K_{uR} such that

$$\Gamma^* = K_L^\dagger\Gamma K_{dR}\ \wedge\ \Delta^* = K_L^\dagger\Delta K_{uR}. \tag{2.89}$$

Using both conditions, it is trivial to prove that CP invariance requires the following relation to hold

$$[H_d, H_u]^T = -K_L^\dagger [H_d, H_u] K_L, \tag{2.90}$$

where $H_q = D_q^0 (D_q^0)^\dagger$ is an Hermitian matrix. This relation can be used to prove that the following trace must be null in order for the SM to be CP invariant [44],

$$\mathrm{Tr}\left[[H_d, H_u]^{2n+1} \right] = 0. \tag{2.91}$$

Notice that this equality is valid for any number of fermionic generations n_g. It can be shown [44] that this trace is always null for $n_g < 3$, implying that at least three generations of matter are required to generate CP violation in the SM.

For $n_g = 3$, we can write the lowest order non-null WBI of (2.91) in the WB in which the down-type mass matrix is diagonal ($K_L = U_{dL}$) as

$$\mathrm{Tr}\left[[H_d, H_u]^3 \right] = 6i(m_b^2 - m_s^2)(m_b^2 - m_d^2)(m_s^2 - m_d^2)\mathrm{Im}\big[(H_u)_{12}(H_u)_{13}^*(H_u)_{23}\big]. \tag{2.92}$$

We conclude from this expression that the SM is CP invariant if the two sectors are diagonalized by the same transformation, i.e., $U_{dL} = U_{uL}$. Thus, CP violation arises in the SM due to a misalignment in flavour space between the two quark sectors. This can be further highlighted by a related invariant[15] to (2.91) [45]

$$J = |\det[H_d, H_u]| = (m_t^2 - m_c^2)(m_t^2 - m_u^2)(m_c^2 - m_u^2)(m_b^2 - m_s^2)(m_b^2 - m_d^2)(m_s^2 - m_d^2)\left|\mathrm{Im}\left[V_{ij}V_{km}V_{im}^*V_{kj}^*\right]\right|. \tag{2.93}$$

In sum, the SM is CP-violating if there are no degenerate quarks (note that we could take such conclusion from (2.91) as well) and if $Q_{ijkm} = V_{ij}V_{km}V_{im}^*V_{kj}^*$ is complex, i.e., $n_g > 2$. Since both conditions hold, we conclude that CP is not conserved in the SM.

2.5.2 Multi-Higgs Doublet Models

When studying CP violation in a MHDM, we will take $\mathcal{L}_{inv} = \mathcal{L} - V(\psi) - \mathcal{L}_Y^q$. Thus, the most generic CP transformations for the spinorial fields are given in (2.87) while for the scalars we write

$$(CP)\phi_a(CP)^\dagger = U_{ab}\phi_b^{\dagger T}. \tag{2.94}$$

It is interesting to distinguish explicit from spontaneous CP violation. It is clear from (2.94) that the unitary matrix U_{ab} must satisfy the following identity after SSB

$$v_a e^{i\alpha_a} = U_{ab} v_b e^{-i\alpha_b}. \tag{2.95}$$

We say that a theory has spontaneous CP violation if there are unitary matrices for which (2.94) is a symmetry of the full Lagrangian, but none of them satisfies (2.95). For the theories in which there are

[15] These WBI are only related for $n_g = 3$. For a different n_g the WBI of (2.93) does not have physical meaning.

no U_{ab} such that (2.94) is a symmetry of the full Lagrangian we say that CP is explicitly violated.

Scalar Potential

The scalar potential of the generic MHDM (see (2.40)) is transformed under the (2.94) to

$$(CP)V(CP)^\dagger = \mu_{ab}U^*_{ac}U_{bd}(\phi^\dagger_d\phi_c) + \lambda_{ab,cd}U^*_{ae}U_{bg}U^*_{cg}U_{dh}(\phi^\dagger_f\phi_e)(\phi^\dagger_h\phi_g). \tag{2.96}$$

This implies that CP is explicitly violated in the scalar potential of a MHDM if there is no unitary matrix U such that

$$\mu^*_{ab} = (U^\dagger)_{ac}\mu_{cd}U_{db}, \;\; \lambda^*_{ab,cd} = (U^\dagger)_{ae}(U^\dagger)_{cg}\lambda_{ef,gh}U_{fb}U_{hd}. \tag{2.97}$$

It is straightforward to prove[16] that imposing CP-conservation in the rephasing invariant sector of the most general scalar potential[17] requires U to be diagonal

$$U_{ab} = e^{i\theta_a}\delta_{ab}. \tag{2.98}$$

This result is generalized to all MHDM that are invariant under an AFS. However, it only holds in the WB in which the symmetry is diagonal since in such WB the rephasing invariant sector of these models is identical to that of the most general MHDM. Thus, the scalar potential of a MHDM invariant under an AFS[18] violates CP-explicitly if there are no θ_i such that

$$\mu^*_{ab} = e^{i(\theta_b-\theta_a)}\mu_{ab}, \;\; \lambda^*_{ab,cd} = e^{i(\theta_d-\theta_c+\theta_b-\theta_a)}\lambda_{ab,cd}. \tag{2.99}$$

Remember that this relation only holds in the WB in which the flavour symmetry is diagonal. After SSB, the coefficents θ_i are determined by (2.95) to be

$$e^{i\theta_a} = e^{2i\alpha_a}. \tag{2.100}$$

As a consequence, a CP-conserving scalar potential must satisfy the following constraints in the WB in which its flavour symmetry is diagonal

$$\tilde{\mu}^*_{ab} = \tilde{\mu}_{ab}, \;\; \tilde{\lambda}^*_{ab,cd} = \tilde{\lambda}_{ab,cd}. \tag{2.101}$$

This result is required to prove (2.54). This was to be expected since (2.54) merely states that the eigenstates of a CP-conserving Hamiltonian are CP-conserving themselves.

To summarize, a potential has explicit CP-violation if (2.99) is not satisfied, spontaneous CP-violation if (2.101) does not hold and no CP-violation if both are true.

[16]Note that there are $N(N+1)$ independent terms in the rephasing invariant sector of the scalar potential of the most general MHDM, but only N^2 parameters in a general $N \cdot N$ unitary matrix. As such, only the trivial solution is possible to ensure CP conservation in such sector.

[17]defined as $V'(\phi_a) = V'(e^{i\beta_a}\phi_a)$.

[18]Notice that the most general MHDM is invariant under the AFS $S = 1$.

Yukawa Sector

The quark sector of the Yukawa Lagrangian in (2.55) is CP transformed by (2.94) to

$$-(CP)\mathcal{L}_Y(CP)^\dagger = U^*_{ab}\bar{q}^0_L\phi_b\left[K^\dagger_L\Gamma'_aK_{dR}\right]^* d^0_R + U_{ab}\bar{q}^0_L\tilde{\phi}_b\left[K^\dagger_L\Delta'_aK_{uR}\right]^* u^0_R + h.c. \tag{2.102}$$

Thus, the Yukawa sector of a MHDM is CP-invariant if there are unitary matrices K_L, K_{dR} and K_{uR} such that

$$(\Gamma'_a)^* = \left[K^\dagger_L\Gamma'_bK_{dR}\right]U_{ba}, \;\; (\Delta'_a)^* = \left[K^\dagger_L\Delta'_bK_{uR}\right]U^*_{ba}. \tag{2.103}$$

In the previous section, we have seen that if a MHDM is invariant under an AFS we can always go into a WB in which (2.98) is valid. Since we saw in Sec. 2.4.3 that in such WB the Yukawa couplings are zero or arbitrary, the K_X matrices must also be diagonal. Thus, CP is explicitly violated in the Yukawa sector of a MHDM invariant under an AFS if there are no θ_a, ξ_L, ξ_d and ξ_u such that

$$(\Gamma'_a)^*_{ij} = e^{i(\xi_{d_j}-\xi_{L_i}+\theta_a)}(\Gamma'_a)_{ij}, \;\; (\Delta'_a)^*_{ij} = e^{i(\xi_{u_j}-\xi_{L_i}-\theta_a)}(\Delta'_a)_{ij}. \tag{2.104}$$

As for spontaneous CP violation, we point out that after SSB the coefficients θ_a are determined by (2.100). Thus, CP is spontaneous violated in the Yukawa sector of a MHDM if (2.104) is satisfied but the following relation is not

$$(\Gamma_a)^*_{ij} = e^{i(\xi_{d_j}-\xi_{L_i})}(\Gamma_a)_{ij}, \;\; (\Delta_a)^*_{ij} = e^{i(\xi_{u_j}-\xi_{L_i})}(\Delta_a)_{ij}. \tag{2.105}$$

Chapter 3

Controlling the Number of Parameters

In Sec. 2.4, we have seen that $36(N-1)$ and $\frac{1}{2}N^2(N^2+3)-2$ new parameters arise in the Yukawa sector and the scalar potential of the general MHDM, respectively. Given such proliferation of free parameters we will focus our analysis on Two-Higgs Doublet Models (2HDM) for the rest of this chapter. Nevertheless, the general 2HDM has 48 more parameters than the SM. As such, they are difficult theories to test, with an elevated number of uncorrelated observables required for their validation [14].

In this chapter, we will develop a principle to build renormalizable 2HDM with a reduced parameter space.

3.1 BGL Models

A solution to this problem was brought forward by Branco, Grimus and Lavoura in the form of BGL models [23]. These are 2HDM that are invariant under the following AFS[1]

$$\mathcal{S}: \; q_{L3} \to \Upsilon q_{L3}, \; u_{R3} \to \Upsilon^2 u_{R3}, \; \phi_2 \to \Upsilon \phi_2, \tag{3.1}$$

with the remaining fields transforming trivially under $\mathcal{S}$ and $|\Upsilon| = 1 \;\; \wedge \;\; \Upsilon^2 \neq 1$. Replacing this symmetry in (2.70), we obtain the following structure for the Yukawa couplings of tBGL models

$$\Gamma_1' = \begin{bmatrix} x & x & x \\ x & x & x \\ 0 & 0 & 0 \end{bmatrix}, \; \Gamma_2' = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ x & x & x \end{bmatrix}, \; \Delta_1' = \begin{bmatrix} x & x & 0 \\ x & x & 0 \\ 0 & 0 & 0 \end{bmatrix}, \; \Delta_2' = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & x \end{bmatrix}. \tag{3.2}$$

The Higgs basis is uniquely defined in a 2HDM [13] since there is only one 2×2 orthogonal matrix satisfying $O_{1a} = \frac{v_a}{v}$, given by

$$O = \frac{1}{v} \begin{bmatrix} v_1 & v_2 \\ v_2 & -v_1 \end{bmatrix} \equiv \begin{bmatrix} c_\beta & s_\beta \\ s_\beta & -c_\beta \end{bmatrix}. \tag{3.3}$$

[1] Note that we selected a generation and one of the quark sectors when implementing this symmetry. This implies that there are six disctinct classes of BGL models that are not continuously linked to eachother. In this section, we will study tBGL models.

Thus, we can write the mass matrices in (2.57) for 2HDM as

$$D_d^0 = \frac{1}{\sqrt{2}}(v_1\Gamma_1 + v_2\Gamma_2), \quad N_d^0 = t_\beta D_d^0 - (t_\beta + t_\beta^{-1})\frac{1}{\sqrt{2}}v_2\Gamma_2,$$
$$D_u^0 = \frac{1}{\sqrt{2}}(v_1\Delta_1 + v_2\Delta_2), \quad N_u^0 = t_\beta D_u^0 - (t_\beta + t_\beta^{-1})\frac{1}{\sqrt{2}}v_2\Delta_2. \tag{3.4}$$

In the previous expression we identified $N_{x1}^0 \equiv D_x^0$ and $N_{x2}^0 \equiv N_x^0$. Inspecting the structure of the Yukawa couplings in (3.2) we obtain the following relations

$$P_3 D_d^0 = \frac{1}{\sqrt{2}}v_2\Gamma_2, \quad P_3 D_u^0 = \frac{1}{\sqrt{2}}v_2\Delta_2, \tag{3.5}$$

where we introduced the projection operators

$$(P_i)_{jk} = \delta_{ij}\delta_{ik}. \tag{3.6}$$

Using (3.5), we write the non-diagonal couplings in (3.4) as

$$(N_d)_{ij} = \left[t_\beta\delta_{ij} - (t_\beta + t_\beta^{-1})V_{ti}^* V_{tj}\right](m_d)_{jj}, \quad N_u = \mathrm{diag}\left\{t_\beta m_u, t_\beta m_c, -t_\beta^{-1} m_t\right\}. \tag{3.7}$$

In the derivation of this expression we used the block diagonal structure of D_u^0 suggested by (3.2) to write $(U_{uL})_{3i} = \delta_{3i}$ and $(U_{dL})_{3i} = V_{3i}$. As a consequence of these features, we observe in (3.7) that the reduction of free parameters in the Yukawa sector of BGL models is maximal (we count t_β as a parameter of the scalar potential since it is determined by it).

Assuming CP to be conserved in the scalar potential of BGL models, we use (2.54) to write the physical states that diagonalize it as follows

$$\begin{bmatrix} h \\ H \end{bmatrix} = \begin{bmatrix} -s_\alpha & c_\alpha \\ c_\alpha & s_\alpha \end{bmatrix}\begin{bmatrix} \rho_1 \\ \rho_2 \end{bmatrix} = \begin{bmatrix} -s_{\beta\alpha} & c_{\beta\alpha} \\ c_{\beta\alpha} & s_{\beta\alpha} \end{bmatrix}\begin{bmatrix} R_1 \\ R_2 \end{bmatrix}, \quad A = J_2, \tag{3.8}$$

where $c_{\beta\alpha} = \cos(\beta - \alpha)$. As such, the couplings in (2.64) are written as follows in any 2HDM with CP-conservation in its scalar potential

$$\tilde{N}_{q1} = s_{\beta\alpha}D_q - c_{\beta\alpha}N_q, \quad \tilde{N}_{q2} = c_{\beta\alpha}D_q + s_{\beta\alpha}N_q, \quad \tilde{N}'_{q2} = N_q. \tag{3.9}$$

Note that $\tilde{N}'_{q1} = D_q$ is not important since it mediates the interactions of the would-be Goldstone G^0 that does not contribute to tree-level results. In order to constrain tBGL models, we begin by replacing these results in (2.82) to find

$$M_{12}^P = -\frac{5m_P^3 f_P^2}{24v^2 m_h^2}c_{\beta\alpha}^2(t_\beta + t_\beta^{-1})^2(V_{ti}V_{tj}^*)^2 \tag{3.10}$$

where we neglected the smallest of the quark masses m_i in relation to the heaviest m_j. It is implicit in this expression that the mixing relates to the down sector, given that there are no tree-level FCNC in the up sector of a tBGL model (see (3.7)). Using the approximations of Sec. A.3, we conclude that tBGL models must satisfy the following experimental bounds

$$\begin{aligned}
c_{\beta\alpha}^2(t_\beta + t_\beta^{-1})^2 &< \frac{12v^2m_h^2}{5m_K^3 f_K^2}\frac{\Delta m_K}{|(V_{td}V_{ts}^*)^2|} \sim 20000,\\
c_{\beta\alpha}^2(t_\beta + t_\beta^{-1})^2 &< \frac{12v^2m_h^2}{5m_{B_d}^3 f_{B_d}^2}\frac{\Delta m_{B_d}}{|(V_{td}V_{tb}^*)^2|} \sim 3,\\
c_{\beta\alpha}^2(t_\beta + t_\beta^{-1})^2 &< \frac{12v^2m_h^2}{5m_{B_s}^3 f_{B_s}^2}\frac{\Delta m_{B_s}}{|(V_{ts}V_{tb}^*)^2|} \sim 4,\\
c_{\beta\alpha}^2(t_\beta + t_\beta^{-1})^2 &< \frac{24v^2m_h^2}{5m_K^3 f_K^2}\frac{\sqrt{2}\Delta m_K|\epsilon|}{|\mathrm{Im}\,[(V_{td}V_{ts}^*)^2]\,|} \sim 180.
\end{aligned} \tag{3.11}$$

While similar constraints can be obtained for the remaining five classes of BGL models [23, 38], we will refrain from doing so here.

In order to complete our analysis of (t)BGL models, we must verify our assumption regarding CP-conservation in their scalar potential. To do so, we begin by writing the most general scalar potential invariant under (3.1)

$$V(\phi) = \mu_{11}^2(\phi_1^\dagger\phi_1) + \mu_{22}^2(\phi_2^\dagger\phi_2) + \lambda_1(\phi_1^\dagger\phi_1)^2 + \lambda_2(\phi_2^\dagger\phi_2)^2 + \lambda_3(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) + \lambda_4(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1), \tag{3.12}$$

where by Hermiticity all parameters are real. Thus, (2.101) is trivially verified, ensuring CP-conservation in the scalar potential of all[2] BGL models. This fact justifies the use of (3.8). Notice that the reduction of parameters in the scalar potential of BGL models is also maximal. Despite four additional parameters, the scalar potential of BGL models is equal to its rephasing invariant sector[3], implying no AFS can remove further parameters.

3.2 Guiding Principle

It is clear that the success of BGL models in the maximal reduction of its free parameters was due to the symmetry applied in (3.1). In the rest of this chapter, we will attempt to find additional models with similar properties, namely with a reduction of the number of free parameters in the Yukawa sector. In order to find such models, we introduce a principle to guide our search:

> *"Each line of the mass matrix of a quark with a given charge should receive contributions from one and only one Higgs doublet."*

If this property is to be stable under renormalization, it must be implemented through a symmetry of the full Lagrangian that can at most be broken softly by a mass term in the scalar potential.

[2]Note that the scalar fields transform equally in every symmetry that generates a class of BGL models.

[3]This scalar potential possesses an accidental global symmetry that will lead to a physical Goldstone boson after SSb. Given that no massless boson has been detected [4], we must remove such symmetry from the potential. While several approaches have already been presented in the literature [23], we will not present them here.

Using the method of Sec. B, we have found that only five 2HDM satisfy this guiding principle: type I and type II 2HDMs [13, 21, 22], BGL models [23] and two new ones [46, 47] which we present in the rest of this chapter.

3.3 gBGL

In this section, we study gBGL models [46]. These are 2HDMs invariant under the following Z_2 symmetry,

$$(q_L^0)_3 \to -(q_L^0)_3,\ (q_L^0)_i \to (q_L^0)_i,\ d_R^0 \to d_R^0,\ u_R^0 \to u_R^0,\ \phi_1 \to \phi_1,\ \phi_2 \to -\phi_2. \tag{3.13}$$

After applying the symmetry of (3.13) to (2.70), we obtain the following structure for the Yukawa couplings of gBGL models in this WB,

$$\Gamma_1 = \begin{bmatrix} x & x & x \\ x & x & x \\ 0 & 0 & 0 \end{bmatrix},\ \Gamma_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ x & x & x \end{bmatrix},\ \Delta_1 = \begin{bmatrix} x & x & \delta_1 \\ x & x & \delta_2 \\ 0 & 0 & 0 \end{bmatrix},\ \Delta_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \tau_1 & \tau_2 & x \end{bmatrix}, \tag{3.14}$$

where x, δ_i and τ_i are arbitrary complex parameters. The δ_i and τ_i entries have been singled out to highlight the fact that gBGL models contain at least one of the BGL classes: (3.2) is obtained by setting $\delta_i = \tau_i = 0$ (tBGL models are a particular case of gBGL models). Since the Yukawa couplings of one sector are independent of the other, it is clear that there will be tree-level FCNC in both sectors of a gBGL model.

Note that (3.14) is WB dependent. As such, we look for a different approach to identify gBGL models. We start by using (3.14) to prove that the following relations hold in the WB in which the symmetry of gBGL models is given by (3.13)

$$P_3\Gamma_1 = P_3\Delta_1 = 0,\ \ P_3\Gamma_2 = \Gamma_2,\ \ P_3\Delta_2 = \Delta_2. \tag{3.15}$$

Guided by these WB dependent conditions, we write the following set of WBI conditions

$$\Gamma_2^\dagger\Gamma_1 = \Gamma_2^\dagger\Delta_1 = \Delta_2^\dagger\Delta_1 = \Delta_2^\dagger\Gamma_1 = 0,\ \ \Gamma_1 \neq 0,\ \ \Gamma_2 \neq 0. \tag{3.16}$$

It is straightforward to prove that these conditions are necessarily satisfied by a gBGL model by evaluating them in the WB in which (3.14) hold

$$\Delta_2^\dagger\Delta_1 = (P_3\Delta_2)^\dagger\Delta_1 = \Delta_2^\dagger(P_3\Delta_1) = 0,\ \ \det(D_d^0) \neq 0 \Leftrightarrow P_3 D_d^0 \neq 0 \Leftrightarrow P_3(v_1\Gamma_1 + v_2\Gamma_2) \neq 0 \Leftrightarrow \Gamma_2 \neq 0. \tag{3.17}$$

Identical arguments can be made for the remaining conditions. Proving sufficiency is a more intricate matter. We begin by using the polar decomposition of any complex matrix to write

$$\Gamma_i = W_{d_i} D_{d_i} U_{d_i}^\dagger,\ \ \Delta_i = W_{u_i} D_{u_i} U_{u_i}^\dagger, \tag{3.18}$$

where D_{x_i} are diagonal matrices with positive entries and W_{x_i} and U_{x_i} are unitary matrices. Using (3.16), we obtain

$$[\Delta_2\Delta_2^\dagger, \Delta_1\Delta_1^\dagger] = \Delta_2(\Delta_2^\dagger\Delta_1)\Delta_1^\dagger - \Delta_1(\Delta_2^\dagger\Delta_1)^\dagger\Delta_2^\dagger = 0, \tag{3.19}$$

implying that

$$W_{u_1} = W_{u_2} = W_u. \tag{3.20}$$

In an identical way, we obtain the relations

$$\begin{aligned} [\Gamma_2\Gamma_2^\dagger, \Gamma_1\Gamma_1^\dagger] = 0 \Rightarrow W_{d_1} = W_{d_2} = W_d, \\ [\Delta_2\Delta_2^\dagger, \Gamma_1\Gamma_1^\dagger] = 0 \Rightarrow W_d = W_u = W. \end{aligned} \tag{3.21}$$

Replacing these relations on (3.16) results in a system of equations for the diagonal matrices D_{x_i},

$$D_{d_2}D_{d_1} = D_{d_2}D_{u_1} = D_{u_2}D_{u_1} = 0. \tag{3.22}$$

Since $\Gamma_1 \neq 0$, $\Gamma_2 \neq 0$ and $\det(\Gamma_1 + \Gamma_2) \neq 0$, the first of these equations has only one solution[4]

$$D_{d_1} = \mathrm{diag}\{d_1, d_2, 0\}, \quad D_{d_2} = \mathrm{diag}\{0, 0, d_3\}. \tag{3.23}$$

It is now trivial to obtain the expression for the remaining diagonal matrices

$$D_{u_1} = \mathrm{diag}\{u_1, u_2, 0\}, \quad D_{u_2} = \mathrm{diag}\{0, 0, u_3\}. \tag{3.24}$$

To complete the proof, we must show that there is always a WB in which these couplings satisfy (3.14). In a generic WB, they are given by

$$\begin{aligned} \Gamma_1 = W D_{d_1} U_{d_1}^\dagger = W \begin{bmatrix} x & x & x \\ x & x & x \\ 0 & 0 & 0 \end{bmatrix}, \ \Gamma_2 = W D_{d_2} U_{d_2}^\dagger = W \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ x & x & x \end{bmatrix}, \\ \Delta_1 = W D_{u_1} U_{u_1}^\dagger = W \begin{bmatrix} x & x & x \\ x & x & x \\ 0 & 0 & 0 \end{bmatrix}, \ \Delta_2 = W D_{u_2} U_{u_2}^\dagger = W \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ x & x & x \end{bmatrix}. \end{aligned} \tag{3.25}$$

It is now straightforward to check that the Yukawa textures of (3.14) can be obtained by making the WBT $q_L \to W q_L$. As such, we have shown that the conditions in (3.16) are sufficient to ensure that a given texture corresponds to a gBGL model.

Finally, we point out that the choice of generation in (3.23) is not physical. Had we selected a different one, we could still generate the Yukawa textures of (3.14) by performing a WBT with the corresponding permutation matrix.

[4]Notice that the overall Higgs label is not physical, i.e., the models obtained from imposing $\psi_1 \to -\psi_1$ rather than $\psi_2 \to -\psi_2$ are identical. Only the relative Higgs label between different couplings has physical meaning.

We have seen in (3.4) how the couplings that control the FCNC in a general 2HDM are written. Using (3.15), we obtain their expression for gBGL models

$$N_d^0 = \left[t_\beta - (t_\beta + t_\beta^{-1})P_3\right] D_d^0, \;\; N_u^0 = \left[t_\beta - (t_\beta + t_\beta^{-1})P_3\right] D_u^0. \tag{3.26}$$

After diagonalizing the mass matrices D_x^0, we obtain the expressions

$$N_d = \left[t_\beta - (t_\beta + t_\beta^{-1})P_3^{dL}\right] D_d, \;\; N_u = \left[t_\beta - (t_\beta + t_\beta^{-1})P_3^{uL}\right] D_u, \tag{3.27}$$

where we introduced the projection operators

$$P_i^X = U_X^\dagger P_i U_X \Leftrightarrow (P_i^X)_{jk} = (U_X)_{ij}^*(U_X)_{ik}. \tag{3.28}$$

Despite (3.30) suggesting the need for two unitary vectors in order to parametrize the tree-level FCNC of a gBGL model, that is not the case. They are in fact related by the CKM matrix

$$P_i^{uL} V = V P_i^{dL}. \tag{3.29}$$

Thus, the couplings in (3.27) are parametrized by one unitary vector $n_i = (U_{uL})_{3i}$

$$\begin{aligned} (N_u)_{ij} &= \left[t_\beta \delta_{ij} - (t_\beta + t_\beta^{-1}) n_i^* n_j\right] (m_u)_{jj}, \\ (N_d)_{ij} &= \left[t_\beta \delta_{ij} - (t_\beta + t_\beta^{-1}) V_{mi}^* V_{nj} n_m^* n_n\right] (m_d)_{jj}. \end{aligned} \tag{3.30}$$

Since n only appears in the rephasing invariant combination $n_i^* n_h$, we can parametrize it as

$$n = \left(\cos\theta, e^{i\varphi_1}\sin\theta\cos\psi, e^{i\varphi_2}\sin\theta\sin\psi\right). \tag{3.31}$$

As such, the Yukawa sector of gBGL models is parametrized by two real angles and two complex phases. While the reduction is not maximal (as was the case for BGL models), it is still significant when compared to the 36 free parameters in the Yukawa sector of the general 2HDM.

Notice also that despite (3.14) suggesting that only two classes of BGL models are contained in gBGL models (tBGL and bBGL), this is not the case. All six classes are particular implementations of gBGL models, being implemented by the following n,

$$\begin{aligned} n_d = (V_{ud}^*, V_{cd}^*, V_{td}^*), \;\; n_s = (V_{us}^*, V_{cs}^*, V_{ts}^*), \;\; n_b = (V_{ub}^*, V_{cb}^*, V_{tb}^*), \\ n_u = (1,0,0), \;\; n_c = (0,1,0), \;\; n_t = (0,0,1). \end{aligned} \tag{3.32}$$

Imposing the scalar potential of (2.40) to be invariant under (3.13) forces it to be

$$V = \mu_{11}^2(\phi_1^\dagger\phi_1) + \mu_{22}^2(\phi_2^\dagger\phi_2) + \lambda_1(\phi_1^\dagger\phi_1)^2 + \lambda_2(\phi_2^\dagger\phi_2)^2 + \lambda_3(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) + \lambda_4(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1) + \lambda_5\left[e^{i\delta_5}(\phi_1^\dagger\phi_2)^2 + h.c.\right]. \tag{3.33}$$

Notice that unlike the scalar potential of BGL models ((3.12)), there is no accidental global symmetry in (3.33) (the term in λ_5 breaks the $U(1)$ of (3.12)). Thus, we do not have to worry about physical Goldstone bosons.

Regarding CP violation, (2.101) requires that

$$\lambda_5^* = \lambda_5 e^{4i(\alpha_2-\alpha_1)} \tag{3.34}$$

must hold if CP is to be conserved in the scalar potential of a gBGL model. Meanwhile, the phases α_i are determined by minimizing the scalar potential

$$\frac{\partial \langle 0|V|0\rangle}{\partial \alpha_1} = 0 \Leftrightarrow \lambda_5^* = \lambda_5 e^{4i(\alpha_2-\alpha_1)}. \tag{3.35}$$

Combining both results, we conclude that CP is conserved in the scalar potential of gBGL models. Thus, the relation between the physical massive scalars and the Higgs basis is still described by (3.8).

Note that the tree-level FCNC of gBGL models given by (3.30) are equivalent to those of BGL models (see (3.4)) provided we replace V_{ti} by n_i. Thus, it is straightforward to obtain the contribution from the neutral scalar to M_{12}^P in gBGL models from (3.10) to be[5]

$$M_{12}^P = -\frac{5m_P^3 f_P^2}{24v^2 m_h^2} c_{\beta\alpha}^2 (t_\beta + t_\beta^{-1})^2 (n_i n_j^*)^2. \tag{3.36}$$

From this expression, we read the following constraint on gBGL models

$$c_{\beta\alpha}^2 (t_\beta + t_\beta^{-1})^2 |n_i n_j^*|^2 < \frac{12v^2 m_h^2 \Delta m_P}{5m_P^3 f_P^2}. \tag{3.37}$$

Out of all available meson states[6], the most stringent of these constraints arises from the $D^0 - \bar{D}^0$ mixing. From the maximum value[7] $|n_1 n_2^*| = \frac{1}{2}$, we conclude that gBGL models are in agreement with experimental results provided

$$|c_{\beta\alpha}(t_\beta + t_\beta^{-1})| < 0.02\ . \tag{3.38}$$

The top and bottom classes of BGL models are the only renormalizable MHDM that verify the MFV in all of the different versions available in the literature [13, 24]. In this subsection, we analyse the properties of gBGL models that are near the tBGL model and the bBGL model.

Starting by the tBGL models, we note that their tree-level FCNC are controlled by

$$(N_d)_{ij} \propto \left(V^T t\right)_i^* \left(V^T t\right)_j m_{d_j}, \quad (N_u)_{ij} \propto t_i^* t_j m_{u_j}, \tag{3.39}$$

where we introduced

$$t = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T \Rightarrow V^T t = \begin{bmatrix} V_{td} & V_{ts} & V_{tb} \end{bmatrix}^T. \tag{3.40}$$

[5]In this expression we assume meson mixing in the up sector. For describing mixing in the down sector we should replace n_j by $n_k V_{kj}$.

[6]Which are K^0, D^0, B_d^0 and B_s^0 since there are tree-level FCNC in both sectors of a gBGL model.

[7]The constraint in (3.38) is relaxed in all regions of the parameter space in which this equality does not hold.

Consider small deviations from tBGL models parametrized by an unitary vector $\tilde{t}$ such that

$$V^T\tilde{t} = \begin{bmatrix} V_{td}(1+\delta_d) & V_{ts}(1+\delta_s) & V_{tb}(1+\delta_b) \end{bmatrix}^T, \tag{3.41}$$

where the complex vector $\vec{\delta} = (\delta_d, \delta_s, \delta_b)$ is assumed to be small ($|\vec{\delta}| \ll 1$). From (3.36) we obtain the contributions to neutral meson mixing in this model

$$M_{12}^K \propto (V_{td}^* V_{ts})^2[1 + 2(\delta_2 + \delta_d^*)] + \sigma(\delta^2), \;\; M_{12}^{B_d} \propto (V_{td}^* V_{tb})^2[1 + 2(\delta_b + \delta_d^*)] + \sigma(\delta^2). \tag{3.42}$$

From these relations we conclude that the near-top gBGL models provide the same contributions to neutral meson mixing as the tBGL models provided $\vec{\delta}$ is such that [46]

$$\mathrm{Re}\,[\delta_d] \sim \mathrm{Re}\,[\delta_s] \sim \mathrm{Im}\,[\delta_s] \leq \mathcal{O}(\lambda^2), \;\; \mathrm{Im}\,[\delta_d] \sim \mathrm{Im}\,[\delta_b] \leq \mathcal{O}(\lambda^3), \tag{3.43}$$

where λ is the Wolfenstein parameter [48]. We must still determine if these models (now well defined by (3.43)) produce tree-level FCNC in the up sector that are incompatible with $D^0 - \bar{D}^0$ mixing. Such couplings are controlled in near-top gBGL models by

$$\tilde{t} = V^* \left(V^T \tilde{t}\right) \approx \begin{bmatrix} V_{tb} V_{ub}^* \delta_b & V_{cb}^* V_{tb}\, \delta_b & 1 + \delta_b \end{bmatrix}^T, \tag{3.44}$$

implying that M_{12}^D is suppressed by a factor of

$$M_{12}^D \propto |V_{tb}|^4 (V_{cb}^* V_{ub})^2 |\delta_b|^4 \sim \lambda^{18}, \tag{3.45}$$

which is much smaller than the contributions which arise in any of the three dBGL models [38]. Thus, we conclude that there is a continuous the region of parameter space of gBGL models linked to tBGL for which the strength of neutral meson mixing is essentially constant.

Regarding bBGL models, we note that their tree-level FCNC are controlled by

$$(N_d)_{ij} \propto b_i^* b_j m_{d_j}, \;\; (N_u)_{ij} \propto (V^* b)_i \,(V^* b)_j \, m_{u_j}, \tag{3.46}$$

where we introduced

$$b = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T, \;\; V^* b = \begin{bmatrix} V_{ub}^* & V_{cb}^* & V_{tb}^* \end{bmatrix}^T. \tag{3.47}$$

Consider small deviations from bBGL models parametrized by an unitary vector $\tilde{b}$ such that

$$V^* \tilde{b} = \begin{bmatrix} V_{ub}^*(1+\delta_u) & V_{cb}^*(1+\delta_c) & V_{tb}^*(1+\delta_t) \end{bmatrix}^T, \tag{3.48}$$

where $\vec{\delta} = (\delta_u, \delta_c, \delta_t)$ is a complex vector assumed small ($|\vec{\delta}| \approx \lambda^2$). The contributions to $D^0 - \bar{D}^0$ mixing in the near-bottom gBGL models are controlled by

$$M_{12}^D \propto (V_{cb}^* V_{ub})^2 [1 + 2(\delta_c + \delta_u^*)] + \sigma(\delta^2) \sim \lambda^{10}. \tag{3.49}$$

This results suggests no significant enhancement to the bBGL prediction. Regarding the tree-level FCNC that arise in the down sector of such models, we note that they are controlled by[8]

$$\tilde{b} = V^T \left(V^* \tilde{b}\right) \approx \left[V_{tb}^* V_{td}\delta_t \quad V_{tb}^* V_{ts}\delta_t \quad 1+\delta_t\right]^T. \tag{3.50}$$

Using this expression, we obtain suppressions for M_{12}^{K,B_d} of the order of

$$M_{12}^K \propto |V_{tb}|^2 (V_{td}^* V_{ts})^2 |\delta_t|^4 \sim \lambda^{18}, \;\; M_{12}^{B_d} \propto (V_{td}^* V_{tb}\delta_t)^2 \sim \lambda^8, \tag{3.51}$$

which are smaller than any of the contributions that arise in any of the three uBGL models [38]. Thus, the situation regarding near-bottom gBGL models mimics our findings for near-top gBGL models.

The existence of additional sources of flavour and CP violation in the gBGL models has the potential to enhance the contributions to the BAU with respect to SM expectations [9]. In particular, a WBI of dimension four with a non-zero imaginary part arises [46, 49, 50]

$$\mathrm{Im}\left[\mathrm{Tr}\left[N_u^0 D_u^{0\dagger} D_d^0 D_d^{0\dagger}\right]\right] = \mathrm{Im}\left[\mathrm{Tr}\left[N_u D_u V D_d^2 V^\dagger\right]\right]. \tag{3.52}$$

Using (3.30), we develope this invariant as

$$\mathrm{Im}\left[\mathrm{Tr}\left[N_u D_u V D_d^2 V^\dagger\right]\right] = -(t_\beta + t_\beta^{-1})\mathrm{Im}\left[\mathrm{Tr}\left[P_3^{uL} D_u^2 V D_d^2 V^\dagger\right]\right] = \frac{i}{2}(t_\beta + t_\beta^{-1}) n_i^* n_j V_{ik}^* V_{jk} m_{d_k}^2 (m_{u_j}^2 - m_{u_i}^2). \tag{3.53}$$

In order to estimate the enhancement to the BAU generated by this WBI, we retain only the leading contributions in terms of the quark masses and powers of the Wolfenstein parameter λ [48],

$$\mathrm{Im}\left[\mathrm{Tr}\left[N_u D_u V D_d^2 V^\dagger\right]\right] \propto m_c^2 n_1^* n_2 (m_s^2 \lambda + m_b^2 \lambda^5) + m_t^2 n_1^* n_3 (m_s^2 \lambda^3 + m_b^2 \lambda^3) + m_t^2 n_2^* n_3 (m_s^2 \lambda^2 + m_b^2 \lambda^2). \tag{3.54}$$

Thus, the leading contribution to the BAU in a gBGL model is given by

$$BAU_{gBGL} \sim (t_\beta + t_\beta^{-1}) \frac{m_t^2 m_b^2}{E^4} \mathrm{Im}\left[n_3^* n_2 V_{tb}^* V_{cb}\right], \tag{3.55}$$

where $E \sim 100\ GeV$ is an energy of the order of the electroweak scale [8, 10]. Meanwhile, we use (2.93) to obtain the prediction for the BAU in the SM

$$BAU_{SM} \sim \frac{m_t^4 m_b^4 m_c^2 m_s^2}{E^{12}} \mathrm{Im}\left[Q_{ijkm}\right]. \tag{3.56}$$

As such, we conclude that gBGL models have room for an enhancement of the BAU in comparison to the SM result of the order of

$$\frac{BAU_{gBGL}}{BAU_{SM}} \sim 10^{16} (t_\beta + t_\beta^{-1}) |n_2^* n_3| \sin\alpha, \tag{3.57}$$

[8]We only present the terms required to indicate the order of magnitude of each entry.

where $\alpha = \arg(n_3^* n_2 V_{tb}^* V_{cb})$. However, note that this WBI is null for BGL models

$$\mathrm{Im}\left[\mathrm{Tr}\left[N_u^0 D_u^{0\dagger} D_d^0 D_d^{0\dagger}\right]\right] = (t_\beta + t_\beta^{-1}) m_t^2 m_b^2 \mathrm{Im}\left[|V_{tb}|^2 |V_{cb}|^2\right] = 0, \tag{3.58}$$

implying that we must look for higher dimension WBI with a non-zero contribution to the BAU in BGL models. The smallest of such invariants is of dimension eight in mass [50], being given by

$$\mathrm{Im}\left[\mathrm{Tr}\left[D_u^0 N_u^{0\dagger} D_u^0 D_u^{0\dagger} D_d^0 D_d^{0\dagger} D_u^0 D_u^{0\dagger}\right]\right] = -(t_\beta + t_\beta^{-1})(m_t^2 - m_c^2)(m_t^2 - m_u^2)(m_c^2 - m_u^2)(m_s^2 - m_d^2) \mathrm{Im}\left[V_{cs}^* V_{ts} V_{tb}^* V_{cb}\right]. \tag{3.59}$$

Thus, there is no significant enhancement of the BAU in tBGL models in respect to the SM

$$\frac{BAU_{bBGL}}{BAU_{SM}} = (t_\beta + t_\beta^{-1}) \frac{E^4}{m_t^4} \sim 1. \tag{3.60}$$

Meanwhile, the near-top gBGL models defined by (3.41) produce an enhancement of

$$\frac{BAU_{nearb}}{BAU_{SM}} \sim 10^{16} (t_\beta + t_\beta^{-1}) |V_{tb}^* V_{ts}| \mathrm{Im}\left[\delta_b + \delta_s^*\right] \sim 10^{12}. \tag{3.61}$$

As such, we conclude that the near-top (and near-bottom) gBGL models have FCNC with comparable intensity to the tBGL (and bBGL) models, but significantly enhanced CP violation.

3.4 jBGL

We present jBGL models [47] as a ”flipped” alternative to gBGL models. They are obtained through the implementation of the following AFS

$$\phi_2 \to \Upsilon \phi_2, \;\; q_{L3}^0 \to \Upsilon^{-1} q_{L3}^0, \;\; d_R^0 \to \Upsilon^{-1} d_R^0, \tag{3.62}$$

where $|\Upsilon| = 1$. Replacing (3.62) in (2.70) we obtain the structure of the Yukawa couplings in this WB to be

$$\Gamma_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ x & x & x \end{bmatrix}, \;\; \Gamma_2 = \begin{bmatrix} x & x & x \\ x & x & x \\ 0 & 0 & 0 \end{bmatrix}, \;\; \Delta_1 = \begin{bmatrix} x & x & x \\ x & x & x \\ 0 & 0 & 0 \end{bmatrix}, \;\; \Delta_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ x & x & x \end{bmatrix}. \tag{3.63}$$

It is straightforward to derive from these the following WB dependent relations

$$P_3 \Gamma_1 = \Gamma_1, \;\; P_3 \Delta_2 = \Delta_2, \;\; P_3 \Gamma_2 = P_3 \Delta_1 = 0. \tag{3.64}$$

As an alternative to the WB dependent textures of (3.63), we introduce the WBI conditions

$$\Gamma_2^\dagger \Gamma_1 = \Delta_2^\dagger \Delta_1 = \Gamma_2^\dagger \Delta_2 = \Delta_1^\dagger \Gamma_1 = 0, \;\; \Gamma_1 \neq 0, \;\; \Gamma_2 \neq 0, \tag{3.65}$$

that are necessary and sufficient to ensure that a given texture belongs to a jBGL model. Given that the demonstration behind such claim is identical to the one in Sec. **??**, we will not repeat it here.

We have seen in (3.4) the couplings which control the tree-level FCNC in any 2HDM. Using the relations of (3.64) we are able to write the couplings of jBGL models in the following way

$$N_d^0 = \left[-t_\beta^{-1} + (t_\beta + t_\beta^{-1})P_3\right] D_d^0, \;\; N_u^0 = \left[t_\beta - (t_\beta + t_\beta^{-1})P_3\right] D_u^0. \tag{3.66}$$

After diagonalizing the mass matrices, these couplings become

$$N_d = \left[-t_\beta^{-1} + (t_\beta + t_\beta^{-1})P_3^{dL}\right] D_d, \;\; N_u = \left[t_\beta - (t_\beta + t_\beta^{-1})P_3^{uL}\right] D_u. \tag{3.67}$$

Remembering (3.29), we describe the tree-level FCNC in both sectors of jBGL models as

$$\begin{aligned} (N_u)_{ij} &= \left[t_\beta \delta_{ij} - (t_\beta + t_\beta^{-1}) n_i^* n_j\right] (m_u)_{jj}, \\ (N_d)_{ij} &= \left[-t_\beta^{-1} \delta_{ij} + (t_\beta + t_\beta^{-1}) V_{mi}^* V_{nj} n_m^* n_n\right] (m_d)_{jj}, \end{aligned} \tag{3.68}$$

where the unitary vector $n = (U_{uL})_{3i}$ may be parametrized by (3.31). As such, jBGL models are completely determined by two real angles and two complex phases (as were gBGL models). While there is always a choice of n such that one of the sectors of jBGL models is identical to gBGL models, both sectors cannot be set equal simultaneously. Thus, jBGL models are a distinct class of 2HDMs than gBGL models.

Finally, we can read from (3.68) that none of the six classes of BGL models is also a jBGL model (as was expected since they are gBGL models).

The most general scalar potential that is invariant under (3.62) is given by

$$V(\phi) = \mu_{11}^2(\phi_1^\dagger\phi_1) + \mu_{22}^2(\phi_2^\dagger\phi_2) + \mu_{12}^2\left[e^{i\delta}(\phi_1^\dagger\phi_2) + h.c.\right] + \lambda_1(\phi_1^\dagger\phi_1)^2 + \lambda_2(\phi_2^\dagger\phi_2)^2 + \lambda_3(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) + \lambda_4(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1), \tag{3.69}$$

where all parameters are real due to Hermiticity. We have softly broken the symmetry through the addition of the μ_{12}^2 term in order to avoid a physical Goldstone boson after SSB.

Using (2.101), we conclude that the following condition must hold if the potential is to conserve CP

$$\mu_{12}^2 = \mu_{12}^2 e^{2i(\delta + \alpha_2 - \alpha_1)}, \tag{3.70}$$

where the phases α_i are determined through the minimization of the scalar potential

$$\frac{\partial \langle 0|V|0\rangle}{\partial \alpha_1} = 0 \Leftrightarrow \mu_{12}^2 = \mu_{12}^2 e^{2i(\delta + \alpha_2 - \alpha_1)}. \tag{3.71}$$

As such, we conclude that the scalar potential of a softly broken jBGL model is CP invariant, implying that (3.8) may be used for these models.

As stated before, gBGL and jBGL models are indistinguishable from each other when considering only one sector of the theory. Given that the constraints applied to gBGL models involved only their up sector, (3.38) must also be satisfied by jBGL models, i.e.,

$$|c_{\beta\alpha}(t_\beta + t_\beta^{-1})| < 0.02 \; . \tag{3.72}$$

The interesting question relates the distinction of jBGL and gBGL models. It is clear that such goal can only be achieved by relating observables of both sectors of the theory. In order to understand them, we write the sector of the Lagrangian which controls the diagonal couplings between the (125) scalar h and the physical fermions[9]

$$\begin{aligned} -\mathcal{L}^h_{yuk} = & \frac{h}{v}(m_u)_{ii}\bar{u}_i \left[s_{\beta\alpha} - c_{\beta\alpha} \left\{ t_\beta - (t_\beta + t_\beta^{-1})|n_i|^2 \right\} \right] u_i \\ & + \frac{h}{v}(m_d)_{ii}\bar{d}_i \left[s_{\beta\alpha} + c_{\beta\alpha} \left\{ t_\beta^{-1} - (t_\beta + t_\beta^{-1}) V^*_{mi} V_{ni} n^*_m n_n \right\} \right] d_i. \end{aligned} \tag{3.73}$$

Using this Lagrangian, we compute the ratio between the Branching ratios $h \to \bar{c}c$ and $h \to \bar{s}s$ in jBGL models

$$\left.\frac{\mathrm{BR}(h \to \bar{c}c)}{\mathrm{BR}(h \to \bar{s}s)}\right|_{jBGL} \approx \left(\frac{m_c}{m_s}\right)^2 \left\{ \frac{s_{\beta\alpha} - c_{\beta\alpha}\left[t_\beta - (t_\beta + t_\beta^{-1})|n_2|^2\right]}{s_{\beta\alpha} + c_{\beta\alpha}\left[t_\beta^{-1} - (t_\beta + t_\beta^{-1})|n_2|^2\right]} \right\}^2 , \tag{3.74}$$

where we neglected m_c and m_s with respect to $m - H$ and considered $V_{jb} \approx \delta_{j3}$. We should also note that this ratio differs from the tree-level SM prediction of $\left(\frac{m_c}{m_s}\right)^2$, implying that this channel may be used in future collider experiments to distinguish jBGL models from the SM.

Using the same approximations, we compute the same ratio in gBGL models to be

$$\left.\frac{\mathrm{BR}(h \to \bar{c}c)}{\mathrm{BR}(h \to \bar{s}s)}\right|_{gBGL} \approx \left(\frac{m_c}{m_s}\right)^2 . \tag{3.75}$$

As such, we have discovered two observables whose measurement allows for the distinction between jBGL and gBGL models. It is also important to mention that this ratio does not allow for the distinction between gBGL models and the SM.

Given the similar structures of gBGL and jBGL models, the lowest order WBI of a jBGL model with a non-zero imaginary part is given by

$$\mathrm{Im}\left[\mathrm{Tr}\left[N_u^0 D_u^{0\dagger} D_d^0 D_d^{0\dagger}\right]\right] . \tag{3.76}$$

After using (3.68) and repeating the procedure applied in Sec. **??**, we obtain the leading contribution to the BAU in jBGL models

[9] Note that we can obtain the gBGL result from this expression by replacing $t_\beta \to -t_\beta^{-1}$ in the down sector of the theory.

$$BAU_{jBGL} \approx (t_\beta + t_\beta^{-1}) \frac{m_t^2 m_b^2}{E^4} \mathrm{Im}\left[n_3^* n_2 V_{tb}^* V_{cb}\right]. \tag{3.77}$$

As such, the enhancement of the BAU in comparison to the SM predicitons is similar in jBGL models as in gBGL models[10]

$$\frac{BAU_{jBGL}}{BAU_{SM}} \approx 10^{16} (t_\beta + t_\beta^{-1}) |n_3 n_2^*| \sin\alpha. \tag{3.78}$$

[10] This was expected since the contributions to the BAU in gBGL models involved only one sector of the theory.

Chapter 4

Controlling FCNC

In Sec. 2.4.4, we have found that the most stringent constraints which must be satisfied by a MHDM to be in agreement with experimental results are

$$\left|\chi_D^+(A_{u1}^+)_{cu}^2 - \chi_D^-(A_{u1}^-)_{cu}^2\right| < 2\times 10^{-3}\ \mathrm{GeV}^2, \quad \left|\chi_K(A_{d1}^+)_{sd}^2 - \chi_K(A_{d1}^-)_{sd}^2\right| < 7\times 10^{-3}\ \mathrm{GeV}^2. \tag{4.1}$$

The right-hand side of these relations can be shown through the use of the Cauchy-Schwarz and triangle inequalities to be smaller than

$$\left|\chi^+(A_{q1}^+)_{ji}^2 - \chi^-(A_{q1}^-)_{ji}^2\right| \leq (\chi^+ + \chi^-)\sum_b \Big[|(N_{qb})_{ij}|^2 + |(N_{qb})_{ji}|^2\Big] + 2(\chi^+ - \chi^-)\sum_b \mathrm{Re}\Big[(N_{qb})_{ij}(N_{qb})_{ji}\Big]. \tag{4.2}$$

Assuming Yukawa couplings of the order one, we write from (2.57)

$$\sum_b \Big[|(N_{qb})_{ij}|^2 + |(N_{qb})_{ji}|^2\Big] \sim m_t^2\, c_1, \;\; 2\sum_b \mathrm{Re}\Big[(N_{qb})_{ij}(N_{qb})_{ji}\Big] \sim m_t^2\, c_2, \tag{4.3}$$

with c_i is a function of couplings of order one since $\frac{1}{\sqrt{2}}v \approx m_t$. As such, we conclude that the natural MHDM is in agreement with experimental results provided

$$1.1c_1^u - c_2^u < 3\times 10^{-9}, \;\; 1.2c_1^d - c_2^d < 9\times 10^{-10}. \tag{4.4}$$

Thus, we conclude that a cancellation of the order of 10^{-9} is required in order for the natural MHDM to be in agreement with experiment. In the rest of this chapter, we will construct a subset of MHDMs with a Yukawa sector that naturally reduces the required cancellation.

4.1 Unitary Flavour Violation

A model is said to have Unitary Flavour Violation (UFV) if its couplings satisfy

$$N_{da}^0 = L_d^a D_d^0 R_d^a, \;\; N_{ua}^0 = L_u^a D_u^0 R_u^a, \tag{4.5}$$

where the matrices L_q^a and R_q^a must be determined by the scalar potential of the theory, but are otherwise arbitrary[1]. Furthermore, the structure of (4.5) must be protected by an AFS [36, 37].

We will now focus on the down sector of the theory[2]. Using (2.57) on the UFV condition of (4.5) implies that the Yukawa couplings of a model with UFV satisfy

$$O_{ab}(\Gamma_b)_{ij} - O_{1b}(L_d^a)_{im}(\Gamma_b)_{mn}(R_d^a)_{nj} = 0, \tag{4.6}$$

where we made the fermionic flavour indices explicit and implicitly sum over them. Since in a MHDM invariant under an AFS the Yukawa couplings are arbitrary, it is clear that they cannot be determined from its scalar potential. Therefore, L_d^a and R_d^a cannot be a function of $(\Gamma_a)_{ij}$. Thus, there cannot be any cancellations between different Γ_a in (4.6), which implies

$$[O_{ab} - O_{1b}(L_d^a)_{ii}(R_d^a)_{jj}](\Gamma_b)_{ij} - O_{1b}\sum_{m\neq n}(L_d^a)_{im}(\Gamma_b)_{mn}(R_d^a)_{nj} = 0, \tag{4.7}$$

where no sum is implied. A similar argument implies that there cannot be cancellations between diferent entries of the same couplings. As such, (4.7) implies that L_d^a and R_d^a must be given by

$$(L_d^a)_{ij} = (l_d^a)_k(P_k)_{ij} \;\wedge\; (R_d^a)_{ij} = (r_d^a)_k(P_k)_{ij}. \tag{4.8}$$

Thus, L_d^a and R_d^a are diagonal matrices determined by

$$(\Gamma_b)_{ij} = 0 \;\vee\; (l_d^a)_i(r_d^a)_j = \frac{O_{ab}}{O_{1b}}. \tag{4.9}$$

Since the up sector of MHDMs with UFV is controlled by an identical relation and only the product $(l_x^a)_i(r_x^a)_i$ is determined by the theory, we conclude that MHDMs with UFV are identified by $10(N-1)$ parameters[3] distributed by $4(N-1)$ vectors, $\vec{l}_q^a$ and $\vec{r}_q^a$. Given that O is a real orthogonal matrix, we always have the possibility to choose $\vec{l}_q^a$ and $\vec{r}_q^a$ real as well.

Consider two couplings Γ_{b_1} and Γ_{b_2} of a model with UFV satisfying $(\Gamma_{b_1})_{ij}(\Gamma_{b_2})_{ij} \neq 0$. Then, (4.9) implies the following relation

$$\frac{O_{ab_1}}{O_{1b_1}} = \frac{O_{ab_2}}{O_{1b_2}} \Leftrightarrow \frac{O_{ab_1}}{O_{ab_2}} = \frac{O_{1b_1}}{O_{1b_2}}. \tag{4.10}$$

Since in a model with UFV this condition must hold for al a, it implies that the columns b_1 and b_2 of O are proportional to each other. Given that there is no orthogonal matrix with that property, the Yukawa couplings of any MHDM with UFV must obey the relation

$$(\Gamma_a)_{ij}(\Gamma_b)_{ij} = 0, \;\forall_{a\neq b}. \tag{4.11}$$

[1]Notice that all MHDMs would have UFV if we had not required L_q^a and R_q^a to be determined by the scalar potential of the theory since $L_q^a = N_{qa}^0$ and $R_q^a = (D_q^0)^{-1}$ would always be allowed.

[2]The analysis is translated directly to the up sector.

[3]We can aways set $(l_x^a)_1 = 1$ and clearly $\vec{l}_x^1 = \vec{r}_x^1 = (1, 1, 1)$.

Since the Yukawa couplings are determined by an AFS, we conclude from this expression that all scalars that couple to fermions must transform differently under the AFS which protects the MHDM with UFV. Note that while necessary, (4.11) is not sufficient to ensure UFV given that a maximum of nine independent equations remain to solve in (4.9) for five parameters.

Before proceeding, we must prove that UFV is a physical property of a given model. Such question arises since we are yet to define UFV in a WB independent way for MHDM with $N > 2$ ((4.9) is written in the Higgs basis, which is not uniquely defined in MHDMs with $N > 2$). Given that Higgs basis are related by $O' = XO$ with X an orthogonal matrix satisfying $X_{1b} = \delta_{1b}$, we must show that UFV is independent of X. Computing

$$(l_q'^a)_i (r_q'^a)_j = \frac{O'_{ab}}{O'_{1b}} = X_{ac}\frac{O_{cb}}{O_{1b}} = X_{ac}(l_q^c)_i (r_q^c)_j \tag{4.12}$$

implies that if a MHDM has UFV in one Higgs basis, it will also have it in all Higgs bases. Thus, we have proven UFV to be a physical property of MHDMs.

Finally, we write the non-diagonal couplings of a MHDM with UFV as

$$(N^0_{qa})_{ij} = (l_q^a)_i (r_q^a)_j (D^0_q)_{ij}. \tag{4.13}$$

4.1.1 Experimental Constraints

In this subsection, we compute the experimental constraints in (4.1) for MHDMs with UFV. To do so, we begin by writing their non-diagonal couplings (see (4.13)) as

$$N_{qa} = (l_q^a)_m (r_q^a)_n P_m^{qL} D_q P_n^{qR} \Rightarrow (N_{qa})_{ij} = (l_q^a)_m (r_q^a)_n (m_q)_{kk} (U_{qL})^*_{mi} (U_{qL})_{mk} (U_{qR})^*_{nk} (U_{qR})_{nj}. \tag{4.14}$$

The term UFV is justified by this expression since the intensity of FCNC in these models is controlled by unitary matrices U_{qX}. This implies that, unlike the FCNC of a general MHDM, the FCNC of a MHDM with UFV cannot be arbitrary large. Nevertheless, this does not imply that the constraints in (4.1) are relaxed for MHDM with UFV in comparison to the natural MHDM. Since $P_i^{\Lambda X} = P_i^X$ with $\Lambda = \mathrm{diag}(e^{i\theta_1}, e^{i\theta_2}, e^{i\theta_3})$ and $P_i^{dL} = V^\dagger P_i^{uL} V$, there are only a maximum of eighteen new parameters in the Yukawa sector of a MHDM with UFV.

Using the triangle and the Cauchy-Schwarz inequalities on (4.1) results in

$$\left|\chi^+ (A^+_{q1})^2_{ji} - \chi^- (A^-_{q1})^2_{ji}\right| \le c^2_{\beta\alpha} \left\{ \chi^+ \left[\sum_b \left| (N_{qb})_{ji} + (N^\dagger_{qb})_{ji} \right|^2 \right] + \chi^- \left[\sum_b \left| (N_{qb})_{ji} - (N^\dagger_{qb})_{ji} \right|^2 \right] \right\}, \tag{4.15}$$

where we used $\sum_b R^2_{1b} = c^2_{\beta\alpha}$. In order to progress, we must understand how to control the quantities

$$\left| (N_{qa})_{ji} \pm (N^\dagger_{qa})_{ji} \right|^2 \tag{4.16}$$

in the context of MHDMs with UFV. Replacing (4.14) on this relation and subsequently applying the

Cauchy-Schwarz inequality produces

$$\begin{aligned}\left|(N_{qa})_{ji} \pm (N_{qa}^{\dagger})_{ji}\right|^2 = \left|(l_q^a)_m (r_q^a)_n (m_q)_{kk}\, g_{ijkmn}\right|^2 \leq m_{q3}^2 \left[\sum_{k=1}^{3} \left|(l_q^a)_m (r_q^a)_n g_{ijkmn}\right|^2\right] \\ \leq m_{q3}^2 \left[\sum_{m=1}^{3} (l_q^a)_m^2\right] \left[\sum_{n=1}^{3} (r_q^a)_n^2\right] \left[\sum_{k,m,n=1}^{3} |g_{ijkmn}|^2\right].\end{aligned} \tag{4.17}$$

where m_{q3} is the heaviest mass of the q sector (m_t for the up sector and m_b for the down sector) and we introduced

$$g_{ijkmn} = (U_{qL})_{mj}^* (U_{qL})_{mk} (U_{qR})_{nk}^* (U_{qR})_{ni} \pm (U_{qL})_{mi} (U_{qL})_{mk}^* (U_{qR})_{nk} (U_{qR})_{nj}^*. \tag{4.18}$$

Using the result $\sum_{k,m,n=1}^{3} |g_{ij}|^2 \leq 20$ of Sec. C, we obtain the inequality

$$c_{\beta\alpha}^2 \left[\sum_{a=2}^{N} \sum_{m=1}^{3} (l_q^a)_m^2\right] \left[\sum_{a=2}^{N} \sum_{n=1}^{3} (r_q^a)_n^2\right] \leq \frac{3v^2 m_h^2 (m_i + m_j)^2 \Delta m_P}{5 m_{q3}^2 m_P f_P^2 [6 m_P^2 - (m_i + m_j)^2]}. \tag{4.19}$$

These relations should be read in the following way: any MHDM with UFV that satisfies (4.19) is in agreement in experiment. However, a MHDM with UFV that does not satisfy (4.19) may still be in agreement with the experimental results (this is a consequence of the use of the Cauchy-Schwarz and triangle inequalities in the derivation of (4.19)). As was the case for the general MHDM, it turns out that the most stringent constraints of this type arise from the $D^0 - \bar{D}^0$ mixing (up sector) and the $K^0 - \bar{K}^0$ mixing (down sector)

$$c_{\beta\alpha}^2 \left[\sum_{a=2}^{N} \sum_{m=1}^{3} (l_u^a)_m^2\right] \left[\sum_{a=2}^{N} \sum_{n=1}^{3} (r_u^a)_n^2\right] \leq 10^{-10}, \quad c_{\beta\alpha}^2 \left[\sum_{a=2}^{N} \sum_{m=1}^{3} (l_d^a)_m^2\right] \left[\sum_{a=2}^{N} \sum_{n=1}^{3} (r_d^a)_n^2\right] \leq 6 \times 10^{-8}. \tag{4.20}$$

Before comparing these results to (4.4), we must point out that they are multiplied by a factor of 20 since $\sum_{k,m,n=1}^{3} |g_{ijkmn}|^2 \leq 20$. However, it is more natural for such factor to be of the order of one, i.e., in most MHDMs with UFV these bounds are indeed relaxed by this factor of 20.

After getting rid of such factor, we conclude that the constraint applied on the up sector of MHDMs with UFV is not relaxed in comparison to the ”natural” MHDMs. This was to be expected, since N_{ua} is expected to be of the order of m_t in MHDMs with UFV. Thus, the success of UFV is only seen on the down sector of the theory, where the constraints are relaxed by a factor of $\frac{m_t^2}{m_b^2} \sim 10^3 - 10^4$ given that N_{da} is controlled by m_b in MHDMs with UFV.

4.1.2 2HDMs

Regarding 2HDMs with UFV, we use (3.4) and (4.11) to prove the relations

$$\begin{aligned}(\Gamma_1)_{ij} \neq 0 \Rightarrow (D_d^0)_{ij} = \frac{1}{\sqrt{2}} v_1 (\Gamma_1)_{ij} \ \wedge \ (N_d^0)_{ij} = \frac{1}{\sqrt{2}} v_2 (\Gamma_1)_{ij} = t_\beta (D_d^0)_{ij}, \\ (\Gamma_2)_{ij} \neq 0 \Rightarrow (D_d^0)_{ij} = \frac{1}{\sqrt{2}} v_2 (\Gamma_2)_{ij} \ \wedge \ (N_d^0)_{ij} = -\frac{1}{\sqrt{2}} v_1 (\Gamma_2)_{ij} = -t_\beta^{-1} (D_d^0)_{ij},\end{aligned} \tag{4.21}$$

As such, the vectors $\vec{r}_q$ and $\vec{l}_q$ that define a 2HDM with UFV can be computed from its Yukawa textures according to the relations

$$(\Gamma_1)_{ij} \neq 0 \Rightarrow (l_d)_i(r_d)_j = t_\beta, \;\; (\Gamma_2)_{ij} \neq 0 \Rightarrow (l_d)_i(r_d)_j = -t_\beta^{-1}, \\ (\Gamma_1)_{ij} \neq 0 \Rightarrow (l_u)_i(r_u)_j = t_\beta, \;\; (\Gamma_2)_{ij} \neq 0 \Rightarrow (l_u)_i(r_u)_j = -t_\beta^{-1}, \tag{4.22}$$

As such, the coefficients $(l_d)_i$ and $(r_d)_i$ in the context of 2HDM can only be powers of t_β. The full list of 2HDMs with UFV is given in Sec. D.

4.2 Left Models

In this section, we define left models has MHDMs with UFV defined by $R_q^a = 1$. From (4.14) we write the non-diagonal couplings of left models [47] as

$$N_{qa} = (l_q^a)_m P_m^{qL} D_q \Rightarrow (N_{qa})_{ij} = (l_q^a)_m (U_{qL})^*_{mi} (U_{qL})_{mj} (m_q)_{jj}. \tag{4.23}$$

Thus, left models are identified through $2(N-1)$ vectors $\vec{l}_q^a$. Since $P_i^{\Lambda X} = P_i^X$ with Λ a diagonal unitary matrix, there are only a maximum of six new parameters in the Yukawa sector of a left model (remember that $\vec{l}_q^a$ is a parameter of the scalar potential and $P_i^{dL} = V^\dagger P_i^{uL} V$) regardless of the number of scalars in the theory.

Consider a left model with two Yukawa couplings Γ_{b_1} and Γ_{b_2} such that $(\Gamma_{b_1})_{ij}(\Gamma_{b_2})_{ik} \neq 0$. Then, (4.9) with $r_q^a = (1,1,1)$ implies

$$(l_d^a)_i = \frac{O_{ab_1}}{O_{1b_1}} = \frac{O_{ab_2}}{O_{1b_2}} \Leftrightarrow \frac{O_{ab_1}}{O_{ab_2}} = \frac{O_{1b_1}}{O_{1b_2}}, \tag{4.24}$$

suggesting that O should be an orthogonal matrix with two proportional columns. Given that no such matrix exists, we conclude that the Yukawa couplings of a left model satisfy

$$(\Gamma_a)_{ij}(\Gamma_b)_{ik} = 0, \;\; (\Delta_a)_{ij}(\Delta_b)_{ik} = 0, \; \forall_{j\neq k}. \tag{4.25}$$

Using these relations, we can build the following set of WBI conditions that are necessary and sufficient to ensure that a given texture belongs to a left model

$$\Gamma_a^\dagger \Gamma_b = \Delta_a^\dagger \Delta_b = \Gamma_a^\dagger \Delta_{p_a} = 0. \tag{4.26}$$

The coefficients p_a are model dependent (in fact, they identify the specific left model) and should be considered as all possible labels with exception of at most one.

Note that (**??**) implies that each line of the mass matrix of a quark of a given charge may receive contributions from one and only one Higgs doublet. Thus, we conclude that imposing the Guiding Principle introduced in Sec. 3.2 is equivalent to requesting left models.

4.2.1 Experimental Constraints

In this subsection, we compute the experimental constraints in (4.1) for left model. Thus, we begin by using the Cauchy-Schwarz and triangle inequalities to write

$$\left|\chi^+(A^+_{q1})^2_{ji} - \chi^-(A^-_{q1})^2_{ji}\right| \leq c^2_{\beta\alpha}\left\{\chi^+\left[\sum_b\left|(N_{qb})_{ji} + (N^\dagger_{qb})_{ji}\right|^2\right] + \chi^-\left[\sum_b\left|(N_{qb})_{ji} - (N^\dagger_{qb})_{ji}\right|^2\right]\right\}. \tag{4.27}$$

In order to progress, we must know how to control the following quantities in the context of left models

$$\left|(N_{qa})_{ji} \pm (N^\dagger_{qa})_{ji}\right|^2. \tag{4.28}$$

Replacing (4.23) on these relations and using the Cauchy-Schwarz inequalities results in

$$\begin{aligned}\left|(N_{qa})_{ji} \pm (N^\dagger_{qa})_{ji}\right|^2 &= \left|(l^a_q)_m (U_{qL})^*_{mj}(U_{qL})_{mi}\left[(m_q)_{ii} \pm (m_q)_{jj}\right]\right|^2 \\ &\leq \left[(m_q)_{ii} \pm (m_q)_{jj}\right]^2\left[\sum_{m=1}^3 (l^a_q)^2_m\right]\left[\sum_{m=1}^3\left|(U_{qL})^*_{mj}(U_{qL})_{mi}\right|^2\right].\end{aligned} \tag{4.29}$$

Neglecting the smallest of the quark masses (m_i) in the meson state and using the results of Sec. C determines

$$\left|(N_{qa})_{ji} \pm (N^\dagger_{qa})_{ji}\right|^2 \leq m_j^2\left[\sum_{m=1}^3 (l^a_q)^2_m\right]. \tag{4.30}$$

As such, we find the following relations which should be applied as experimental constraints on left models in the same way (4.19) was on MHDMs with UFV

$$c^2_{\beta\alpha}\left[\sum_{a=2}^N\sum_{m=1}^3 (l^a_q)^2_m\right] \leq \frac{12v^2 m_h^2 \Delta m_P}{m_P f_P^2(6m_P^2 - m_j^2)}. \tag{4.31}$$

Unlike the previous cases, it turns out that the most stringent of these constraints in the down sector of the theory arises from $B^0_d - \bar{B}^0_d$ mixing. As for the up sector, the $D^0 - \bar{D}^0$ mixing remains the most constraining results to left models, resulting in

$$c^2_{\beta\alpha}\left[\sum_{a=2}^N\sum_{m=2}^3 (l^a_u)^2_m\right] < 5\times10^{-5},\ \ c^2_{\beta\alpha}\left[\sum_{a=2}^N\sum_{m=1}^3 (l^a_d)^2_m\right] < 2\times10^{-4}. \tag{4.32}$$

Before comparing (4.32) to (4.4) and (4.19), we note that this result does not have a factor similar to the one found for MHDMs with UFV. This implies that its best to compare (4.32) to (4.4) and ((4.19))$/20$.

Focusing on the up sector of the theory, we realize that left models have succeeded where MHDMs with UFV failed - a reduction of the order of $\frac{m_t^2}{m_c^2} \sim 10^4$ was obtained. Such feature is a consequence of the fact that N_{qa} is controlled by the heaviest of the quark masses inside the meson state in left models. Since such behaviour translates to the down sector, we conclude that $B^0_x - \bar{B}^0_x$ remains of the same order as in MHDMs with UFV whole $K^0 - \bar{K}^0$ mixing is suppressed by a factor of $\frac{m_b^2}{m_s^2} \sim 10^3$. The strength of this factor is such that $B_d^0 - \bar{B}_d^0$ mixing becomes more important than $K^0 - \bar{K}^0$ mixing in left models.

4.2.2 2HDMs

As for left models in the context of 2HDM, (3.4) and (4.26) imply that

$$\begin{aligned}(\Gamma_1)_{ij} \neq 0 \Rightarrow (D_d^0)_{ik} = \frac{1}{\sqrt{2}} v_1 (\Gamma_1)_{ik} \ \wedge \ (N_d^0)_{ik} = \frac{1}{\sqrt{2}} v_2 (\Gamma_1)_{ik} = t_\beta (D_d^0)_{ik}, \\ (\Gamma_2)_{ij} \neq 0 \Rightarrow (D_d^0)_{ik} = \frac{1}{\sqrt{2}} v_2 (\Gamma_2)_{ik} \ \wedge \ (N_d^0)_{ik} = -\frac{1}{\sqrt{2}} v_1 (\Gamma_2)_{ik} = -t_\beta^{-1} (D_d^0)_{ik}.\end{aligned} \tag{4.33}$$

Thus, the coefficients $(l_q)_i$ can be determined from the Yukawa textures of left models as

$$\begin{aligned}(\Gamma_1)_{ij} \neq 0 \Rightarrow (l_d)_i = t_\beta, \ (\Gamma_2)_{ij} \neq 0 \Rightarrow (l_d)_i = -t_\beta^{-1}, \\ (\Delta_1)_{ij} \neq 0 \Rightarrow (l_u)_i = t_\beta, \ (\Delta_2)_{ij} \neq 0 \Rightarrow (l_u)_i = -t_\beta^{-1}.\end{aligned} \tag{4.34}$$

Since left models are the result of implementing the Guiding Principle of Sec. 3.2, we have already studied all Two-Higgs left models. In this new formalism, such models are defined in the next table. BGL models can be distinguished from gBGL models since one of its sectors also satisfies right relations (for example, for tBGL models we have $N_u = D_u R_u$ with $R_u = \mathrm{diag}\{t_\beta, t_\beta, -t_\beta^{-1}\}$).

Modelo	$\vec{l}_d$	$\vec{l}_u$
Type I	$(t_\beta, t_\beta, t_\beta)$	$(t_\beta, t_\beta, t_\beta)$
Type II	$(t_\beta, t_\beta, t_\beta)$	$(-t_\beta^{-1}, -t_\beta^{-1}, -t_\beta^{-1})$
BGL	$(t_\beta, t_\beta, -t_\beta^{-1})$	$(t_\beta, t_\beta, -t_\beta^{-1})$
gBGL	$(t_\beta, t_\beta, -t_\beta^{-1})$	$(t_\beta, t_\beta, -t_\beta^{-1})$
jBGL	$(-t_\beta^{-1}, -t_\beta^{-1}, t_\beta)$	$(t_\beta, t_\beta, -t_\beta^{-1})$

Table 4.1: Identification of Two-Higgs left models.

4.3 Right Models

In this section, we define right models as MHDMs with UFV defined by $L_q^a = 1$. We write the non-diagonal couplings of right models [47] from (4.14) as

$$N_{qa} = (r_q^a)_n D_q P_n^{qR} \Rightarrow (N_{qa})_{ij} = (r_q^a)_n (U_{qR})^*_{ni} (U_{qR})_{nj} (m_q)_{ii}. \tag{4.35}$$

As such, right models are identified by $2(N-1)$ vectors $\vec{r}_q^a$. Given that $P_i^{\Lambda X} = P_i^X$ with Λ a diagonal unitary matrix, there are only a maximum of twelve new parameters (note that P_i^{dR} and P_i^{uR} are related by an unphysical V_R) in the Yukawa sector of a right model regardless of the number of scalars in the theory.

Consider a right model with two Yukawa couplings Γ_{b_1} and Γ_{b_2} such that $(\Gamma_{b_1})_{ji} (\Gamma_{b_2})_{ki} \neq 0$. Applying (4.11) for right models implies

$$(r_q^a)_i = \frac{O_{ab_1}}{O_{1b_1}} = \frac{O_{ab_2}}{O_{1b_2}} \Leftrightarrow \frac{O_{ab_2}}{O_{ab_1}} = \frac{O_{1b_2}}{O_{1b_1}}. \tag{4.36}$$

As such, O must be an orthogonal matrix with two proportional columns (b_1 and b_2). Since that is impossible, we conclude that the Yukawa couplings of a right model must satisfy

$$(\Gamma_a)_{ij}(\Gamma_a)_{kj} = (\Delta_a)_{ij}(\Delta_a)_{kj} = 0,\ \forall_{i\neq k}. \tag{4.37}$$

From these expressions, we conclude that in right models each column of the mass matrix of a quark of a given charge may only receive contributions from one and only one Higgs doublet. It can also be shown that the following WBI conditions are sufficient and necessary to ensure that a given texture belongs to a right model

$$\Gamma_a \Gamma_b^\dagger = \Delta_a \Delta_b^\dagger = 0. \tag{4.38}$$

4.3.1 Experimental Constraints

In the previous two sections, we have seen that understanding the experimental constraints that must be satisfied by a model requires controlling the quantities

$$\left|(N_{qa})_{ji} \pm (N_{qa}^\dagger)_{ji}\right|^2. \tag{4.39}$$

Using (4.35), and the Cauchy-Schwarz inequalities we compute

$$\begin{aligned} \left|(N_{qa})_{ji} \pm (N_{qa}^\dagger)_{ji}\right|^2 &= \left[(m_q)_{jj} \pm (m_q)_{ii}\right]^2 \left|(r_q^a)_n (U_{qR})^*_{nj}(U_{qR})_{ni}\right|^2 \\ &\leq \left[(m_q)_{jj} \pm (m_q)_{ii}\right]^2 \left[\sum_{n=1}^{3}(r_q^a)_n^2\right]\left[\sum_{n=1}^{3}\left|(U_{qR})^*_{nj}(U_{qR})_{ni}\right|^2\right]. \end{aligned} \tag{4.40}$$

Notice that after neglecting the smallest of the quark masses this expression is identical to (4.29) provided we replace r_q^a by l_q^a. This implies that the bounds in (4.32) can be directly translated to right models

$$c_{\beta\alpha}^2 \left[\sum_{a=2}^{N}\sum_{n=1}^{3}(r_u^a)_n^2\right] < 5\times 10^{-5},\ \ c_{\beta\alpha}^2 \left[\sum_{a=2}^{N}\sum_{n=1}^{3}(r_d^a)_n^2\right] < 2\times 10^{-4}. \tag{4.41}$$

Thus, right models are identical to left models in regards to the cancellations required to make them valid theories. Despite the strength of (4.41), it is more useful to write these bounds for the next subsection[4] in the following form

$$c_{\beta\alpha}^2 \left|(r_u)_n (U_{uR})^*_{n2}(U_{uR})_{n1}\right|^2 < 5\times 10^{-5},\ \ c_{\beta\alpha}^2 \left|(r_d)_n (U_{dR})^*_{n3}(U_{dR})_{n1}\right|^2 < 2\times 10^{-4}. \tag{4.42}$$

4.3.2 2HDMs

Regarding right models in the context of 2HDM, (3.4) and (4.38) imply that

[4] Since the next subsection deals with 2HDM, there is only one unitary vector r_q for each sector.

$$
\begin{aligned}
(\Gamma_1)_{ij} \neq 0 \Rightarrow (D_d^0)_{ij} &= \frac{1}{\sqrt{2}} v_1 (\Gamma_1)_{ij} \ \wedge \ (N_d^0)_{ij} = \frac{1}{\sqrt{2}} v_2 (\Gamma_1)_{ij} = t_\beta (D_d^0)_{ij}, \\
(\Gamma_2)_{ij} \neq 0 \Rightarrow (D_u^0)_{ij} &= \frac{1}{\sqrt{2}} v_2 (\Delta_1)_{ij} \ \wedge \ (N_u^0)_{ij} = -\frac{1}{\sqrt{2}} v_1 (\Delta_1)_{ij} = -t_\beta^{-1} (D_u^0)_{ij}.
\end{aligned}
\tag{4.43}
$$

As such, the coefficients $(r_q)_i$ can be determined from the Yukawa textures of right models as

$$
\begin{aligned}
(\Gamma_1)_{ij} \neq 0 \Rightarrow (r_d)_j = t_\beta, \ \ (\Gamma_2)_{ij} \neq 0 \Rightarrow (l_d)_j = -t_\beta^{-1}, \\
(\Delta_1)_{ij} \neq 0 \Rightarrow (r_u)_j = t_\beta, \ \ (\Delta_2)_{ij} \neq 0 \Rightarrow (l_u)_j = -t_\beta^{-1}.
\end{aligned}
\tag{4.44}
$$

We will now go over the properties and phenomenology of all Two-Higgs right models.

Type A

The first right model to be studied in this subsection is the Type A model [47] that is identified by the vectors $\vec{r}_d = (t_\beta, t_\beta, t_\beta)$ and $\vec{r}_u = (t_\beta, t_\beta, -t_\beta^{-1})$. This configuration is protected by the AFS

$$
\phi_2 \to \Upsilon \phi_2, \ \ u_{R3}^0 \to \Upsilon u_{R3}^0, \tag{4.45}
$$

where $|\Upsilon| = 1$ and $\Upsilon \neq 1$. In the rest of this chapter, the fields whose transformation is not explicitly shown when defining a symmetry are invariant under it. The Yukawa textures of type A models are found to be given in this WB by

$$
\Gamma_1 = \begin{bmatrix} x & x & x \\ x & x & x \\ x & x & x \end{bmatrix}, \ \Gamma_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \ \Delta_1 = \begin{bmatrix} x & x & 0 \\ x & x & 0 \\ x & x & 0 \end{bmatrix}, \ \Delta_2 = \begin{bmatrix} 0 & 0 & x \\ 0 & 0 & x \\ 0 & 0 & x \end{bmatrix}. \tag{4.46}
$$

It is now trivial to write the non-diagonal couplings of type A models as

$$
N_d = t_\beta D_d, \ \ (N_u)_{ij} = \left[t_\beta \delta_{ij} - (t_\beta + t_\beta^{-1}) n_i^* n_j \right] (m_u)_{ii}, \tag{4.47}
$$

where $n_i = (U_{uR})_{3i}$. This implies that type A models are parametrized by four parameters (see (3.31)), two real angles and two complex phases.

Regarding the scalar potential of Type A models, we notice that the scalars are transformed by (4.45) in the same fashion as by (3.62). Thus, the analysis of Sec. **??** applies to type A models. As a consequence, the scalar potential of type A models conserves CP, justifying the use of (4.42).

Considering the existence of tree-level FCNC in the up sector of type A models, we may use the corresponding bound in (4.42). Using $\vec{r}_u = (t_\beta, t_\beta, -t_\beta^{-1})$ in it, we obtain[5]

$$
|c_{\beta\alpha}(t_\beta + t_\beta^{-1})| < 0.02 \ . \tag{4.48}
$$

[5] To obtain this bound, we have taken the maximum value of $|n_2^* n_1| = \frac{1}{2}$. Thus, it will be relaxed in all regions of the parameter space in which $|n_2^* n_1|$ does not take its maximum value.

Type B

Type B models [47] are defined by $\vec{r}_d = (t_\beta, t_\beta, t_\beta)$ and $\vec{r}_u = (-t_\beta^{-1}, -t_\beta^{-1}, t_\beta)$. They can be implemented by the AFS

$$\phi_2 \to \Upsilon\phi_2, \;\; u^0_{R1,2} \to \Upsilon u^0_{R1,2}, \tag{4.49}$$

where $|\Upsilon| = 1$ and $\Upsilon \neq 1$. Using (2.70), it is straightforward to obtain the Yukawa textures of Type B models

$$\Gamma_1 = \begin{bmatrix} x & x & x \\ x & x & x \\ x & x & x \end{bmatrix}, \;\; \Gamma_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \;\; \Delta_1 = \begin{bmatrix} 0 & 0 & x \\ 0 & 0 & x \\ 0 & 0 & x \end{bmatrix}, \;\; \Delta_2 = \begin{bmatrix} x & x & 0 \\ x & x & 0 \\ x & x & 0 \end{bmatrix}. \tag{4.50}$$

From these, we write the non-diagonal couplings of type B models as

$$N_d = t_\beta D_d, \;\; (N_u)_{ij} = \left[-t_\beta^{-1}\delta_{ij} + (t_\beta + t_\beta^{-1}) n_i^* n_j\right](m_u)_{ii}, \tag{4.51}$$

where $n_i = (U_{uR})_{3i}$. Thus, Type B models are parametrized by the four parameters of (3.31).

As for the scalar potential of type B models, we notice that the scalars transfom equally under (4.49) as under (4.45). Thus, the scalar potentials of type A and type B models are identical, implying (4.42) can be applied to type B models.

Since type B models have tree-level FCNC in the up sector of the theory that are symmetric to those of type A models, the same constraint applies to both models,

$$|c_{\beta\alpha}(t_\beta + t_\beta^{-1})| < 0.02 \,. \tag{4.52}$$

Type C

In this subsubsection, we analyse Type C models [47] that are defined by $\vec{r}_d = (t_\beta, t_\beta, -t_\beta^{-1})$ and $\vec{r}_u = (t_\beta, t_\beta, t_\beta)$. They are protected under renormalization by the AFS

$$\phi_2 \to \Upsilon\phi_2, \;\; d^0_{R3} \to \Upsilon^{-1} d^0_{R3}, \tag{4.53}$$

where $|\Upsilon| = 1$ and $\Upsilon \neq 1$. Applying (4.53) in (2.70) results in the following Yukawa textures for type C models

$$\Gamma_1 = \begin{bmatrix} x & x & 0 \\ x & x & 0 \\ x & x & 0 \end{bmatrix}, \;\; \Gamma_2 = \begin{bmatrix} 0 & 0 & x \\ 0 & 0 & x \\ 0 & 0 & x \end{bmatrix}, \;\; \Delta_1 = \begin{bmatrix} x & x & x \\ x & x & x \\ x & x & x \end{bmatrix}, \;\; \Delta_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \tag{4.54}$$

from which we write their non-diagonal couplings as

$$(N_d)_{ij} = \left[t_\beta\delta_{ij} - (t_\beta + t_\beta^{-1}) n_i^* n_j\right](m_d)_{ii}, \;\; N_u = t_\beta D_u, \tag{4.55}$$

where $n_i = (U_{dR})_{3i}$. Thus, type C models are still parametrized by the four parameters of (3.31).

Given that the scalars are transformed identically in (4.53) and in (4.45), we conclude that their potentials are identical. Thus, (4.42) may be applied to type C models.

Noting that there are only tree-level FCNC in the down sector of type C models, only the corresponding bound in (4.42) can be used to constrain them.

$$|c_{\beta\alpha}(t_\beta + t_\beta^{-1})| < 0.03\ . \tag{4.56}$$

Type D

Type D models [47] that are defined by $\vec{r}_d = (-t_\beta^{-1}, -t_\beta^{-1}, t_\beta)$ and $\vec{r}_u = (t_\beta, t_\beta, t_\beta)$ are protected by the following AFS

$$\phi_2 \to \Upsilon\phi_2,\ \ d^0_{R1,2} \to \Upsilon^{-1} d^0_{R1,2}, \tag{4.57}$$

where $|\Upsilon| = 1$ and $\Upsilon \neq 1$. It is straightforward to obtain the Yukawa textures of type D models to be

$$\Gamma_1 = \begin{bmatrix} 0 & 0 & x \\ 0 & 0 & x \\ 0 & 0 & x \end{bmatrix},\ \ \Gamma_2 = \begin{bmatrix} x & x & 0 \\ x & x & 0 \\ x & x & 0 \end{bmatrix},\ \ \Delta_1 = \begin{bmatrix} x & x & x \\ x & x & x \\ x & x & x \end{bmatrix},\ \ \Delta_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \tag{4.58}$$

We write the non-diagonal couplings of type D models in this WB as

$$(N_d)_{ij} = \left[-t_\beta^{-1}\delta_{ij} + (t_\beta + t_\beta^{-1}) n_i^* n_j\right](m_d)_{ii},\ \ N_u = t_\beta D_u, \tag{4.59}$$

where $n_i = (U_{dR})_{3i}$. As such, type D models are parametrized by two real angles and two complex phases (see (3.31)).

As for the scalar potential of type D models, we note that the scalars are transformed in the same way under (4.57) as in (4.45), implying that type A and type D models share a similar scalar potential. As a consequence, (4.42) can be used for type D models.

Since type D models only have tree-level FCNC in the down sector of the theory that are symmetric to those of type C models, they are constrained in the same way as them

$$|c_{\beta\alpha}(t_\beta + t_\beta^{-1})| < 0.03\ . \tag{4.60}$$

Type E

In this subsubsection, we analyse Type E models [47]. These are defined by $\vec{r}_d = (t_\beta, t_\beta, -t_\beta^{-1})$ and $\vec{r}_u = (t_\beta, t_\beta, -t_\beta^{-1})$ and can be implemented by the AFS

$$\phi_2 \to \Upsilon\phi_2,\;\; d^0_{R3} \to \Upsilon^{-1} d^0_{R3},\;\; u^0_{R3} \to \Upsilon u^0_{R3}, \tag{4.61}$$

where $|\Upsilon| = 1$ and $\Upsilon \neq 1$. Using (2.70) to obtain the Yukawa textures of these models results in

$$\Gamma_1 = \begin{bmatrix} x & x & 0 \\ x & x & 0 \\ x & x & 0 \end{bmatrix},\;\; \Gamma_2 = \begin{bmatrix} 0 & 0 & x \\ 0 & 0 & x \\ 0 & 0 & x \end{bmatrix},\;\; \Delta_1 = \begin{bmatrix} x & x & 0 \\ x & x & 0 \\ x & x & 0 \end{bmatrix},\;\; \Delta_2 = \begin{bmatrix} 0 & 0 & x \\ 0 & 0 & x \\ 0 & 0 & x \end{bmatrix}. \tag{4.62}$$

From these we write the non-diagonal couplings of type E models as

$$(N_d)_{ij} = \left[t_\beta \delta_{ij} - (t_\beta + t_\beta^{-1})(n^d)^*_i (n^d)_j\right](m_d)_{ii},\;\; (N_u)_{ij} = \left[t_\beta \delta_{ij} - (t_\beta + t_\beta^{-1})(n^u)^*_i (n^u)_j\right](m_u)_{ii}, \tag{4.63}$$

where $(n^d)_i = (U_{dR})_{3i}$ and $(n^u)_i = (U_{uR})_{3i}$ are two independent unitary vectors. As such, type E models are parametrized by eight new parameters - four real angles and four complex phases.

Regarding the scalar potential of type E models, we notice that the scalars are transformed under (4.45) in the same way as under (4.61). Thus, type A and type E models share the same scalar potential, which means that (4.42) can be used to constrain type E models.

Given that the most stringent constraint applicable to type E models arises from the up sector of the theory which is identical in type A and type E models, the same constraint applies to both theories

$$|c_{\beta\alpha}(t_\beta + t_\beta^{-1})| < 0.02\ . \tag{4.64}$$

Type F

We now study Type F models [47] which are defined by $\vec{r}_d = (t_\beta, t_\beta, -t_\beta^{-1})$ and $\vec{r}_u = (-t_\beta^{-1}, -t_\beta^{-1}, t_\beta)$. Such models can be implemented by the AFS

$$\phi_2 \to \Upsilon\phi_2,\;\; d^0_{R3} \to \Upsilon^{-1} d^0_{R3},\;\; u^0_{R1,2} \Upsilon u^0_{R1,2}, \tag{4.65}$$

where $|\Upsilon| = 1$ and $\Upsilon \neq 1$. Implementing (4.65) on (2.70) determines the Yukawa textures of type F models to be

$$\Gamma_1 = \begin{bmatrix} x & x & 0 \\ x & x & 0 \\ x & x & 0 \end{bmatrix},\;\; \Gamma_2 = \begin{bmatrix} 0 & 0 & x \\ 0 & 0 & x \\ 0 & 0 & x \end{bmatrix},\;\; \Delta_1 = \begin{bmatrix} 0 & 0 & x \\ 0 & 0 & x \\ 0 & 0 & x \end{bmatrix},\;\; \Delta_2 = \begin{bmatrix} x & x & 0 \\ x & x & 0 \\ x & x & 0 \end{bmatrix}. \tag{4.66}$$

Using these textures, it is straightforward to write their non-diagonal couplings as

$$(N_d)_{ij} = \left[t_\beta \delta_{ij} - (t_\beta + t_\beta^{-1})(n^d)^*_i (n^d)_j\right](m_d)_{ii},\;\; (N_u)_{ij} = \left[-t_\beta^{-1} \delta_{ij} + (t_\beta + t_\beta^{-1})(n^u)^*_i (n^u)_j\right](m_u)_{ii}, \tag{4.67}$$

where $(n^d)_i = (U_{dR})_{3i}$ and $(n^u)_i = (U_{uR})_{3i}$. Thus, type F models are parametrized by two unitary vectors, implying a total of eight new parameters.

Since the scalar fields transform equally under (4.65) and (4.45), the scalar potential of type F models is identical to that of type A models. Thus, (4.42) can be applied to type F models.

Considering that the most stringent experimental bound applicable to type F models arises from their up sector and that the corresponding couplings are identical to those of type B models, type F models must satisfy

$$|c_{\beta\alpha}(t_\beta + t_\beta^{-1})| < 0.02 \; . \tag{4.68}$$

4.3.3 WBI Identification

In Sec. 4.3.2, we have found six non-trivial right models in addition to the trivial Type I and II 2HDMs.

Modelo	$\vec{r}_d$	$\vec{r}_u$	Modelo	$\vec{r}_d$	$\vec{r}_u$
Type I	$(t_\beta, t_\beta, t_\beta)$	$(t_\beta, t_\beta, t_\beta)$	Type A	$(t_\beta, t_\beta, t_\beta)$	$(t_\beta, t_\beta, -t_\beta^{-1})$
Type II	$(t_\beta, t_\beta, t_\beta)$	$(-t_\beta^{-1}, -t_\beta^{-1}, -t_\beta^{-1})$	Type B	$(t_\beta, t_\beta, t_\beta)$	$(-t_\beta^{-1}, -t_\beta^{-1}, t_\beta)$
Type C	$(t_\beta, t_\beta, -t_\beta^{-1})$	$(t_\beta, t_\beta, t_\beta)$	Type E	$(t_\beta, t_\beta, -t_\beta^{-1})$	$(t_\beta, t_\beta, -t_\beta^{-1})$
Type D	$(-t_\beta^{-1}, -t_\beta^{-1}, t_\beta)$	$(t_\beta, t_\beta, t_\beta)$	Type F	$(t_\beta, t_\beta, -t_\beta^{-1})$	$(-t_\beta^{-1}, -t_\beta^{-1}, t_\beta)$

Table 4.2: Identification of Two-Higgs right models.

Noting that the Higgs basis is uniquely defined in 2HDMs, this table provides a WBI identification of right models. However, it is clear that a similar table will not identify right models in MHDMs with more than two scalar doublets. For this reason, we must find a different way to define right models.

Unlike left models, (4.38) cannot distinguish right models (all right models satisfy it). Furthermore, the natural expansion of (4.38) by introducing the relations $\Gamma_a \Delta_{p_a}^\dagger = 0$ is not valid given that such equality is not WBI. With that being said, the rank of a coupling is a WBI that can be used to identify all right models. Note that situations such as BGL and gBGL models (which are not distinguished by rank) cannot occur in right models since distinction between lines is provided by q_L, which necessarily implies the same structure for both sectors of the theory.

	$(3,0)$	$(0,3)$	$(2,1)$	$(1,2)$
$(3,0)$	Type I	Type II	Type A	Type B
$(0,3)$	Type II	Type I	Type B	Type A
$(2,1)$	Type C	Type D	Type E	Type F
$(1,2)$	Type D	Type C	Type F	Type E

Table 4.3: Identification of Two-Higgs right models through their ranks. Rows contain rank Γ_k and columns rank Δ_k.

Chapter 5

Conclusion

In this MSc thesis, we have shown the existence of a simple principle that reduces the 36 new free parameters in the Yukawa sector of the most general 2HDM to a maximum of four. Besides the already known Type I, Type II and BGL models, we have found two additional classes of models which satisfy the principle. The first of them, gBGL, can be seen as a generalization of BGL models outside of the MFV framework. Contrary to BGL models, gBGL models contain tree-level FCNC in both sectors that are parametrized by four additional flavour parameters beyond the CKM matrix. Nevertheless, we have shown that its FCNC were significantly suppressed, rendering gBGL models plausible extensions of the SM. We have also analysed the low order CP-odd invariants that arise in gBGL models, having discovered that they are capable of generating a much higher BAU than the SM. The second of the non-trivial 2HDMs which satisfies our principle is that of jBGL models. While they cannot be seen as a generalization of BGL models, significant comparisons can be made to gBGL models. In fact, we found them to be indistinguishable when analysing only one sector of the theory. Thus, jBGL models mantain the controlled FCNC and enhanced BAU of gBGL models. However, we were still capable of finding a channel that may be reached in future collider experiments capable of making a clear distinction between the two models.

We have also introduced the condition of UFV in order to obtain a large class of MHDMs with controlled tree-level FCNC. In particular, we have analysed extensively MHDMs with UFV in the context of 2HDMs, having presented the full list of these models that are protected by an AFS. In this study, we have encountered two important subclasses of UFV. The first, populated by left models, was found to be equivalent to the class of 2HDMs that satisfies our principle to reduce the number of free parameters in the Yukawa sector. Meanwhile, the second subclass, populated by right models, included six previously undiscovered symmetry protected classes of 2HDMs. While by construction the tree-level FCNC of all MHDMS with UFV are controlled, we found a stronger suppresion in both left and right models. As such, those models are the most plausible extensions of the SM in the context of MHDMs.

Having determined the set of MHDMs with controlled Yukawa sectors, we leave as future work a more detailed analysis of their phenomenological properties. In particular, we are interested in describing the new signals predicted by such models that may be detected in the next generation of detectors.

While the study of CP violation in the scalar potential of MHDMs with UFV was discussed extensively through out this MSc thesis, we have not discussed further its properties. As such, we have left as future work the control of such sector in view of reducing the intensity of tree-level FCNC in those models.

In this MSc thesis, we have required the fulfillment of strict conditions in Sec. B in order to consider a MHDM as valid. A future avenue of work is the analysis of the models that arise when we soften such requirements. In particular, we are interested in the generation of the lightest quark masses and the CKM structure from the breaking of an AFS.

Bibliography

[1] Michael E Peskin. *An introduction to quantum field theory*. CRC Press, 2018.

[2] David Bailin and Alexander Love. *Introduction to Gauge Field Theory Revised Edition*. CRC Press, 1993.

[3] Jorge C Romao. Advanced quantum field theory. *Lecture Notes for the PhD program, Phys. Dept., Inst. Sup. Tec., Lisboa*, 2013.

[4] C. Patrignani et al. Review of Particle Physics. *Chin. Phys.*, C40(10):100001, 2016.

[5] Gustavo C Branco, Luis Lavoura, and Joao Paulo Silva. *CP violation*, volume 103. Oxford University Press, 1999.

[6] Joao P. Silva. Phenomenological aspects of CP violation. In *Central European School in Particle Physics Prague, Czech Republic, September 14-24, 2004*, 2004.

[7] A. D. Sakharov. Violation of CP Invariance, C asymmetry, and baryon asymmetry of the universe. *Pisma Zh. Eksp. Teor. Fiz.*, 5:32–35, 1967. [Usp. Fiz. Nauk161,no.5,61(1991)].

[8] M. E. Shaposhnikov. Baryon Asymmetry of the Universe in Standard Electroweak Theory. *Nucl. Phys.*, B287:757–775, 1987.

[9] M. B. Gavela, P. Hernandez, J. Orloff, and O. Pene. Standard model CP violation and baryon asymmetry. *Mod. Phys. Lett.*, A9:795–810, 1994.

[10] V. A. Rubakov and M. E. Shaposhnikov. Electroweak baryon number nonconservation in the early universe and in high-energy collisions. *Usp. Fiz. Nauk*, 166:493–537, 1996. [Phys. Usp.39,461(1996)].

[11] Georges Aad et al. Observation of a new particle in the search for the Standard Model Higgs boson with the ATLAS detector at the LHC. *Phys. Lett.*, B716:1–29, 2012.

[12] Serguei Chatrchyan et al. Observation of a new boson at a mass of 125 GeV with the CMS experiment at the LHC. *Phys. Lett.*, B716:30–61, 2012.

[13] G. C. Branco, P. M. Ferreira, L. Lavoura, M. N. Rebelo, Marc Sher, and Joao P. Silva. Theory and phenomenology of two-Higgs-doublet models. *Phys. Rept.*, 516:1–102, 2012.

[14] Igor P. Ivanov. Building and testing models with extended Higgs sectors. *Prog. Part. Nucl. Phys.*, 95:160–208, 2017.

[15] TD Lee. A theory of spontaneous t violation. *Physical Review D*, 8(4):1226, 1973.

[16] James M. Cline and Pierre-Anthony Lemieux. Electroweak phase transition in two Higgs doublet models. *Phys. Rev.*, D55:3873–3881, 1997.

[17] Lars Fromme, Stephan J. Huber, and Michael Seniuch. Baryogenesis in the two-Higgs doublet model. *JHEP*, 11:038, 2006.

[18] G. Belanger, F. Boudjema, A. Pukhov, and A. Semenov. micrOMEGAs: Version 1.3. *Comput. Phys. Commun.*, 174:577–604, 2006.

[19] Aleksandra Drozd, Bohdan Grzadkowski, John F. Gunion, and Yun Jiang. Extending two-Higgs-doublet models by a singlet scalar field - the Case for Dark Matter. *JHEP*, 11:105, 2014.

[20] R. D. Peccei and Helen R. Quinn. CP Conservation in the Presence of Instantons. *Phys. Rev. Lett.*, 38:1440–1443, 1977. [,328(1977)].

[21] Sheldon L. Glashow and Steven Weinberg. Natural Conservation Laws for Neutral Currents. *Phys. Rev.*, D15:1958, 1977.

[22] E. A. Paschos. Diagonal Neutral Currents. *Phys. Rev.*, D15:1966, 1977.

[23] G. C. Branco, W. Grimus, and L. Lavoura. Relating the scalar flavor changing neutral couplings to the CKM matrix. *Phys. Lett.*, B380:119–126, 1996.

[24] F. J. Botella, G. C. Branco, and M. N. Rebelo. Minimal Flavour Violation and Multi-Higgs Models. *Phys. Lett.*, B687:194–200, 2010.

[25] David Tong. Lectures on quantum field theory. *Part III Cambridge University Mathematics Tripos, Michaelmas*, 2006.

[26] Howard Georgi. *Lie algebras in particle physics: from isospin to unified theories*. CRC Press, 2018.

[27] Robert N Cahn. *Semi-simple Lie algebras and their representations*. Courier Corporation, 2014.

[28] F. Abe et al. A Measurement of the W boson mass. *Phys. Rev. Lett.*, 65:2243–2246, 1990.

[29] Morad Aaboud et al. Measurement of the W-boson mass in pp collisions at $\sqrt{s} = 7$ TeV with the ATLAS detector. *Eur. Phys. J.*, C78(2):110, 2018.

[30] Peter W. Higgs. Broken symmetries, massless particles and gauge fields. *Phys. Lett.*, 12:132–133, 1964.

[31] F. Englert and R. Brout. Broken Symmetry and the Mass of Gauge Vector Mesons. *Phys. Rev. Lett.*, 13:321–323, 1964. [,157(1964)].

[32] G. S. Guralnik, C. R. Hagen, and T. W. B. Kibble. Global Conservation Laws and Massless Particles. *Phys. Rev. Lett.*, 13:585–587, 1964. [,162(1964)].

[33] Jeffrey Goldstone, Abdus Salam, and Steven Weinberg. Broken symmetries. *Physical Review*, 127(3):965, 1962.

[34] D. Gotz, S. Covino, A. Fernandez-Soto, P. Laurent, and Z . Bosnjak. The polarized Gamma-Ray Burst GRB 061122. *Mon. Not. Roy. Astron. Soc.*, 431:3550, 2013.

[35] V. Vasileiou, A. Jacholkowska, F. Piron, J. Bolmont, C. Couturier, J. Granot, F. W. Stecker, J. Cohen-Tanugi, and F. Longo. Constraints on Lorentz Invariance Violation from Fermi-Large Area Telescope Observations of Gamma-Ray Bursts. *Phys. Rev.*, D87(12):122001, 2013.

[36] P. M. Ferreira and Joao P. Silva. Abelian symmetries in the two-Higgs-doublet model with fermions. *Phys. Rev.*, D83:065026, 2011.

[37] H. Serôdio. Yukawa sector of Multi Higgs Doublet Models in the presence of Abelian symmetries. *Phys. Rev.*, D88(5):056015, 2013.

[38] F. J. Botella, G. C. Branco, M. Nebot, and M. N. Rebelo. Flavour Changing Higgs Couplings in a Class of Two Higgs Doublet Models. *Eur. Phys. J.*, C76(3):161, 2016.

[39] Georges Aad et al. Study of the spin and parity of the Higgs boson in diboson decays with the ATLAS detector. *Eur. Phys. J.*, C75(10):476, 2015. [Erratum: Eur. Phys. J.C76,no.3,152(2016)].

[40] Serguei Chatrchyan et al. Study of the Mass and Spin-Parity of the Higgs Boson Candidate Via Its Decays to Z Boson Pairs. *Phys. Rev. Lett.*, 110(8):081803, 2013.

[41] Morad Aaboud et al. Search for flavor-changing neutral currents in top quark decays $t \to Hc$ and $t \to Hu$ in multilepton final states in proton–proton collisions at $\sqrt{s} = 13$ TeV with the ATLAS detector. 2018.

[42] Vardan Khachatryan et al. Searches for heavy Higgs bosons in two-Higgs-doublet models and for $t \to ch$ decay using multilepton and diphoton final states in pp collisions at 8 TeV. *Phys. Rev.*, D90:112013, 2014.

[43] Vardan Khachatryan et al. Search for Higgs boson off-shell production in proton-proton collisions at 7 and 8 TeV and derivation of constraints on its total decay width. *JHEP*, 09:051, 2016.

[44] J. Bernabeu, G. C. Branco, and M. Gronau. CP Restrictions on Quark Mass Matrices. *Phys. Lett.*, 169B:243–247, 1986.

[45] Cecilia Jarlskog. Commutator of the quark mass matrices in the standard electroweak model and a measure of maximal cp nonconservation. *Physical Review Letters*, 55(10):1039, 1985.

[46] Joao M. Alves, Francisco J. Botella, Gustavo C. Branco, Fernando Cornet-Gomez, and Miguel Nebot. Controlled Flavour Changing Neutral Couplings in Two Higgs Doublet Models. *Eur. Phys. J.*, C77(9):585, 2017.

[47] João M. Alves, Francisco J. Botella, Gustavo C. Branco, Fernando Cornet-Gomez, Miguel Nebot, and João P. Silva. Symmetry Constrained Two Higgs Doublet Models. 2018.

[48] Lincoln Wolfenstein. Parametrization of the Kobayashi-Maskawa Matrix. *Phys. Rev. Lett.*, 51:1945, 1983.

[49] F. J. Botella, G. C. Branco, and M. N. Rebelo. Invariants and Flavour in the General Two-Higgs Doublet Model. *Phys. Lett.*, B722:76–82, 2013.

[50] Gustavo C. Branco, M. N. Rebelo, and J. I. Silva-Marcos. CP-odd invariants in models with several Higgs doublets. *Phys. Lett.*, B614:187–194, 2005.

[51] Victor Weisskopf and Eugene Wigner. Berechnung der natürlichen linienbreite auf grund der diracschen lichttheorie. *Zeitschrift für Physik*, 63(1-2):54–73, 1930.

[52] M Fierz. M. fierz, z. phys. 104, 553 (1937). *Z. Phys.*, 104:553, 1937.

[53] Celso C Nishi. Simple derivation of general fierz-type identities. *American journal of physics*, 73(12):1160–1163, 2005.

[54] Yosef Nir. Cp violation: The ckm matrix and new physics. *arXiv preprint hep-ph/0208080*, 2002.

[55] Yosef Nir. CP violation: A New era. In *Heavy flavor physics: Theory and experimental results in heavy quark physics and CP violation. Proceedings, 55th Scottish Universities Summer School in Physics, SUSSP 2001, St. Andrews, UK, August 7-23, 2001*, pages 147–200, 2001.

[56] John F Donoghue, Eugene Golowich, Barry R Holstein, and Josip Trampetić. Dispersive effects in d 0-d 0 mixing. *Physical Review D*, 33(1):179, 1986.

[57] Howard Georgi. D- d mixing in heavy quark effective field theory. *Physics Letters B*, 297(3-4):353–357, 1992.

Appendix A

Meson Mixing

A.1 Phenomenology

Throughout this MSc thesis, we used meson-antimeson mixing[1] to constrain MHDM. In this Annex, we go over the phenomenology of such systems. In particular, we must understand the detection of CP violation in these systems [6].

To achieve our goal, we begin by studying the action of CP on the physical (meson) states, rather than on individual quarks. Given that classically $(CP)^2 = 1$, we use the correspondence principle to set the quantum operator describing $(CP)^2$ to be the identity[2]. Using such fact we write the way in which CP acts on physical states

$$(CP)\,|f\rangle = e^{i\xi_f}\,|\bar{f}\rangle\,,\;\; (CP)\,|\bar{f}\rangle = e^{-i\xi_f}\,|f\rangle\,,\;\; (CP)\,|g\rangle = \eta_g\,|g\rangle\,,\; \eta_g^2 = 1, \tag{A.1}$$

where g is an eigenstate of CP and $\bar{f}$ is the physical antiparticle of f. Using these transformations laws, we can relate the following transition amplitudes

$$\mathcal{M}_{if} = \langle f|\mathcal{M}|i\rangle = e^{i(\xi_i-\xi_f)}\,\langle \bar{f}|\mathcal{M}_{CP}|\bar{i}\rangle = e^{i(\xi_i-\xi_f)}(\mathcal{M}_{CP})_{\bar{i}\bar{f}}, \tag{A.2}$$

where $\mathcal{M}_{CP} = (CP)\mathcal{M}(CP)^\dagger$. While this expression is always valid, application to theories with CP conservation ($\mathcal{M}_{CP} = \mathcal{M}$) results in the relations

$$|\mathcal{M}_{if}| = |\mathcal{M}_{\bar{i}\bar{f}}| \;\;\Rightarrow\;\; \sigma(i\to f) = \sigma(\bar{i}\to\bar{f}) \;\vee\; \Gamma(i\to f) = \Gamma(\bar{i}\to\bar{f}). \tag{A.3}$$

This is the first relation between observables that may be used as a sign of CP violation. Since it applies to cross sections and decays, violation of this conditions is usually referred to as direct CP violation.

[1] We will use the notation $P^0 - \bar{P}^0$ for such mixing, where $P^0 = i\bar{j}$ and $\bar{P}^0 = \bar{i}j$.

[2] The correspondence principle only sets $(CP)^2 = e^{i\alpha}$ with α an arbitrary phase. However, we can simply rephase the physical states to absorve α, thus obtaining $(CP)^2 = 1$ without loss of generality.

The remaining experimental methods of detecting CP violation rely on free propagation of CP conjugated states. To study such systems, we work in the Wigner-Weisskopf approximation [51]. In it, the beam is described by the wave function

$$|\psi(t)\rangle = \psi_1(t)\,|P^0\rangle + \psi_2(t)\,|\bar{P}^0\rangle\,, \tag{A.4}$$

whose time evolution is controlled by a Schrödinger-like equation

$$i\frac{\mathrm{d}}{\mathrm{d}t}\,|\psi(t)\rangle = H\,|\psi(t)\rangle\,. \tag{A.5}$$

Since the mesonic states P^0 and $\bar{P}^0$ may decay, the Hamiltonian H is not Hermitian. Nevertheless, we decompose it as $H = M - \frac{i}{2}\Gamma$, where each component is Hermiitan and computed as follows,

$$\begin{aligned} M = \frac{1}{2}(H + H^\dagger) \leftrightarrow M_{ij} &= m_0\,\delta_{ij} + \mathcal{M}_{ij} + \sum_n \frac{\mathcal{M}_{in}\mathcal{M}_{nj}}{m_0 - E_n}, \\ \Gamma = i(H - H^\dagger) \leftrightarrow \Gamma &= 2\pi \sum_n \delta(m_0 - E_n)\mathcal{M}_{in}\mathcal{M}_{nj}. \end{aligned} \tag{A.6}$$

Since the Hamiltonian is not Hermitian, its eigenvalues μ_H and μ_L are complex observables from which we define $\Delta\mu = \mu_H - \mu_L = \Delta m - \frac{i}{2}\Delta\Gamma$. It can easily be shown [6] that these parameters are obtained from the Hamiltonian as

$$(\Delta m)^2 = \frac{4|M_{12}|^2 - \delta^2|\Gamma_{12}|^2}{1+\delta^2}, \;\; (\Delta\Gamma)^2 = \frac{4|\Gamma_{12}|^2 - 16\delta^2|M_{12}|^2}{1+\delta^2}, \tag{A.7}$$

where we introduced the parameter

$$\delta = \frac{|H_{12}| - |H_{21}|}{|H_{12}| + |H_{21}|} = \frac{2\mathrm{Im}\,[M_{12}^*\Gamma_{12}]}{(\Delta m)^2 + |\Gamma_{12}|^2}. \tag{A.8}$$

Using the transformation laws in (A.1), it is straightforward to prove that CP invariance requires $\delta = 0$. Thus, we have encountered a second type of CP violation, appropriately named CP violation in mixing.

Meanwhile, the eigenstates of the Hamiltonian are given in our convention as

$$|P_H\rangle = p\,|P^0\rangle + q\,|\bar{P}^0\rangle\,, \;\; |P_L\rangle = p\,|P^0\rangle - q\,|\bar{P}^0\rangle\,, \tag{A.9}$$

where we assumed CPT invariance[3] and introduced the parameters p and q as

$$\frac{q}{p} = \sqrt{\frac{H_{21}}{H_{12}}} = \frac{2H_{21}}{\Delta\mu}. \tag{A.10}$$

Finally, we introduce the parameter

$$\lambda_f = \frac{q}{p}\frac{\langle f|\mathcal{M}|\bar{P}^0\rangle}{\langle f|\mathcal{M}|P^0\rangle} \tag{A.11}$$

[3] This assumption is justified by the notion that all local QFTs are invariant under such transformation.

which, if CP is conserved, must satisfy

$$\arg \lambda_f + \arg \lambda_{\bar{f}} = 0. \tag{A.12}$$

We have found the final type of CP violation. Since it mixes propagation ($\frac{q}{p}$) with decays ($\langle f|\mathcal{M}|P^0\rangle$), it is often referred to as interference CP violation.

Out of the three types of CP violation, the most relevant to meson-antimeson mixing is that of CP violation in the mixing.

A.2 Hadronic Matrix Elements

From (A.8) we note that CP violation in the mixing requires the computation of Hadronic Matrix Elements (HMEs) of the form

$$\langle \bar{P}^0 | \left(\bar{j}\Gamma^a i\right)^2 |P^0\rangle \,, \quad \langle \bar{P}^0 | \left[\bar{j}(a+b\gamma_5)i\right]^2 |P^0\rangle \,, \tag{A.13}$$

where Γ^a stands for the elements of an ordered basis of the Dirac space, $\{1, \gamma^\mu, \sigma^{\mu\nu}, \gamma^\mu\gamma_5, \gamma_5\}$. In the 2^{nd} Quantization Formalism [1, 3], we write a quark operator $q = q^+ + q^-$ where q^+ creates an antiquark and q^- destroys a quark. In the same way, we can decompose $\bar{q} = \bar{q}^+ + \bar{q}^-$ with $\bar{q}^+$ adding a quark and $\bar{q}^-$ removing an antiquark when acting on a state. Given that the flavour eigenstates P^0 and P^0 are described using quark operators as $P^0 = i\bar{j}$ and $\bar{P}^0 = \bar{i}j$, it is useful to rewrite the previous HMEs in the following way

$$\begin{aligned}
\langle \bar{P}^0|(\bar{j}\Gamma^a i)^2|P^0\rangle &= 2\,\langle \bar{P}^0|(\bar{j}^+\Gamma^a i^+)(\bar{j}^-\Gamma_a i^-) + (\bar{j}^+\Gamma^a i^-)(\bar{j}^-\Gamma_a i^+)|P^0\rangle \,, \\
\langle \bar{P}^0|[\bar{j}(a+b\gamma_5)i]^2|P^0\rangle &= 2\,\langle \bar{P}^0|[\bar{j}^+(a+b\gamma_5)i^+][\bar{j}^-(a+b\gamma_5)i^-] + [\bar{j}^+(a+b\gamma_5)i^-][\bar{j}^-(a+b\gamma_5)i^+]|P^0\rangle \,.
\end{aligned} \tag{A.14}$$

The Fierz reshuffling theorem [52] for Dirac matrices states

$$(\Gamma^a)_{\alpha\beta}(\Gamma_a)_{\gamma\delta} = \sum_b F_{ab}(\Gamma^b)_{\alpha\delta}(\Gamma_b)_{\gamma\beta}, \tag{A.15}$$

where we introduced F as

$$F = \frac{1}{8}\begin{pmatrix} 2 & 2 & 1 & -2 & 2 \\ 8 & -4 & 0 & -4 & -8 \\ 24 & 0 & -4 & 0 & 24 \\ -8 & -4 & 0 & -4 & 8 \\ 2 & -2 & 1 & 2 & 2 \end{pmatrix}. \tag{A.16}$$

With this relation we can derive the following Fierz identity for matrices of the type $a + b\gamma_5$

$$(a+b\gamma_5)_{\alpha\beta}(a+b\gamma_5)_{\gamma\delta} = \frac{a^2+b^2}{4}\left[\delta_{\alpha\delta}\delta_{\gamma\beta} + (\gamma_5)_{\alpha\delta}(\gamma_5)_{\gamma\beta}\right] + \frac{a^2-b^2}{4}\left[(\gamma^\mu)_{\alpha\delta}(\gamma_\mu)_{\gamma\beta} - (\gamma^\mu\gamma_5)_{\alpha\delta}(\gamma_\mu\gamma_5)_{\gamma\beta}\right]$$
$$+ \frac{a^2+b^2}{8}(\sigma^{\mu\nu})_{\alpha\delta}(\sigma_{\mu\nu})_{\gamma\beta} + \frac{ab}{2}\left[(\gamma_5)_{\alpha\delta}\delta_{\gamma\beta} + \delta_{\alpha\delta}(\gamma_5)_{\gamma\beta}\right] + \frac{ab}{8}\left[(\sigma^{\mu\nu}\gamma_5)_{\alpha\delta}(\sigma_{\mu\nu})_{\gamma\beta} + (\sigma^{\mu\nu})_{\alpha\delta}(\sigma_{\mu\nu}\gamma_5)_{\gamma\beta}\right]. \quad \text{(A.17)}$$

We can also obtain similar Fierz identities for the Gell-Mann matrices [53]. The most important of such relations is

$$\delta_{xy}\delta_{zq} = \frac{1}{3}\delta_{xq}\delta_{zy} + \frac{1}{2}\sum_{c=1}^{8}\lambda^c_{xq}\lambda^c_{zy} \approx \frac{1}{3}\delta_{xq}\delta_{zy}, \quad \text{(A.18)}$$

where we considered that the term $\lambda^c \otimes \lambda^c$ can be neglected. Combining the three types of Fierz identities, we find the relations

$$(\bar{j}^+\Gamma^a i^-)(\bar{j}^-\Gamma_a i^+) = -\frac{1}{3}\sum_b F_{ab}(\bar{j}^+\Gamma^b i^+)(\bar{j}^-\Gamma_b i^-),$$
$$[\bar{j}^+(a+b\gamma_5)i^-][\bar{j}^-(a+b\gamma_5)i^+] = -\frac{a^2+b^2}{12}\Big[(\bar{j}^+i^+)(\bar{j}^-i^-) + (\bar{j}^+\gamma_5 i^+)(\bar{j}^-\gamma_5 i^-)\Big] + \frac{b^2-a^2}{12}\Big[(\bar{j}^+\gamma^\mu i^+)(\bar{j}^-\gamma_\mu i^-)$$
$$- (\bar{j}^+\gamma^\mu\gamma_5 i^+)(\bar{j}^-\gamma_\mu\gamma_5 i^-)\Big] - \frac{a^2+b^2}{24}(\bar{j}^+\sigma^{\mu\nu}i^+)(\bar{j}^-\sigma_{\mu\nu}i^-) - \frac{ab}{6}\Big[(\bar{j}^+\gamma_5 i^+)(\bar{j}^-i^-) + (\bar{j}^+i^+)(\bar{j}^-\gamma_5 i^-)\Big]$$
$$- \frac{ab}{24}\Big[(\bar{j}^+\sigma^{\mu\nu}\gamma_5 i^+)(\bar{j}^-\sigma_{\mu\nu}i^-) + (\bar{j}^+\sigma^{\mu\nu}i^+)(\bar{j}^-\sigma_{\mu\nu}\gamma_5 i^-)\Big], \quad \text{(A.19)}$$

which we use to simplify the initial HMEs

$$\langle\bar{P}^0|(\bar{j}\Gamma^a i)^2|P^0\rangle \approx 2\sum_b\left(\delta_{ab} - \frac{1}{3}F_{ab}\right)\langle\bar{P}^0|\bar{j}\Gamma^a i|0\rangle\langle 0|\bar{j}\Gamma_a i|P^0\rangle,$$
$$\langle\bar{P}^0|\Big[\bar{j}(a+b\gamma_5)i\Big]^2|P^0\rangle \approx \frac{11a^2-b^2}{6}\langle\bar{P}^0|\bar{j}i|0\rangle\langle 0|\bar{j}i|P^0\rangle + \frac{11b^2-a^2}{6}\langle\bar{P}^0|\bar{j}\gamma_5 i|0\rangle\langle 0|\bar{j}\gamma_5 i|P^0\rangle$$
$$+ \frac{5}{3}ab\Big[\langle\bar{P}^0|\bar{j}\gamma_5 i|0\rangle\langle 0|\bar{j}i|P^0\rangle + \langle\bar{P}^0|\bar{j}i|0\rangle\langle 0|\bar{j}\gamma_5 i|P^0\rangle\Big] + \frac{b^2-a^2}{6}\Big[\langle\bar{P}^0|\bar{j}\gamma^\mu i|0\rangle\langle 0|\bar{j}\gamma_\mu i|P^0\rangle \quad \text{(A.20)}$$
$$- \langle\bar{P}^0|\bar{j}\gamma^\mu\gamma_5 i|0\rangle\langle 0|\bar{j}\gamma_\mu\gamma_5 i|P^0\rangle\Big] - \frac{a^2+b^2}{12}\langle\bar{P}^0|\bar{j}\sigma^{\mu\nu}i|0\rangle\langle 0|\bar{j}\sigma_{\mu\nu}|P^0\rangle - \frac{ab}{12}\Big[\langle\bar{P}^0|\bar{j}\sigma^{\mu\nu}\gamma_5|0\rangle$$
$$\cdot\langle 0|\sigma_{\mu\nu}|P^0\rangle + \langle\bar{P}^0|\bar{j}\sigma^{\mu\nu}|0\rangle\langle 0|\sigma_{\mu\nu}\gamma_5|P^0\rangle\Big].$$

We have used the Vacuum Insertion Approximation (VIA) in the derivation of these expressions. This implies that we have neglected all the decay states of P^0 to write $1 = |0\rangle\langle 0|$. Note that such approximation would not be allowed by the $\lambda^c \otimes \lambda^c$ term in (A.18). As such, these are the two key approximations required to compute the HMEs of (A.13).

In order to complete our task, we must understand how to calculate the following entities

$$\langle 0|\bar{j}\Gamma^a i|P^0\rangle, \quad \langle\bar{P}^0|\bar{j}\Gamma^a i|0\rangle. \quad \text{(A.21)}$$

Since we cannot build an antisymmetric tensor with two indices using the metric $g^{\mu\nu}$, the Levi-Civita tensor $\epsilon^{\mu\nu\rho\sigma}$ and the four-momentum of the meson p^μ, we conclude that

$$\langle 0|\bar{j}\sigma^{\mu\nu}i|P^0(p^\alpha)| = \langle\bar{P}^0(p^\alpha)|\bar{j}\sigma^{\mu\nu}i|0\rangle = 0. \quad \text{(A.22)}$$

It can easily be shown that the fermionic operators are transformed under the most general parity transformation [5] to

$$\mathcal{P}q\mathcal{P}^\dagger = e^{i\beta_q}\gamma^0 q. \tag{A.23}$$

Given that the parity of an antiparticle is opposite to that of its particle,

$$\mathcal{P}(CqC^\dagger)\mathcal{P}^\dagger = -e^{-i\beta_q}\gamma^0(CqC^\dagger), \tag{A.24}$$

we obtain the action of the Parity operator in a bound meson state to be[4]

$$\mathcal{P}\,|P^0\rangle = (-1)^{L+1}\,|P^0\rangle\,, \tag{A.25}$$

where L is the orbital angular momentum of the bound state of quarks. Since the most stable configuration of the stable mesons (K^0, D^0, B_d^0 and B_s^0 has $L=0$ [4], we consider P^0 to have negative parity in the rest of this section.

Given that the strong interactions are invariant under parity [4] and the weak interactions were already integrated over in the derivation of the effective Hamiltonian of the system, it is straightforward to use Eqs. A.23-A.25 to prove

$$\langle 0|\bar{j}i|P^0\rangle = \langle 0|\bar{j}\gamma^\mu i|P^0\rangle = \langle \bar{P}^0|\bar{j}i|0\rangle = \langle \bar{P}^0|\bar{j}\gamma^\mu i|0\rangle = 0. \tag{A.26}$$

Since none of the remaining discrete symmetries of the strong interactions (C and T) may be used to force the remaining HMEs to be null, we will consider them different from zero. Thus, we define the fundamental HME

$$\langle 0|\bar{j}\gamma^\mu\gamma_5 i|P^0(p^\alpha)\rangle = -e^{i\varphi}p^\mu f_P, \tag{A.27}$$

where f_P is a real positive constant[5] and φ an abitrary phase. Under the approximation $p^\mu = p_i^\mu + p_{\bar{j}}^\mu$, we compute the remaining HME by contracting (A.31) with p^μ,

$$\langle 0|\bar{j}\gamma_5 i|P^0(p^\alpha)\rangle = -e^{i\varphi}\frac{m_P^2 f_P}{m_i+m_j}. \tag{A.28}$$

Due to the CP invariance of the strong interactions, we can use (2.87)[6] to compute

$$\langle \bar{P}^0(p^\alpha)|\bar{j}\gamma^\mu\gamma_5 i|0\rangle = e^{i(\xi_P+\xi_i-\xi_j-\varphi)}p^\mu f_P,\ \ \langle \bar{P}^0(p^\alpha)|\bar{j}\gamma_5 i|0\rangle = e^{i(\xi_P+\xi_i-\xi_j-\varphi)}\frac{m_P^2 f_P}{m_i+m_j}, \tag{A.29}$$

with which we calculate the combinations of HMEs that appeared in (A.20),

$$\begin{aligned} V_1 &= \langle \bar{P}^0|\bar{j}\gamma_5 i|0\rangle\,\langle 0|\bar{j}\gamma_5 i|P^0\rangle = -e^{i(\xi_P+\xi_i-\xi_j)}\frac{m_P^4 f_P^2}{2m_P(m_i+m_j)^2}, \\ V_2 &= \langle \bar{P}^0|\bar{j}\gamma^\mu\gamma_5 i|0\rangle\,\langle 0|\bar{j}\gamma_\mu\gamma_5 i|P^0\rangle = -e^{i(\xi_P+\xi_i-\xi_j)}\frac{m_P^2 f_P^2}{2m_P}. \end{aligned} \tag{A.30}$$

[4]We have chosen the parity transformation $\beta_i = \beta_j$ in order to remove complex phasis from (A.25)
[5]In order to determine f_P from the theory we would have to solve the strong interactions at low energies.
[6]We must take $K_{ab} = e^{i\theta_a}\delta_{ab}$ since we apply the transformation on the physical quarks that are distinguished by their mass.

We have inserted a denominator of $2m_P$ in the previous expression to correct the normalization of the states, which in QFT is $2E$ particles per unit volume. Finally, we compute (A.13) to be

$$\begin{aligned}\langle \bar{P}^0|(\bar{j}\Gamma^a i)^2|P^0\rangle &\approx -m_P f_P^2\left[\delta_{a4}-\frac{1}{3}F_{a4}+\frac{m_P^2}{(m_i+m_j)^2}\left(\delta_{a5}-\frac{1}{3}F_{a5}\right)\right],\\ \langle \bar{P}^0|\left[\bar{j}(a+b\gamma_5)i\right]^2|P^0\rangle &\approx \frac{m_P f_P^2}{12}\left[b^2-a^2+\frac{m_P^2}{(m_i+m_j)^2}(a^2-11b^2)\right],\end{aligned} \tag{A.31}$$

where we selected the CP transformations such that $\xi_P = \xi_j - \xi_i$.

A.2.1 Determination of f_P

In order to predict the parameters in (A.31) from a theory we must understand how to compute f_P from experience. From the previous section we obtained the identity

$$\langle 0|\bar{j}\gamma^\mu\gamma_L i|P^0(p^\alpha)\rangle = e^{i\varphi}p^\mu\frac{f_P}{2}. \tag{A.32}$$

Using the isospin symmetry of the strong interactions, we use it to obtain

$$\langle 0|\bar{j}\gamma^\mu\gamma_L u_i|P^+(p^\alpha)\rangle = e^{i\varphi}p^\mu\frac{f_P}{2}, \tag{A.33}$$

where u_i is the up-type quark of the generation of the down-type quark i[7]. After normalizing the states with the conventional $2m_P$ factor, we can compute f_P from the following decays

$$\Gamma(P^\pm\to\mu^\pm\nu)+\Gamma(P^\mu\to\mu^\mu\nu\gamma)=\frac{G_F^2|V_{ij}|^2}{8\pi}f_P^2 m_\nu^2 m_P\left(1-\frac{m_\nu^2}{m_P^2}\right)^2. \tag{A.34}$$

Taking into account the experimental values of these decay widths [4], we determine f_P for all stable mesons. The uncertainty on f_{B_d} and f_{B_s} arises from lack of experimental detection of the given branching ratio. Nevertheless, we will assume $f_{B_{d,s}} = f_K$ in this MSc thesis given that we are only interested in its order of magnitude.

Meson	$\Gamma_{P^\pm}$ (MeV)	BR	f_P (MeV)	Δm (MeV)	m^0 (MeV)
K^0 $(d\bar{s})$	5.317×10^{-14}	64.2%	157	3.484×10^{-12}	497.6
D^0 $(c\bar{u})$	6.329×10^{-10}	3.82×10^{-4}	207	6.253×10^{-12}	1865
B_d^0 $(d\bar{b})$	4.018×10^{-10}	$<10^{-5}$	<860	3.337×10^{-10}	5280
B_s^0 $(s\bar{b})$	4.018×10^{-10}	$<10^{-5}$	<860	1.169×10^{-8}	5367

Table A.1: Proprieties of stable meson states.

A.3 Approximations

In the kaon sector, $\Delta m_K \approx -\frac{1}{2}\Delta\Gamma_K$ by accident. Since CP violation is small in the $K^0-\bar{K}^0$ mixing ($\delta_L\sim10^{-3}$), we obtain the following relations from (A.7)

[7] We assumed that both i and j were down-type quarks in the derivation of (A.33). The opposite assumption results in a similar relation with d_i and P^- replacing u_i and P^+, respectively.

$$\Delta m_K \approx 2|M_{12}| \approx -\frac{1}{2}\Delta\Gamma_K \approx |\Gamma_{12}|. \tag{A.35}$$

Replacing these approximations in (A.8) results in

$$\delta_K \approx \frac{\mathrm{Im}\,[M_{12}^*\Gamma_{12}]}{(\Delta m_K)^2}. \tag{A.36}$$

The information contained in δ_K is often presented in ϵ [5], which is given by[8]

$$|\epsilon| \approx \left|\frac{\mathrm{Im}\,[M_{12}]}{\sqrt{2}\Delta m_K}\right| \approx 2.228 \times 10^{-3}, \tag{A.37}$$

where we neglected the ξ_0 phase of the isospin-zero amplitude of two-pion decays of the kaons.

For the B_d and B_s systems, it may be argued that $|\Gamma_{12}| \ll |M_{12}|$ [54, 55]. Thus, we extract the eigenvalues of the B systems from (A.7) to be [6]

$$\Delta m_B \approx 2|M_{12}|, \quad \Delta\Gamma_B \approx 2\frac{\mathrm{Re}\,[M_{12}^*\Gamma_{12}]}{|M_{12}|}, \quad \delta_B \approx \frac{1}{2}\mathrm{Im}\left[\frac{\Gamma_{12}}{M_{12}}\right]. \tag{A.38}$$

Regarding the D^0 meson, the previous approximation ($|\Gamma_{12}| \ll |M_{12}$) does not hold. In fact, its mixing is expected to be dominated by long-range interactions that are mediated by intermediate hadronic states [56, 57]. Nevertheless, new physics are constrained by requiring that the contributions arising from the new short-range interactions do not exceed the experimental bound, i.e., neglecting the long-range interactions. Thus, we are constraining new models by assuming them in a region in which $|M_{12}|\gamma|\Gamma_{12}|$, implying that we may apply $\Delta m_D \approx 2|M_{12}|$ when constraining them.

[8] This result was obtained by working in a basis in which $\mathrm{Im}\,[\Gamma_{12}] = 0$.

Appendix B

Generation of Symmetry Protected Models

In Sec. 2.4.3, we studied the general proprieties of AFS. In particular, we found that there is always a WB in which such symmetries can be written as

$$\phi_a \to e^{i\theta_a}\phi_a, \;\; q_{Li} \to e^{i\alpha_i} q_{Li}, \;\; d_{Rj} \to e^{i\beta_j} d_{Rj}, \;\; u_{Rj} \to e^{i\gamma_j} u_{Rj}, \tag{B.1}$$

where α_i, β_j and γ_j leave in the linear space $\mathcal{F}$ spanned by the θ_a coefficients. As a result of implementing AFS, the Yukawa couplings are forced to satisfy

$$(\Gamma_a)_{ij} = e^{i(\beta_j - \alpha_i + \theta_a)}(\Gamma_a)_{ij}, \;\; (\Delta_a)_{ij} = e^{i(\gamma_j - \alpha_i - \theta_a)}(\Delta_a)_{ij} \tag{B.2}$$

Defining the composed coefficients $\theta_{ij} = \alpha_i - \beta_j$ and $\bar{\theta}_{ij} = \alpha_i - \gamma_j$, we build the matrices

$$\Theta_d = \begin{bmatrix} \theta_{11} & \theta_{12} & \theta_{13} \\ \alpha_2 + \theta_{11} & \alpha_2 + \theta_{12} & \alpha_2 + \theta_{13} \\ \alpha_3 + \theta_{11} & \alpha_3 + \theta_{12} & \alpha_3 + \theta_{13} \end{bmatrix}, \;\; \Theta_u = \begin{bmatrix} \bar{\theta}_{11} & \bar{\theta}_{12} & \bar{\theta}_{13} \\ \alpha_2 + \bar{\theta}_{11} & \alpha_2 + \bar{\theta}_{12} & \alpha_2 + \bar{\theta}_{13} \\ \alpha_3 + \bar{\theta}_{11} & \alpha_3 + \bar{\theta}_{12} & \alpha_3 + \bar{\theta}_{13} \end{bmatrix}. \tag{B.3}$$

The flavour textures of MHDMs invariant under AFS are obtained as

$$(\Theta_d)_{ij} \neq \theta_a + 2\pi k \Rightarrow (\Gamma_a)_{ij} = 0, \quad (\Theta_u)_{ij} \neq -\theta_a + 2\pi k \Rightarrow (\Delta)_{ij} = 0, \;\; k \in \mathbb{Z}. \tag{B.4}$$

From (2.69) we determine the need for eight parameters in order to identify an AFS. Considering only models in which there are no massless quarks restricts those parameters to live in $\mathcal{F}$[1].

An additional reduction of the available parameter space is obtained by requiring $V_{ij} \neq 0$ [4]. Noting that the implementation of an AFS results in

[1]When we are interested in finding models with UFV, we take all θ_a to be different. Thus, $\mathcal{F}$ becomes a linear space of dimension $N-1$. Special cases in which several θ_a differ but are not linearly independent are treated individually.

$$(\Gamma_a\Gamma_b^\dagger)_{ij} = e^{i(\theta_a-\theta_b+\alpha_j-\alpha_i)}(\Gamma_a\Gamma_b^\dagger)_{ij}, \;\; (\Delta_a\Delta_b^\dagger)_{ij} = e^{i(\theta_b-\alpha_a+\alpha_j-\alpha_i)}(\Gamma_a\Gamma_b^\dagger)_{ij}, \tag{B.5}$$

we conclude that the off-diagonal terms of H_q are null if $\alpha_j-\alpha_i \neq \pm(\theta_b-\alpha_a)$. Given that V has always a zero if both H_d and H_u are block diagonal, we conclude that at least two $\alpha_j - \alpha_i$ must equal $\pm(\theta_b - \alpha_a)$ in order for the resulting model to be in agreement with experiment. Applying both restrictions ($V_{ij} \neq 0$ and $\Pi_i m_{q_i} \neq 0$) produces a sufficiently small space of potentially valid symmetry protected 2HDM[2] to cover computationally. As such, we have obtained the full set of 2HDM that arise from the application of an AFS[3].

[2]While these restrictions are obviously appliable to all MHDM, we only studied 2HDM since the allowed space grows with N.
[3]We consider models related by a permutation of q_L^0, d_R^0, u_R^0 and/or ψ_a to be identical.

Appendix C

Mathematical Relations

In this Annex, we demonstrate the following mathematical proprieties

$$f_i = \sum_{j,k=1}^{3} \left|U^*_{ki} U_{kj}\right|^2 = 1, \quad g_{ij} = \sum_{k,m,n=1}^{3} \left|U^*_{mi} U_{mk} V^*_{nk} V_{nj} \pm U_{mj} U^*_{mk} V_{nk} V^*_{ni}\right|^2 \leq 20, \tag{C.1}$$

where U and V are 3×3 unitary matrices. The second relation was instrumental in the derivation of (4.19), while the first is needed for its demonstration and the derivation of (4.32). In the rest of this Annex, we will use the followign parametrization of the most general 3×3 unitary matrix

$$W = \begin{bmatrix} e^{i\alpha_1} \cos x \cos y & -e^{i(\alpha_1+\alpha_4)}(\sin x \cos z + e^{i\delta} \cos x \sin y \sin z) & e^{i(\alpha_1+\alpha_5)}(\sin x \sin z - e^{i\delta} \cos x \sin y \cos z) \\ e^{i\alpha_2} \sin x \cos y & e^{i(\alpha_2+\alpha_4)}(\cos x \cos z - e^{i\delta} \sin x \sin y \sin z) & -e^{i(\alpha_2+\alpha_5)}(\cos x \sin z + e^{i\delta} \sin x \sin y \cos z) \\ e^{i(\alpha_3-\delta)} \sin y & e^{i(\alpha_3+\alpha_4)} \cos y \sin z & e^{i(\alpha_3+\alpha_5)} \cos y \cos z \end{bmatrix}. \tag{C.2}$$

Since PW with P a permutation matrix also parametrizes the most general unitary matrix, we only have to compute

$$f_1 = \sum_{j,k=1}^{3} \left|W^*_{k1} W_{kj}\right|^2 \tag{C.3}$$

to demonstrate the first identity of (C.1). In order to make such demonstration easier, we defined the auxiliary quantities

$$f_{1i} = \sum_{j=1}^{3} \left|W^*_{i1} W_{ij}\right|^2, \tag{C.4}$$

with which we compute $f_i = f_{11} + f_{12} + f_{13}$. Using (C.2) we obtain

$$\begin{aligned} |W^*_{11} W_{12}|^2 &= \frac{1}{4}[\sin^2(2x) \cos^2 y \cos^2 z + \cos^4 x \sin^2(2y) \sin^2 z] + \cos\delta \cos^3 x \sin x \cos^2 y \sin y \sin(2z), \\ |W^*_{11} W_{13}|^2 &= \frac{1}{4}[\sin^2(2x) \cos^2 y \sin^2 z + \cos^4 x \sin^2(2y) \cos^2 z] - \cos\delta \cos^3 x \sin x \cos^2 y \sin y \sin(2z), \\ |W^*_{11} W_{11}|^2 &= \cos^4 x \cos^4 y \Rightarrow f_{11} = \cos^4 x \cos^4 y + \frac{1}{4}[\sin^2(2x) \cos^2 y + \cos^4 x \sin^2(2y)] = \cos^2 x \cos^2 y. \end{aligned} \tag{C.5}$$

Meanwhile, we compute f_{12} to be

$$|W_{21}^*W_{22}|^2 = \frac{1}{4}[\sin^2(2x)\cos^2 y\cos^2 z + \sin^4 x\sin^2(2y)\sin^2 z] - \cos\delta\cos x\sin^3 x\cos^2 y\sin y\sin(2z),$$
$$|W_{21}^*W_{23}|^2 = \frac{1}{4}[\sin^2(2x)\cos^2 y\sin^2 z + \sin^4 x\sin^2(2y)\cos^2 z] + \cos\delta\cos x\sin^3 x\cos^2 y\sin y\sin(2z),$$
$$|W_{21}^*W_{21}|^2 = \sin^4 x\cos^4 y \Rightarrow f_{12} = \sin^4 x\cos^4 y + \frac{1}{4}[\sin^2(2x)\cos^2 y + \sin^4 x\sin^2(2y)] = \sin^2 x\cos^2 y. \tag{C.6}$$

Finally, we compute f_{13}

$$\begin{aligned} |W_{31}^*W_{31}|^2 = \sin^4 y,\;\; |W_{31}^*W_{32}|^2 = \frac{1}{4}\sin^2(2y)\sin^2 z,\;\; |W_{31}^*W_{33}|^2 = \frac{1}{4}\sin^2(2y)\cos^2 z, \\ f_{13} = \sin^4 y + \frac{1}{4}\sin^2(2y) = \sin^2 y. \end{aligned} \tag{C.7}$$

Putting everything together, we demonstrate the first identity of (C.1)

$$f_i \sim f_1 = \cos^2 x\cos^2 y + \sin^2 x\cos^2 y + \sin^2 y = 1. \tag{C.8}$$

We now focus on the second relation of (C.1). Using the triangle inequality, we obtain

$$\begin{aligned} g_{ij} &= \sum_{k,m,n=1}^{3} |U_{mi}^*U_{mk}V_{nk}^*V_{nj} \pm U_{mj}U_{mk}^*V_{nk}V_{ni}^*|^2 \le \sum_{k,m,n=1}^{3} [|U_{mi}^*U_{mk}V_{nk}^*V_{nj}| + |U_{mj}U_{mk}^*V_{nk}V_{ni}^*|]^2 \\ &= \sum_{k,m,n=1}^{3} \left[|U_{mi}^*U_{mk}|^2|V_{nk}^*V_{nj}|^2 + |U_{mj}U_{mk}^*|^2|V_{nk}V_{ni}^*|^2 + 2|U_{mi}^*U_{mk}U_{mj}U_{mk}^*||V_{nk}^*V_{nj}V_{nk}V_{ni}^*|\right] \\ &= \left(\sum_{k,m=1}^{3}|U_{mi}^*U_{mk}|^2\right)\left(\sum_{k,n=1}^{3}|V_{nk}^*V_{nj}|^2\right) + \left(\sum_{k,m=1}^{3}|U_{mj}U_{mk}^*|^2\right)\left(\sum_{k,n=1}^{3}|V_{nk}V_{ni}^*|^2\right) \\ &+ 2\left(\sum_{m=1}^{3}|U_{mi}^*U_{mj}|\right)\left(\sum_{n=1}^{3}|V_{ni}^*V_{nj}|\right)\left(\sum_{k,m=1}^{3}|U_{mk}|^2\right)\left(\sum_{k,n=1}^{3}|V_{nk}|^2\right) \Leftrightarrow \\ g_{ij} &\le f_if_j + f_jf_i + 2a_{ij}^2b^2 = 2\left(1 + a_{ij}^2b^2\right), \end{aligned} \tag{C.9}$$

where we introduced

$$a_{ij} = \sum_{m=1}^{3}|W_{mi}^*W_{mj}|,\;\; b = \sum_{i,j=1}^{3}|W_{ij}|^2. \tag{C.10}$$

We compute b trivially by using the unitarity of W

$$b = \sum_{i=1}^{3}\left(\sum_{j=1}^{3}|W_{ij}|^2\right) = \sum_{i=1}^{3}1 = 3. \tag{C.11}$$

Given that P_1WP_2 with P_i permutation matrices parametrizes the most general unitary matrix, we obtain

$$a_{ij}^2 \sim a_{12}^2 = \left(\sum_{m=1}^{3}|W_{m1}^*W_{m2}|\right)^2. \tag{C.12}$$

To proceed we use the parametrization of W given in (C.2)

$$
\begin{aligned}
&|W_{11}| = \cos x \cos y, \quad |W_{21}| = \sin x \cos y, \quad |W_{31}| = \sin y, \quad |W_{32}| = \cos y \sin z \\
&|W_{12}| = \sqrt{\sin^2 x \cos^2 z + \cos^2 x \sin^2 y \sin^2 z + \frac{1}{2} \cos \delta \sin(2x) \sin y \sin(2z)} \\
&|W_{22}| = \sqrt{\cos^2 x \cos^2 z + \sin^2 x \sin^2 y \sin^2 z - \frac{1}{2} \cos \delta \sin(2x) \sin y \sin(2z)} \\
&a_{12}^2 = \Big(\cos y \sin y \sin z + \cos x \cos y \sqrt{\sin^2 x \cos^2 z + \cos^2 x \sin^2 y \sin^2 z + \frac{1}{2} \cos \delta \sin(2x) \sin y \sin(2z)} \\
&\quad + \sin x \cos y \sqrt{\cos^2 x \cos^2 z + \sin^2 x \sin^2 y \sin^2 z - \frac{1}{2} \cos \delta \sin(2x) \sin y \sin(2z)} \Big)^2 .
\end{aligned}
\tag{C.13}
$$

Using this expression, we have numerically majorated a_{12}^2 to find

$$
a_{ij}^2 \sim a_{12}^2 \leq 1, \tag{C.14}
$$

with which we demonstrate the final relation of (C.1)

$$
g_{ij} \leq 20. \tag{C.15}
$$

Appendix D

List of 2HDM

We present now the full list of symmetry protected 2HDMs which satisfy Eq. **??**. We do not show the models which can be obtained by these from swapping the couplings of the up with the couplings of the down sector. We also indicate the existence of a model where the couplings are swapped in just one sector[1] by placing a star after the name of the model.

D.1 Models with UFV

D.1.1 Left Models

BGL models

BGL models are left models identified by $\vec{l}_d = \alpha(1, 1, \beta)$, $\vec{r}_d = \alpha^{-1}(t_\beta, t_\beta, -\beta^{-1} t_\beta^{-1})$, $\vec{l}_u = \gamma(t_\beta, t_\beta, -t_\beta^{-1})$ and $\vec{r}_u = \gamma^{-1}(1, 1, 1)$. They can be implemented by the following symmetry,

$$q_L^0 \to \mathrm{diag}(1, 1, \Upsilon^{-1}) q_L^0, \quad d_R^0 \to \mathrm{diag}(1, 1, \Upsilon^{-2}) d_R^0, \quad u_R^0 \to u_R^0, \quad \phi_2 \to \Upsilon \phi_2, \tag{D.1}$$

where $\Upsilon^2 \neq 1$. After using (2.70) we obtain the Yukawa textures of these models in this WB,

$$\Gamma_1 = \begin{bmatrix} x & x & 0 \\ x & x & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \Gamma_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & x \end{bmatrix}, \quad \Delta_1 = \begin{bmatrix} x & x & x \\ x & x & x \\ 0 & 0 & 0 \end{bmatrix}, \quad \Delta_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ x & x & x \end{bmatrix}, \tag{D.2}$$

from which it is straightforward to write their non-diagonal couplings as

$$N_d = D_d [t_\beta - (t_\beta + t_\beta^{-1}) P_3], \quad N_u = [t_\beta - (t_\beta + t_\beta^{-1}) V P_3 V^\dagger] D_u. \tag{D.3}$$

Thus, there are no new parameters in the Yukawa sector of BGL models.

[1] $\Gamma_1 \to \Gamma_1$, $\Gamma_2 \to \Gamma_2$, $\Delta_1 \to \Delta_2$, $\Delta_2 \to \Delta_1$.

jBGL Models*

jBGL models are left models defined by $\vec{l}_d = \alpha(t_\beta, t_\beta, -t_\beta^{-1})$, $\vec{r}_d = \alpha^{-1}(1,1,1)$, $\vec{l}_u = \gamma(-t_\beta^{-1}, -t_\beta^{-1}, t_\beta)$ and $\vec{r}_u = \gamma^{-1}(1,1,1)$. They can be implemented by the following symmetry,

$$q_L^0 \to \mathrm{diag}(1,1,\Upsilon)q_L^0, \quad d_R^0 \to d_R^0, \quad u_R^0 \to \Upsilon u_R^0, \quad \phi_2 \to \Upsilon\phi_2, \tag{D.4}$$

where $\Upsilon \neq 1$. It is now trivial to obtain the Yukawa textures of these models in this WB,

$$\Gamma_1 = \begin{bmatrix} x & x & x \\ x & x & x \\ 0 & 0 & 0 \end{bmatrix}, \quad \Gamma_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ x & x & x \end{bmatrix}, \quad \Delta_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ x & x & x \end{bmatrix}, \quad \Delta_2 = \begin{bmatrix} x & x & x \\ x & x & x \\ 0 & 0 & 0 \end{bmatrix}, \tag{D.5}$$

from which we write their non-diagonal couplings,

$$N_d = [t_\beta - (t_\beta + t_\beta^{-1})V^\dagger P_3^{uL} V]D_d, \quad N_u = [-t_\beta^{-1} + (t_\beta + t_\beta^{-1})P_3^{uL}]D_u. \tag{D.6}$$

Thus, there are four new parameters in the Yukawa sector of jBGL models.

D.1.2 Right Models

Type I*

Type I models are left and right models which are identified by $\vec{l}_d = \alpha(1,1,1)$, $\vec{r}_d = \alpha^{-1}t_\beta(1,1,1)$, $\vec{l}_u = \beta(1,1,1)$ and $\vec{r}_u = \beta^{-1}t_\beta(1,1,1)$. They are protected by the following symmetry,

$$q_L^0 \to q_L^0, \quad d_R^0 \to d_R^0, \quad u_R^0 \to u_R^0, \quad \phi_2 \to \Upsilon\phi_2. \tag{D.7}$$

with $\Upsilon \neq 1$. By using (2.70) we obtain the Yukawa textures of these models in this WB,

$$\Gamma_1 = \begin{bmatrix} x & x & x \\ x & x & x \\ x & x & x \end{bmatrix}, \quad \Gamma_2 = 0, \quad \Delta_1 = \begin{bmatrix} x & x & x \\ x & x & x \\ x & x & x \end{bmatrix}, \quad \Delta_2 = 0, \tag{D.8}$$

from which it is straightforward to write their non-diagonal couplings as

$$N_d = t_\beta D_d, \quad N_u = t_\beta D_u. \tag{D.9}$$

Thus, no new parameters arise in the Yukawa sector of Type I models.

Type A*

Type A models are right models which are identified by $\vec{l}_d = \alpha(1,1,1)$, $\vec{r}_d = \alpha^{-1}t_\beta(1,1,1)$, $\vec{l}_u = \gamma(1,1,1)$ and $\vec{r}_u = \gamma^{-1}(t_\beta, t_\beta, -t_\beta^{-1})$. They are protected by the following symmetry,

$$q_L^0 \to q_L^0, \quad d_R^0 \to d_R^0, \quad u_R^0 \to \mathrm{diag}(1,1,\Upsilon)u_R^0, \quad \phi_2 \to \Upsilon\phi_2, \tag{D.10}$$

with $\Upsilon \neq 1$. It is now trivial to obtain the Yukawa textures of these models in this WB,

$$\Gamma_1 = \begin{bmatrix} x & x & x \\ x & x & x \\ x & x & x \end{bmatrix}, \quad \Gamma_2 = 0, \quad \Delta_1 = \begin{bmatrix} x & x & 0 \\ x & x & 0 \\ x & x & 0 \end{bmatrix}, \quad \Delta_2 = \begin{bmatrix} 0 & 0 & x \\ 0 & 0 & x \\ 0 & 0 & x \end{bmatrix}, \tag{D.11}$$

from which it is straightforward to write their non-diagonal couplings as

$$N_d = t_\beta D_d, \quad N_u = D_u[t_\beta - (t_\beta + t_\beta^{-1})P_3^{uR}]. \tag{D.12}$$

As such, only four new parameters arise in the Yukawa sector of Type A models.

Type E*

Type E models are right models which are identified by $\vec{l}_d = \alpha(1,1,1)$, $\vec{r}_d = \alpha^{-1}(t_\beta, t_\beta, -t_\beta^{-1})$, $\vec{l}_u = \gamma(1,1,1)$ and $\vec{r}_u = \gamma^{-1}(t_\beta, t_\beta, -t_\beta^{-1})$. Such configuration is protected by the following symmetry,

$$q_L^0 \to q_L^0, \quad d_R^0 \to \mathrm{diag}(1,1,\Upsilon^{-1})d_R^0, \quad u_R^0 \to \mathrm{diag}(1,1,\Upsilon)u_R^0, \quad \phi_2 \to \Upsilon\phi_2, \tag{D.13}$$

where $\Upsilon \neq 1$. We obtain the Yukawa textures of these models in this WB from (2.70),

$$\Gamma_1 = \begin{bmatrix} x & x & 0 \\ x & x & 0 \\ x & x & 0 \end{bmatrix}, \quad \Gamma_2 = \begin{bmatrix} 0 & 0 & x \\ 0 & 0 & x \\ 0 & 0 & x \end{bmatrix}, \quad \Delta_1 = \begin{bmatrix} x & x & 0 \\ x & x & 0 \\ x & x & 0 \end{bmatrix}, \quad \Delta_2 = \begin{bmatrix} 0 & 0 & x \\ 0 & 0 & x \\ 0 & 0 & x \end{bmatrix}, \tag{D.14}$$

from which we write their non-diagonal couplings as

$$N_d = D_d[t_\beta - (t_\beta + t_\beta^{-1})P_3^{dR}], \quad N_u = D_u[t_\beta - (t_\beta + t_\beta^{-1})P_3^{uR}]. \tag{D.15}$$

Thus, eight new parameters arise in the Yukawa sector of a Type E model.

D.1.3 General models with UFV

Type U1*

Type U1 models are identified by $\vec{l}_d = \alpha(1, t_\beta^{-4}, -t_\beta^{-2})$, $\vec{r}_d = \alpha^{-1}(t_\beta, -t_\beta^{-1}, -t_\beta^3)$, $\vec{l}_u = \gamma(1, t_\beta^4, -t_\beta^2)$ and $\vec{r}_u = \gamma^{-1}(t_\beta, t_\beta^{-3}, -t_\beta^{-1})$. They are protected by the following symmetry,

$$q_L^0 \to \mathrm{diag}(1,\Upsilon^2,\Upsilon)q_L^0, \quad d_R^0 \to \mathrm{diag}(1,\Upsilon^{-1},\Upsilon)d_R^0, \quad u_R^0 \to \mathrm{diag}(1,\Upsilon^2,\Upsilon)u_R^0, \quad \phi_2 \to \Upsilon\phi_2, \tag{D.16}$$

with $\Upsilon^2 \neq 1$ and $\Upsilon^3 \neq 1$. Using (2.70) we obtain the Yukawa textures of these models in this WB,

$$\Gamma_1 = \begin{bmatrix} x & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & x \end{bmatrix}, \quad \Gamma_2 = \begin{bmatrix} 0 & x & 0 \\ 0 & 0 & x \\ x & 0 & 0 \end{bmatrix}, \quad \Delta_1 = \begin{bmatrix} x & 0 & 0 \\ 0 & x & 0 \\ 0 & 0 & x \end{bmatrix}, \quad \Delta_2 = \begin{bmatrix} 0 & 0 & x \\ 0 & 0 & 0 \\ 0 & x & 0 \end{bmatrix}, \tag{D.17}$$

from which it is straightforward to write their non-diagonal couplings as

$$\begin{aligned} N_d &= [V^\dagger P_1^{uL} V + t_\beta^{-4} V^\dagger P_2^{uL} V - t_\beta^{-2} V^\dagger P_3^{uL} V] D_d [t_\beta P_1^{dR} - t_\beta^{-1} P_2^{dR} - t_\beta^3 P_3^{dR}], \\ N_u &= [P_1^{uL} + t_\beta^4 P_2^{uL} - t_\beta^2 P_3^{uL}] D_u [t_\beta P_1^{uR} + t_\beta^{-3} P_2^{uR} - t_\beta^{-1} P_3^{uR}]. \end{aligned} \tag{D.18}$$

Thus, eighteen new parameters arise in the Yukawa sectors of Type U1 models.

Type U2*

Type U2 models are identified by $\vec{l}_d = \alpha(1, t_\beta^{-4}, -t_\beta^{-2})$, $\vec{r}_d = \alpha^{-1}(t_\beta, -t_\beta^{-1}, -t_\beta^3)$, $\vec{l}_u = \gamma(\beta, 1, -t_\beta^{-2})$ and $\vec{r}_u = \gamma^{-1}(\beta^{-1} t_\beta, t_\beta, -t_\beta^{-1})$. Such models are protected by the symmetry,

$$q_L^0 \to \mathrm{diag}(1, \Upsilon^2, \Upsilon) q_L^0, \quad d_R^0 \to \mathrm{diag}(1, \Upsilon^{-1}, \Upsilon) d_R^0, \quad u_R^0 \to \mathrm{diag}(1, \Upsilon^2, \Upsilon^3) u_R^0, \quad \phi_2 \to \Upsilon \phi_2, \tag{D.19}$$

where $\Upsilon^2 \neq 1$ and $\Upsilon^3 \neq 1$. It is then straightforward to write the Yukawa textures in this WB,

$$\Gamma_1 = \begin{bmatrix} x & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & x \end{bmatrix}, \quad \Gamma_2 = \begin{bmatrix} 0 & x & 0 \\ 0 & 0 & x \\ x & 0 & 0 \end{bmatrix}, \quad \Delta_1 = \begin{bmatrix} x & 0 & 0 \\ 0 & x & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \Delta_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & x \\ 0 & x & 0 \end{bmatrix}, \tag{D.20}$$

from which we write their non-diagonal couplings as

$$\begin{aligned} N_d &= [V^\dagger P_1 V + t_\beta^{-4} V^\dagger P_2^{uL} V - t_\beta^{-2} V^\dagger P_3^{uL}] D_d [t_\beta P_1^{dR} - t_\beta^{-1} P_2^{dR} - t_\beta^3 P_3^{dR}], \\ N_u &= [1 - (1 + t_\beta^{-2}) P_3^{uL}] D_u [t_\beta - (t_\beta + t_\beta^{-1}) P_3^{uR}]. \end{aligned} \tag{D.21}$$

The block diagonal form of H_u implies that there are only ten new parameters in the Yukawa sector of Type U2 models.

Type U3*

Type U3 models are identified by $\vec{l}_d = \alpha(1, t_\beta^{-4}, -t_\beta^{-2})$, $\vec{r}_d = \alpha^{-1}(t_\beta, -t_\beta^{-1}, -t_\beta^3)$, $\vec{l}_u = \gamma(-t_\beta^{-2}, -t_\beta^2, 1)$ and $\vec{r}_u = \gamma^{-1}(t_\beta, t_\beta, -t_\beta^{-1})$. They are implemented through the following symmetry,

$$q_L^0 \to \mathrm{diag}(1, \Upsilon^2, \Upsilon) q_L^0, \quad d_R^0 \to \mathrm{diag}(1, \Upsilon^{-1}, \Upsilon) d_R^0, \quad u_R^0 \to \mathrm{diag}(\Upsilon, \Upsilon, \Upsilon^2) u_R^0, \quad \phi_2 \to \Upsilon \phi_2, \tag{D.22}$$

where $\Upsilon^2 \neq 1$ and $\Upsilon^3 \neq 1$. From (2.70) we obtain the Yukawa textures of these models in this WB,

$$\Gamma_1 = \begin{bmatrix} x & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & x \end{bmatrix}, \quad \Gamma_2 = \begin{bmatrix} 0 & x & 0 \\ 0 & 0 & x \\ x & 0 & 0 \end{bmatrix}, \quad \Delta_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & x \\ x & x & 0 \end{bmatrix}, \quad \Delta_2 = \begin{bmatrix} x & x & 0 \\ 0 & 0 & 0 \\ 0 & 0 & x \end{bmatrix}, \tag{D.23}$$

from which we write their non-diagonal couplings as

$$\begin{aligned} N_d &= [V^\dagger P_1^{uL} V + t_\beta^{-4} V^\dagger P_2^{uL} V - t_\beta^{-2} V^\dagger P_3^{uL} V] D_d [t_\beta P_1^{dR} - t_\beta^{-1} P_2^{dR} - t_\beta^3 P_3^{dR}], \\ N_u &= [-t_\beta^{-2} P_1^{uL} - t_\beta^2 P_2^{uL} + P_3^{uL}] D_u [t_\beta - (t_\beta + t_\beta^{-1}) P_3^{uR}]. \end{aligned} \tag{D.24}$$

Thus, sixteen new parameters arise in the Yukawa sector of Type U3 models.

Type U4*

Type U4 models are identified by $\vec{l}_d = \alpha(1, t_\beta^{-4}, -t_\beta^{-2})$, $\vec{r}_d = \alpha^{-1}(t_\beta, -t_\beta^{-1}, -t_\beta^3)$, $\vec{l}_u = \gamma(-t_\beta^{-1}, -\beta t_\beta^{-1}, t_\beta)$ and $\vec{r}_u = \gamma^{-1}(1, 1, \beta^{-1})$. They are protected by the following symmetry,

$$q_L^0 \to \mathrm{diag}(1, \Upsilon^2, \Upsilon) q_L^0, \quad d_R^0 \to \mathrm{diag}(1, \Upsilon^{-1}, \Upsilon) d_R^0, \quad u_R^0 \to \mathrm{diag}(\Upsilon, \Upsilon, \Upsilon^3) u_R^0, \quad \phi_2 \to \Upsilon \phi_2, \tag{D.25}$$

where $\Upsilon^2 \neq 1$ and $\Upsilon^3 \neq 1$. It is now trivial to obtain the Yukawa textures of these models in this WB,

$$\Gamma_1 = \begin{bmatrix} x & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & x \end{bmatrix}, \quad \Gamma_2 = \begin{bmatrix} 0 & x & 0 \\ 0 & 0 & x \\ x & 0 & 0 \end{bmatrix}, \quad \Delta_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ x & x & 0 \end{bmatrix}, \quad \Delta_2 = \begin{bmatrix} x & x & 0 \\ 0 & 0 & x \\ 0 & 0 & 0 \end{bmatrix}, \tag{D.26}$$

from which it is straightforward to write their non-diagonal couplings as

$$\begin{aligned} N_d &= [V^\dagger P_1^{uL} V + t_\beta^{-4} V^\dagger P_2^{uL} V - t_\beta^{-2} V^\dagger P_3^{uL}] D_d [t_\beta P_1^{dR} - t_\beta^{-1} P_2^{dR} - t_\beta^3 P_3^{dR}], \\ N_u &= [-t_\beta^{-1} + (t_\beta + t_\beta^{-1}) P_3^{uL}] D_u. \end{aligned} \tag{D.27}$$

The block diagonal structure of H_u implies that there are only eight new parameters in the Yukawa sector of Type U4 models.

Type U5*

Type U5 models are identified by $\vec{l}_d = \alpha(1, \beta, -t_\beta^{-2})$, $\vec{r}_d = \alpha^{-1}(t_\beta, -t_\beta^{-1}, \beta^{-1} t_\beta)$, $\vec{l}_u = \gamma(-t_\beta^{-2}, -t_\beta^2, 1)$ and $\vec{r}_u = \gamma^{-1}(t_\beta, t_\beta, -t_\beta^{-1})$. They are implemented by the symmetry

$$q_L^0 \to \mathrm{diag}(1, \Upsilon^2, \Upsilon) q_L^0, \quad d_R^0 \to \mathrm{diag}(1, \Upsilon^{-1}, \Upsilon^2) d_R^0, \quad u_R^0 \to \mathrm{diag}(\Upsilon, \Upsilon, \Upsilon^2) u_R^0, \quad \phi_2 \to \Upsilon \phi_2, \tag{D.28}$$

with $\Upsilon^2 \neq 1$ and $\Upsilon^3 \neq 1$. From (2.70) we obtain the Yukawa textures of these models in this WB,

$$\Gamma_1 = \begin{bmatrix} x & 0 & 0 \\ 0 & 0 & x \\ 0 & 0 & 0 \end{bmatrix}, \quad \Gamma_2 = \begin{bmatrix} 0 & x & 0 \\ 0 & 0 & 0 \\ x & 0 & 0 \end{bmatrix}, \quad \Delta_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & x \\ x & x & 0 \end{bmatrix}, \quad \Delta_2 = \begin{bmatrix} x & x & 0 \\ 0 & 0 & 0 \\ 0 & 0 & x \end{bmatrix}, \tag{D.29}$$

from which it is trivial to write their non-diagonal couplings as

$$\begin{aligned} N_d &= [1 - (1 + t_\beta^{-2}) P_3^{dL}] D_d [t_\beta - (t_\beta + t_\beta^{-1}) P_2^{dR}], \\ N_u &= [-t_\beta^{-2} V P_1^{dL} V^\dagger - t_\beta^2 V P_2 V^\dagger + V P_3^{dL} V^\dagger] D_u [t_\beta - (t_\beta + t_\beta^{-1}) P_3^{uR}]. \end{aligned} \tag{D.30}$$

The block diagonal structure of H_d implies that there are only ten new parameters in the Yukawa sector of Type U5 models.

Type U6*

Type U6 models are identified by $\vec{l}_d = \alpha(1, -t_\beta^2, -t_\beta^{-2})$, $\vec{r}_d = \alpha^{-1}(t_\beta, t_\beta, -t_\beta^{-1})$, $\vec{l}_u = \gamma(1, -t_\beta^{-2}, -t_\beta^2)$, $\vec{r}_u = \gamma^{-1}(-t_\beta^{-1}, -t_\beta^{-1}, t_\beta)$. They are protected by the symmetry

$$q_L^0 \to \mathrm{diag}(1, \Upsilon^{-1}, \Upsilon) q_L^0, \quad d_R^0 \to \mathrm{diag}(1, 1, \Upsilon^{-1}) d_R^0, \quad u_R^0 \to \mathrm{diag}(\Upsilon, \Upsilon, 1) u_R^0, \quad \phi_2 \to \Upsilon \phi_2, \tag{D.31}$$

where $\Upsilon^2 \neq 1$. It is now trivial to obtain the Yukawa textures of these models in this WB,

$$\Gamma_1 = \begin{bmatrix} x & x & 0 \\ 0 & 0 & x \\ 0 & 0 & 0 \end{bmatrix}, \quad \Gamma_2 = \begin{bmatrix} 0 & 0 & x \\ 0 & 0 & 0 \\ x & x & 0 \end{bmatrix}, \quad \Delta_1 = \begin{bmatrix} 0 & 0 & x \\ 0 & 0 & 0 \\ x & x & 0 \end{bmatrix}, \quad \Delta_2 = \begin{bmatrix} x & x & 0 \\ 0 & 0 & x \\ 0 & 0 & 0 \end{bmatrix}, \tag{D.32}$$

from which we write their non-diagonal couplings as

$$\begin{aligned} N_d &= [V^\dagger P_1^{uL} V - t_\beta^2 V^\dagger P_2^{uL} V - t_\beta^{-2} V^\dagger P_3^{uL} V] D_d [t_\beta - (t_\beta + t_\beta^{-1}) P_3^{dR}] \\ N_u &= [P_1^{uL} - t_\beta^{-2} P_2^{uL} - t_\beta^2 P_3^{uL}] D_u [-t_\beta^{-1} + (t_\beta + t_\beta^{-1}) P_3^{uR}]. \end{aligned} \tag{D.33}$$

As such, there are fourteen new parameters in the Yukawa sector of Type U6 models.

Type U7*

Type U7 models are identified by $\vec{l}_d = \alpha(-t_\beta^2, -t_\beta^{-2}, 1)$, $\vec{r}_d = \alpha^{-1}(-t_\beta^{-1}, -t_\beta^{-1}, t_\beta)$, $\vec{l}_u = \gamma(-t_\beta^{-1}, -\beta t_\beta^{-1}, t_\beta)$, $\vec{r}_u = \gamma^{-1}(1, 1, -\beta^{-1} t_\beta^2)$. They are protected by the following symmetry,

$$q_L^0 \to \mathrm{diag}(1, \Upsilon^2, \Upsilon) q_L^0, \quad d_R^0 \to \mathrm{diag}(1, 1, \Upsilon) d_R^0, \quad u_R^0 \to \mathrm{diag}(\Upsilon, \Upsilon, \Upsilon^{-1}) u_R^0, \quad \phi_2 \to \Upsilon \phi_2, \tag{D.34}$$

where $\Upsilon^2 \neq 1$ and $\Upsilon^3 \neq 1$. Using (2.70), we obtain the Yukawa textures of these models,

$$\Gamma_1 = \begin{bmatrix} x & x & 0 \\ 0 & 0 & 0 \\ 0 & 0 & x \end{bmatrix}, \quad \Gamma_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & x \\ x & x & 0 \end{bmatrix}, \quad \Delta_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & x \\ x & x & 0 \end{bmatrix}, \quad \Delta_2 = \begin{bmatrix} x & x & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \tag{D.35}$$

from which it is straightforward to write their non-diagonal couplings as

$$\begin{aligned} N_d &= [-t_\beta^2 V^\dagger P_1^{uL} V - t_\beta^{-2} V^\dagger P_2^{uL} V + V^\dagger P_3^{uL} V] D_d [-t_\beta^{-1} + (t_\beta + t_\beta^{-1}) P_3^{dR}], \\ N_u &= [t_\beta - (t_\beta + t_\beta^{-1}) P_1^{uL}] D_u. \end{aligned} \tag{D.36}$$

Thus, we conclude that six new parameters arise in the Yukawa sector of Type U7 models.

Type U8

Type U8 models are identified by $\vec{l}_d = \alpha(1,1,\beta)$, $\vec{r}_d = \alpha^{-1}(t_\beta, t_\beta, -\beta^{-1}t_\beta^{-1})$, $\vec{l}_u = \gamma(t_\beta, t_\beta, -t_\beta^{-1})$ and $\vec{r}_u = \gamma^{-1}(1, -t_\beta^{-2}, -t_\beta^2)$. They are protected by the following Abelian flavour symmetry,

$$q_L^0 \to \mathrm{diag}(1,1,\Upsilon^{-1})q_L^0, \quad d_R^0 \to \mathrm{diag}(1,1,\Upsilon^{-2})d_R^0, \quad u_R^0 \to \mathrm{diag}(1,\Upsilon,\Upsilon^{-1})u_R^0, \quad \phi_2 \to \Upsilon\phi_2, \tag{D.37}$$

where $\Upsilon^2 \neq 1$. It is now trivial to obtain the Yukawa textures of these models in this WB,

$$\Gamma_1 = \begin{bmatrix} x & x & 0 \\ x & x & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \Gamma_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & x \end{bmatrix}, \quad \Delta_1 = \begin{bmatrix} x & 0 & 0 \\ x & 0 & 0 \\ 0 & 0 & x \end{bmatrix}, \quad \Delta_2 = \begin{bmatrix} 0 & x & 0 \\ 0 & x & 0 \\ x & 0 & 0 \end{bmatrix}, \tag{D.38}$$

from which we write their non-diagonal couplings as

$$\begin{aligned} N_d &= D_d[t_\beta - (t_\beta + t_\beta^{-1})P_3] \\ N_u &= [t_\beta - (t_\beta + t_\beta^{-1})VP_3V^\dagger]D_u[P_1^{uR} - t_\beta^{-2}P_2^{uR} - t_\beta^2 P_3^{uR}]. \end{aligned} \tag{D.39}$$

Thus, six new parameters arise in the Yukawa sector of Type U8 models.

Type U9

Type U9 models are identified by $\vec{l}_d = \alpha(1,1,\beta)$, $\vec{r}_d = \alpha^{-1}(t_\beta, t_\beta, -\beta^{-1}t_\beta^{-1})$, $\vec{l}_u = \gamma(1,1,-t_\beta^{-2})$ and $\vec{r}_u = \gamma^{-1}(-t_\beta^{-1}, -t_\beta^{-1}, t_\beta)$. They can be implemented through the following symmetry,

$$q_L^0 \to \mathrm{diag}(1,1,\Upsilon^{-1})q_L^0, \quad d_R^0 \to \mathrm{diag}(1,1,\Upsilon^{-2})d_R^0, \quad u_R^0 \to \mathrm{diag}(\Upsilon,\Upsilon,1)u_R^0, \quad \phi_2 \to \Upsilon\phi_2, \tag{D.40}$$

where $\Upsilon^2 \neq 1$. Using (2.70) we obtain the Yukawa textures of these models in this WB,

$$\Gamma_1 = \begin{bmatrix} x & x & 0 \\ x & x & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \Gamma_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & x \end{bmatrix}, \quad \Delta_1 = \begin{bmatrix} 0 & 0 & x \\ 0 & 0 & x \\ 0 & 0 & 0 \end{bmatrix}, \quad \Delta_2 = \begin{bmatrix} x & x & 0 \\ x & x & 0 \\ 0 & 0 & x \end{bmatrix}, \tag{D.41}$$

from which it is straightforward to write their non-diagonal couplings as

$$N_d = D_d[t_\beta - (t_\beta + t_\beta^{-1})P_3], \quad N_u = [1 - (1 + t_\beta^{-2})VP_3V^\dagger]D_u[-t_\beta^{-1} + (t_\beta + t_\beta^{-1})P_3^{uR}]. \tag{D.42}$$

Thus, four new parameters arise in the Yukawa sector of a Type U9 models.

Type U10

Type U10 models are identified by $\vec{l}_d = \alpha(1,1,\beta)$, $\vec{r}_d = \alpha^{-1}(t_\beta, t_\beta, -\beta^{-1}t_\beta^{-1})$, $\vec{l}_u = \gamma(-t_\beta^2, -t_\beta^2, 1)$ and $\vec{r}_u = \gamma^{-1}(-t_\beta^{-1}, -t_\beta^{-1}, t_\beta)$. They are implemented by the following symmetry,

$$q_L^0 \to \mathrm{diag}(1,1,\Upsilon^{-1})q_L^0, \quad d_R^0 \to \mathrm{diag}(1,1,\Upsilon^{-2})d_R^0, \quad u_R^0 \to \mathrm{diag}(1,1,\Upsilon^{-1})u_R^0, \quad \phi_2 \to \Upsilon\phi_2, \tag{D.43}$$

where $\Upsilon^2 \neq 1$. Using (2.70) we obtain the Yukawa textures of these models in this WB,

$$\Gamma_1 = \begin{bmatrix} x & x & 0 \\ x & x & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \Gamma_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & x \end{bmatrix}, \quad \Delta_1 = \begin{bmatrix} x & x & 0 \\ x & x & 0 \\ 0 & 0 & x \end{bmatrix}, \quad \Delta_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ x & x & 0 \end{bmatrix}, \tag{D.44}$$

from which it is straightforward to write their non-diagonal couplings as

$$N_d = D_d[t_\beta - (t_\beta + t_\beta^{-1})P_3], \quad N_u = [-t_\beta^2 + (1 + t_\beta^2)VP_3V^\dagger]D_u[-t_\beta^{-1} + (t_\beta + t_\beta^{-1})P_3^{uR}]. \tag{D.45}$$

As such, we conclude that four new parameters arise in the Yukawa sector of Type U10 models.

Type U11

Type U11 models are identified by $\vec{l}_d = \alpha(1, 1, \beta)$, $\vec{r}_d = \alpha^{-1}(t_\beta, t_\beta, -\beta^{-1}t_\beta^{-1})$, $\vec{l}_u = \gamma(1, 1, -t_\beta^{-2})$ and $\vec{r}_u = \gamma^{-1}(t_\beta, t_\beta, -t_\beta^{-1})$. They are implemented by the following symmetry,

$$q_L^0 \to \text{diag}(1, 1, \Upsilon^{-1})q_L^0, \quad d_R^0 \to \text{diag}(1, 1, \Upsilon^{-2})d_R^0, \quad u_R^0 \to \text{diag}(1, 1, \Upsilon)u_R^0, \quad \phi_2 \to \Upsilon\phi_2, \tag{D.46}$$

where $\Upsilon^2 \neq 1$. Using (2.70) we obtain the Yukawa textures of these models in this WB,

$$\Gamma_1 = \begin{bmatrix} x & x & 0 \\ x & x & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \Gamma_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & x \end{bmatrix}, \quad \Delta_1 = \begin{bmatrix} x & x & 0 \\ x & x & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \Delta_2 = \begin{bmatrix} 0 & 0 & x \\ 0 & 0 & x \\ x & x & 0 \end{bmatrix}, \tag{D.47}$$

from which it is straightforward to write their non-diagonal couplings as

$$N_d = D_d[t_\beta - (t_\beta + t_\beta^{-1})P_3], \quad N_u = [1 - (1 + t_\beta^{-2})VP_3V^\dagger]D_u[t_\beta - (t_\beta + t_\beta^{-1})P_3^{uR}]. \tag{D.48}$$

Thus, we see that four new parameters arise in the Yukawa sector of Type U11 models.

Type U12

Type U12 models are identified by $\vec{l}_d = \alpha(t_\beta, t_\beta, -t_\beta^3)$, $\vec{r}_d = \alpha^{-1}(1, -t_\beta^{-2}, t_\beta^{-4})$, $\vec{l}_u = \gamma(t_\beta, t_\beta, -t_\beta^{-1})$ and $\vec{r}_u = \gamma^{-1}(-t_\beta^{-2}, 1, -t_\beta^2)$. These models can be obtained from the following symmetry,

$$q_L^0 \to \text{diag}(1, 1, \Upsilon^{-1})q_L^0, \quad d_R^0 \to \text{diag}(1, \Upsilon^{-1}, \Upsilon^{-2})d_R^0, \quad u_R^0 \to \text{diag}(\Upsilon, 1, \Upsilon^{-1})u_R^0, \quad \phi_2 \to \Upsilon\phi_2, \tag{D.49}$$

where $\Upsilon^2 \neq 1$. From (2.70) we obtain the Yukawa textures of these models in this WB,

$$\Gamma_1 = \begin{bmatrix} x & 0 & 0 \\ x & 0 & 0 \\ 0 & x & 0 \end{bmatrix}, \quad \Gamma_2 = \begin{bmatrix} 0 & x & 0 \\ 0 & x & 0 \\ 0 & 0 & x \end{bmatrix}, \quad \Delta_1 = \begin{bmatrix} 0 & x & 0 \\ 0 & x & 0 \\ 0 & 0 & x \end{bmatrix}, \quad \Delta_2 = \begin{bmatrix} x & 0 & 0 \\ x & 0 & 0 \\ 0 & x & 0 \end{bmatrix}, \tag{D.50}$$

from which we obtain their non-diagonal couplings as follows,

$$
\begin{aligned}
N_d &= [t_\beta - (t_\beta + t_\beta^3)V^\dagger P_3^{uL} V] D_d [P_1^{dR} - t_\beta^{-2} P_2^{dR} + t_\beta^{-4} P_3^{dR}], \\
N_u &= [t_\beta - (t_\beta + t_\beta^{-1}) P_3^{uL}] D_u [-t_\beta^{-2} P_1^{uR} + P_2^{uR} - t_\beta^2 P_3^{uR}].
\end{aligned} \tag{D.51}
$$

Thus, sixteen new parameters arise in the Yukawa sector of Type U12 models.

Type U13

Type U13 models are defined by $\vec{l}_d = \alpha(t_\beta, t_\beta, -t_\beta^3)$, $\vec{r}_d = \alpha^{-1}(1, -t_\beta^{-2}, t_\beta^{-4})$, $\vec{l}_u = \gamma(1, 1, -t_\beta^{-2})$ and $\vec{r}_u = \gamma^{-1}(-t_\beta^{-1}, -t_\beta^{-1}, t_\beta)$. They can be implemented through the following symmetry,

$$
q_L^0 \to \mathrm{diag}(1, 1, \Upsilon^{-1}) q_L^0, \quad d_R^0 \to \mathrm{diag}(1, \Upsilon^{-1}, \Upsilon^{-2}) d_R^0, \quad u_R^0 \to \mathrm{diag}(\Upsilon, \Upsilon, 1) u_R^0, \quad \phi_2 \to \Upsilon \phi_2, \tag{D.52}
$$

where $\Upsilon^2 \neq 1$. Using (2.70) we obtain the Yukawa textures of these models in this WB,

$$
\Gamma_1 = \begin{bmatrix} x & 0 & 0 \\ x & 0 & 0 \\ 0 & x & 0 \end{bmatrix}, \quad \Gamma_2 = \begin{bmatrix} 0 & x & 0 \\ 0 & x & 0 \\ 0 & 0 & x \end{bmatrix}, \quad \Delta_1 = \begin{bmatrix} 0 & 0 & x \\ 0 & 0 & x \\ 0 & 0 & 0 \end{bmatrix}, \quad \Delta_2 = \begin{bmatrix} x & x & 0 \\ x & x & 0 \\ 0 & 0 & x \end{bmatrix}, \tag{D.53}
$$

from which it is straightforward to write their non-diagonal couplings in the following way,

$$
\begin{aligned}
N_d &= [t_\beta - (t_\beta + t_\beta^3)V^\dagger P_3^{uL} V] D_u [P_1^{dR} - t_\beta^{-2} P_2^{dR} + t_\beta^{-4} P_3^{dR}], \\
N_u &= [1 - (1 + t_\beta^{-2}) P_3^{uL}] D_u [-t_\beta^{-1} + (t_\beta + t_\beta^{-1}) P_3^{uR}].
\end{aligned} \tag{D.54}
$$

As such, we conclude that fourteen new parameters arise in the Yukawa texture of Type U13 models.

Type U14

Type U14 models are identified by $\vec{l}_d = \alpha(t_\beta, t_\beta, -t_\beta^3)$, $\vec{r}_d = \alpha^{-1}(1, -t_\beta^{-2}, t_\beta^{-4})$, $\vec{l}_u = \gamma(-t_\beta^2, -t_\beta^2, 1)$ and $\vec{r}_u = \gamma^{-1}(-t_\beta^{-1}, -t_\beta^{-1}, t_\beta)$. They are protected by the following symmetry,

$$
q_L^0 \to \mathrm{diag}(1, 1, \Upsilon^{-1}) q_L^0, \quad d_R^0 \to \mathrm{diag}(1, \Upsilon^{-1}, \Upsilon^{-2}) d_R^0, \quad u_R^0 \to \mathrm{diag}(1, 1, \Upsilon^{-1}) u_R^0, \quad \phi_2 \to \Upsilon \phi_2, \tag{D.55}
$$

where $\Upsilon^2 \neq 1$. As a result, the Yukawa textures of these models are written in this WB as

$$
\Gamma_1 = \begin{bmatrix} x & 0 & 0 \\ x & 0 & 0 \\ 0 & x & 0 \end{bmatrix}, \quad \Gamma_2 = \begin{bmatrix} 0 & x & 0 \\ 0 & x & 0 \\ 0 & 0 & x \end{bmatrix}, \quad \Delta_1 = \begin{bmatrix} x & x & 0 \\ x & x & 0 \\ 0 & 0 & x \end{bmatrix}, \quad \Delta_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ x & x & 0 \end{bmatrix}. \tag{D.56}
$$

From this textures, it is straightforward to write their non-diagonal couplings as

$$
\begin{aligned}
N_d &= [t_\beta - (t_\beta + t_\beta^3)V^\dagger P_3^{uL} V] D_d [P_1^{dR} - t_\beta^{-2} P_2^{dR} + t_\beta^{-4} P_3^{dR}], \\
N_u &= [-t_\beta^2 + (1 + t_\beta^2) P_3^{uL}] D_u [-t_\beta^{-1} + (t_\beta + t_\beta^{-1}) P_3^{uR}],
\end{aligned} \tag{D.57}
$$

implying that fourteen new parameters arise in the Yukawa sector of Type U14 models.

Type U15

Type U15 models are defined by $\vec{l}_d = \alpha(t_\beta, t_\beta, -t_\beta^3)$, $\vec{r}_d = \alpha^{-1}(1, -t_\beta^{-2}, t_\beta^{-4})$, $\vec{l}_u = \gamma(1, 1, -t_\beta^{-2})$ and $\vec{r}_u = \gamma^{-1}(t_\beta, t_\beta, -t_\beta^{-1})$. They can be implemented through the following symmetry,

$$q_L^0 \to \mathrm{diag}(1, 1, \Upsilon^{-1}) q_L^0, \quad d_R^0 \to \mathrm{diag}(1, \Upsilon^{-1}, \Upsilon^{-2}) d_R^0, \quad u_R^0 \to \mathrm{diag}(1, 1, \Upsilon) u_R^0, \quad \phi_2 \to \Upsilon \phi_2, \tag{D.58}$$

where $\Upsilon^2 \neq 1$. By applying (2.70) we obtain the Yukawa textures of these models in this WB,

$$\Gamma_1 = \begin{bmatrix} x & 0 & 0 \\ x & 0 & 0 \\ 0 & x & 0 \end{bmatrix}, \quad \Gamma_2 = \begin{bmatrix} 0 & x & 0 \\ 0 & x & 0 \\ 0 & 0 & x \end{bmatrix}, \quad \Delta_1 = \begin{bmatrix} x & x & 0 \\ x & x & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \Delta_2 = \begin{bmatrix} 0 & 0 & x \\ 0 & 0 & x \\ x & x & 0 \end{bmatrix}, \tag{D.59}$$

from which we write their non-diagonal couplings as

$$\begin{aligned} N_d &= [t_\beta - (t_\beta + t_\beta^3) V^\dagger P_3^{uL} V] D_d [P_1^{dR} - t_\beta^{-2} P_2^{dR} + t_\beta^{-4} P_3^{dR}], \\ N_u &= [1 - (1 + t_\beta^{-2}) P_3^{uL}] D_u [t_\beta - (t_\beta + t_\beta^{-1}) P_3^{uR}]. \end{aligned} \tag{D.60}$$

As such, fourteen new parameters arise in the Yukawa sector of Type U15 models.

Type U16

Type U16 models are defined by $\vec{l}_d = \alpha(t_\beta, t_\beta, -t_\beta^3)$, $\vec{r}_d = \alpha^{-1}(1, -t_\beta^{-2}, t_\beta^{-4})$, $\vec{l}_u = \gamma(t_\beta, t_\beta, -t_\beta^{-1})$ and $\vec{r}_u = \gamma^{-1}(1, 1, 1)$. They are protected by the following Abelian symmetry,

$$q_L^0 \to \mathrm{diag}(1, 1, \Upsilon^{-1}) q_L^0, \quad d_R^0 \to \mathrm{diag}(1, \Upsilon^{-1}, \Upsilon^{-2}) d_R^0, \quad u_R^0 \to u_R^0, \quad \phi_2 \to \Upsilon \phi_2, \tag{D.61}$$

where $\Upsilon^2 \neq 1$. It is now trivial to obtain the Yukawa textures of these models in this WB,

$$\Gamma_1 = \begin{bmatrix} x & 0 & 0 \\ x & 0 & 0 \\ 0 & x & 0 \end{bmatrix}, \quad \Gamma_2 = \begin{bmatrix} 0 & x & 0 \\ 0 & x & 0 \\ 0 & 0 & x \end{bmatrix}, \quad \Delta_1 = \begin{bmatrix} x & x & x \\ x & x & x \\ 0 & 0 & 0 \end{bmatrix}, \quad \Delta_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ x & x & x \end{bmatrix}, \tag{D.62}$$

from which we obtain their non-diagonal couplings to be

$$\begin{aligned} N_d &= [t_\beta - (t_\beta + t_\beta^{-1}) V^\dagger P_3^{uL} V] D_d [P_1^{dR} - t_\beta^{-2} P_2^{dR} - t_\beta^2 P_3^{dR}], \\ N_u &= [t_\beta - (t_\beta + t_\beta^{-1}) P_3^{uL}] D_u. \end{aligned} \tag{D.63}$$

Thus, ten new parameters arise in the Yukawa sector of Type U16 models.

Type U17

Type U17 models are identified by $\vec{l}_d = \alpha(1, 1, -t_\beta^{-2})$, $\vec{r}_d = \alpha^{-1}(t_\beta, -t_\beta^{-1}, -t_\beta^{-1})$, $\vec{l}_u = \gamma(-t_\beta^{-1}, -t_\beta^{-1}, t_\beta)$ and $\vec{r}_u = \gamma^{-1}(-t_\beta^2, -t_\beta^2, 1)$. They can be implemented through the following symmetry,

$$q_L^0 \to \text{diag}(1,1,\Upsilon)q_L^0, \quad d_R^0 \to \text{diag}(1,\Upsilon^{-1},\Upsilon^{-1})d_R^0, \quad u_R^0 \to \text{diag}(1,1,\Upsilon)u_R^0, \quad \phi_2 \to \Upsilon\phi_2, \tag{D.64}$$

where $\Upsilon^2 \neq 1$. From (2.70) we obtain the Yukawa textures of these models in this WB,

$$\Gamma_1 = \begin{bmatrix} x & 0 & 0 \\ x & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \Gamma_2 = \begin{bmatrix} 0 & x & x \\ 0 & x & x \\ x & 0 & 0 \end{bmatrix}, \quad \Delta_1 = \begin{bmatrix} x & x & 0 \\ x & x & 0 \\ 0 & 0 & x \end{bmatrix}, \quad \Delta_2 = \begin{bmatrix} 0 & 0 & x \\ 0 & 0 & x \\ 0 & 0 & 0 \end{bmatrix}, \tag{D.65}$$

from which we write their non-diagonal couplings as

$$\begin{aligned} N_d &= [1-(1+t_\beta^{-2})V^\dagger P_3^{uL} V]D_d[-t_\beta^{-1}+(t_\beta+t_\beta^{-1})P_1^{dR}], \\ N_u &= [-t_\beta^{-1}+(t_\beta+t_\beta^{-1})P_3^{uL}]D_u[-t_\beta^{2}+(1+t_\beta^{2})P_3^{uR}]. \end{aligned} \tag{D.66}$$

Thus, twelve new parameters arise in the Yukawa sector of Type U17 models.

Type U18

Type U18 models are identified by $\vec{l}_d = \alpha(1,1,-t_\beta^{-2})$, $\vec{r}_d = \alpha^{-1}(t_\beta,-t_\beta^{-1},-t_\beta^{-1})$, $\vec{l}_u = \gamma(-t_\beta^{-1},-t_\beta^{-1},t_\beta)$ and $\vec{r}_u = \gamma^{-1}(1,1,-t_\beta^{-2})$. They can be implemented by the following symmetry,

$$q_L^0 \to \text{diag}(1,1,\Upsilon)q_L^0, \quad d_R^0 \to \text{diag}(1,\Upsilon^{-1},\Upsilon^{-1})d_R^0, \quad u_R^0 \to \text{diag}(\Upsilon,\Upsilon,\Upsilon^2)u_R^0, \quad \phi_2 \to \Upsilon\phi_2, \tag{D.67}$$

where $\Upsilon^2 \neq 1$. Using (2.70), we obtain the Yukawa textures of these models in this WB,

$$\Gamma_1 = \begin{bmatrix} x & 0 & 0 \\ x & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \Gamma_2 = \begin{bmatrix} 0 & x & x \\ 0 & x & x \\ x & 0 & 0 \end{bmatrix}, \quad \Delta_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ x & x & 0 \end{bmatrix}, \quad \Delta_2 = \begin{bmatrix} x & x & 0 \\ x & x & 0 \\ 0 & 0 & x \end{bmatrix}, \tag{D.68}$$

from which it is straightforward to write their non-diagonal couplings as

$$\begin{aligned} N_d &= [1-(1+t_\beta^{-2})V^\dagger P_3^{uL} V]D_d[-t_\beta^{-1}+(t_\beta+t_\beta^{-1})P_1^{dR}], \\ N_u &= [-t_\beta^{-1}+(t_\beta+t_\beta^{-1})P_3^{uL}]D_u[1-(1+t_\beta^{-2})P_3^{uR}]. \end{aligned} \tag{D.69}$$

As such, twelve new parameters arise in the Yukawa sector of Type U18 models.

Type U19

Type U19 models are defined by $\vec{l}_d = \alpha(1,1,-t_\beta^{-2})$, $\vec{r}_d = \alpha^{-1}(t_\beta,-t_\beta^{-1},-t_\beta^{-1})$, $\vec{l}_u = \gamma(1,1,-t_\beta^{2})$ and $\vec{r}_u = \gamma^{-1}(-t_\beta^{-1},-t_\beta^{-1},t_\beta)$. They are protected by the following symmetry,

$$q_L^0 \to \text{diag}(1,1,\Upsilon)q_L^0, \quad d_R^0 \to \text{diag}(1,\Upsilon^{-1},\Upsilon^{-1})d_R^0, \quad u_R^0 \to \text{diag}(\Upsilon,\Upsilon,1)u_R^0, \quad \phi_2 \to \Upsilon\phi_2, \tag{D.70}$$

where $\Upsilon^2 \neq 1$. Using (2.70) we obtain the Yukawa textures of these models in this WB,

$$\Gamma_1 = \begin{bmatrix} x & 0 & 0 \\ x & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \Gamma_2 = \begin{bmatrix} 0 & x & x \\ 0 & x & x \\ x & 0 & 0 \end{bmatrix}, \quad \Delta_1 = \begin{bmatrix} 0 & 0 & x \\ 0 & 0 & x \\ x & x & 0 \end{bmatrix}, \quad \Delta_2 = \begin{bmatrix} x & x & 0 \\ x & x & 0 \\ 0 & 0 & 0 \end{bmatrix}, \tag{D.71}$$

from which we write their non-diagonal couplings as

$$\begin{aligned} N_d &= [1 - (1 + t_\beta^{-2}) V^\dagger P_3^{uL} V] D_d [-t_\beta^{-1} + (t_\beta + t_\beta^{-1}) P_1^{dR}], \\ N_u &= [1 - (1 + t_\beta^2) P_3^{uL}] D_u [-t_\beta^{-1} + (t_\beta + t_\beta^{-1}) P_3^{uR}]. \end{aligned} \tag{D.72}$$

Thus, we find twelve new parameters in the Yukawa sector of Type U19 models.

Type U20

Type U20 models are defined by $\vec{l}_d = \alpha(1, 1, -t_\beta^{-2})$, $\vec{r}_d = \alpha^{-1}(t_\beta, -t_\beta^{-1}, -t_\beta^{-1})$, $\vec{l}_u = \gamma(-t_\beta^{-1}, -t_\beta^{-1}, t_\beta)$ and $\vec{r}_u = \gamma^{-1}(1, 1, 1)$. They are implemented by the following symmetry,

$$q_L^0 \to \mathrm{diag}(1, 1, \Upsilon) q_L^0, \quad d_R^0 \to \mathrm{diag}(1, \Upsilon^{-1}, \Upsilon^{-1}) d_R^0, \quad u_R^0 \to \Upsilon u_R^0, \quad \phi_2 \to \Upsilon \phi_2, \tag{D.73}$$

where $\Upsilon^2 \neq 1$. It is now trivial to obtain the Yukawa textures of these models in this WB,

$$\Gamma_1 = \begin{bmatrix} x & 0 & 0 \\ x & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \Gamma_2 = \begin{bmatrix} 0 & x & x \\ 0 & x & x \\ x & 0 & 0 \end{bmatrix}, \quad \Delta_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ x & x & x \end{bmatrix}, \quad \Delta_2 = \begin{bmatrix} x & x & x \\ x & x & x \\ 0 & 0 & 0 \end{bmatrix}, \tag{D.74}$$

from which we write their non-diagonal couplings as

$$N_d = [1 - (1 + t_\beta^{-2}) V^\dagger P_3^{uL} V] D_d [-t_\beta^{-1} + (t_\beta + t_\beta^{-1}) P_1^{dR}], \quad N_u = [-t_\beta^{-1} + (t_\beta + t_\beta^{-1}) P_3^{uL}] D_u. \tag{D.75}$$

As such, there are eight new parameters in the Yukawa sector of Type U20 models.

Type U21

Type U21 models are defined by $\vec{l}_d = \alpha(-t_\beta^2, -t_\beta^2, 1)$, $\vec{r}_d = \alpha^{-1}(-t_\beta^{-1}, -t_\beta^{-1}, t_\beta)$, $\vec{l}_u = \gamma(-t_\beta^{-1}, -t_\beta^{-1}, t_\beta)$ and $\vec{r}_u = \gamma^{-1}(1, 1, -t_\beta^{-2})$. They are protected by the following Abelian symmetry,

$$q_L^0 \to \mathrm{diag}(1, 1, \Upsilon) q_L^0, \quad d_R^0 \to \mathrm{diag}(1, 1, \Upsilon) d_R^0, \quad u_R^0 \to \mathrm{diag}(\Upsilon, \Upsilon, \Upsilon^2) u_R^0, \quad \phi_2 \to \Upsilon \phi_2, \tag{D.76}$$

where $\Upsilon^2 \neq 1$. Using (2.70) we obtain the Yukawa textures of these models in this WB,

$$\Gamma_1 = \begin{bmatrix} x & x & 0 \\ x & x & 0 \\ 0 & 0 & x \end{bmatrix}, \quad \Gamma_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ x & x & 0 \end{bmatrix}, \quad \Delta_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ x & x & 0 \end{bmatrix}, \quad \Delta_2 = \begin{bmatrix} x & x & 0 \\ x & x & 0 \\ 0 & 0 & x \end{bmatrix}, \tag{D.77}$$

from which we write their non-diagonal couplings as

$$N_d = [-t_\beta^2 + (1+t_\beta^2)V^\dagger P_3^{uL} V] D_d [-t_\beta^{-1} + (t_\beta + t_\beta^{-1}) P_3^{dR}],$$
$$N_u = [-t_\beta^{-1} + (t_\beta + t_\beta^{-1}) P_3^{uL}] D_u [1 - (1+t_\beta^{-2}) P_3^{uR}]. \tag{D.78}$$

Thus, twelve new parameters arise in the Yukawa sector of a Type U21 model.

Type U22

Type U22 models are defined by $\vec{l}_d = \alpha(-t_\beta^2, -t_\beta^2, 1)$, $\vec{r}_d = \alpha^{-1}(-t_\beta^{-1}, -t_\beta^{-1}, t_\beta)$, $\vec{l}_u = \gamma(1, 1, -t_\beta^2)$ and $\vec{r}_u = \gamma^{-1}(-t_\beta^{-1}, -t_\beta^{-1}, t_\beta)$. They are implemented through the following symmetry,

$$q_L^0 \to \text{diag}(1,1,\Upsilon) q_L^0, \quad d_R^0 \to \text{diag}(1,1,\Upsilon) d_R^0, \quad u_R^0 \to \text{diag}(\Upsilon,\Upsilon,1) u_R^0, \quad \phi_2 \to \Upsilon \phi_2, \tag{D.79}$$

where $\Upsilon^2 \neq 1$. Using (2.70) we obtain the Yukawa textures of these models in this WB,

$$\Gamma_1 = \begin{bmatrix} x & x & 0 \\ x & x & 0 \\ 0 & 0 & x \end{bmatrix}, \quad \Gamma_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ x & x & 0 \end{bmatrix}, \quad \Delta_1 = \begin{bmatrix} 0 & 0 & x \\ 0 & 0 & x \\ x & x & 0 \end{bmatrix}, \quad \Delta_2 = \begin{bmatrix} x & x & 0 \\ x & x & 0 \\ 0 & 0 & 0 \end{bmatrix}. \tag{D.80}$$

from which it is straightforward to write the non-diagonal couplings as

$$N_d = [-t_\beta^2 + (1+t_\beta^2)V^\dagger P_3^{uL} V] D_d [-t_\beta^{-1} + (t_\beta + t_\beta^{-1}) P_3^{dR}],$$
$$N_u = [1 - (1+t_\beta^2) P_3^{uL}] D_u [-t_\beta^{-1} + (t_\beta + t_\beta^{-1}) P_3^{uR}]. \tag{D.81}$$

Thus, there are twelve new parameters in the Yukawa sector of Type U22 models.

Type U23

Type U23 models are identified by $\vec{l}_d = \alpha(-t_\beta^2, -t_\beta^2, 1)$, $\vec{r}_d = \alpha^{-1}(-t_\beta^{-1}, -t_\beta^{-1}, t_\beta)$, $\vec{l}_u = \gamma(-t_\beta^{-1}, -t_\beta^{-1}, t_\beta)$ and $\vec{r}_u = \gamma^{-1}(1, 1, 1)$. They are implemented by the following symmetry,

$$q_L^0 \to \text{diag}(1,1,\Upsilon) q_L^0, \quad d_R^0 \to \text{diag}(1,1,\Upsilon) d_R^0, \quad u_R^0 \to \Upsilon u_R^0, \quad \phi_2 \to \Upsilon \phi_2, \tag{D.82}$$

where $\Upsilon^2 \neq 1$. It is now trivial to obtain the Yukawa textures of these models in this WB,

$$\Gamma_1 = \begin{bmatrix} x & x & 0 \\ x & x & 0 \\ 0 & 0 & x \end{bmatrix}, \quad \Gamma_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ x & x & 0 \end{bmatrix}, \quad \Delta_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ x & x & x \end{bmatrix}, \quad \Delta_2 = \begin{bmatrix} x & x & x \\ x & x & x \\ 0 & 0 & 0 \end{bmatrix}, \tag{D.83}$$

from which we write their non-diagonal couplings as

$$N_d = [-t_\beta^2 + (1+t_\beta^2)V^\dagger P_3^{uL} V] D_d [-t_\beta^{-1} + (t_\beta + t_\beta^{-1}) P_3^{dR}],$$
$$N_u = [-t_\beta^{-1} + (t_\beta + t_\beta^{-1}) P_3^{uL}] D_u. \tag{D.84}$$

Thus, eight new parameters arise in the Yukawa sector of Type U23 models.

Type U24

Type U24 models are defined by $\vec{l}_d = \alpha(1,1,-t_\beta^2)$, $\vec{r}_d = \alpha^{-1}(t_\beta, t_\beta, -t_\beta^{-1})$, $\vec{l}_u = \gamma(1,1,-t_\beta^2)$ and $\vec{r}_u = \gamma^{-1}(-t_\beta^{-1}, -t_\beta^{-1}, t_\beta)$. They can be obtained from the following symmetry,

$$q_L^0 \to \mathrm{diag}(1,1,\Upsilon)q_L^0, \quad d_R^0 \to \mathrm{diag}(1,1,\Upsilon^{-1})d_R^0, \quad u_R^0 \to \mathrm{diag}(\Upsilon,\Upsilon,1)u_R^0, \quad \phi_2 \to \Upsilon\phi_2, \tag{D.85}$$

where $\Upsilon^2 \neq 1$. After applying (2.70), we obtain the Yukawa textures of these models,

$$\Gamma_1 = \begin{bmatrix} x & x & 0 \\ x & x & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \Gamma_2 = \begin{bmatrix} 0 & 0 & x \\ 0 & 0 & x \\ x & x & 0 \end{bmatrix}, \quad \Delta_1 = \begin{bmatrix} 0 & 0 & x \\ 0 & 0 & x \\ x & x & 0 \end{bmatrix}, \quad \Delta_2 = \begin{bmatrix} x & x & 0 \\ x & x & 0 \\ 0 & 0 & 0 \end{bmatrix}, \tag{D.86}$$

from which it is straightforward to write their non-diagonal couplings as

$$\begin{aligned} N_d &= [1-(1+t_\beta^2)V^\dagger P_3^{uL}V]D_d[t_\beta - (t_\beta + t_\beta^{-1})P_3^{dR}], \\ N_u &= [1-(1+t_\beta^2)P_3^{uL}]D_u[-t_\beta^{-1} + (t_\beta + t_\beta^{-1})P_3^{uR}]. \end{aligned} \tag{D.87}$$

As such, twelve new parameters arise in the Yukawa sector of Type U24 models.

Type U25

Type U25 models are defined by $\vec{l}_d = \alpha(1,1,-t_\beta^{-2})$, $\vec{r}_d = \alpha^{-1}(t_\beta, t_\beta, -t_\beta^{-1})$, $\vec{l}_u = \gamma(-t_\beta^{-1}, -t_\beta^{-1}, t_\beta)$ and $\vec{r}_u = \gamma^{-1}(1,1,1)$. They are protected by the following Abelian flavour symmetry,

$$q_L^0 \to \mathrm{diag}(1,1,\Upsilon)q_L^0, \quad d_R^0 \to \mathrm{diag}(1,1,\Upsilon^{-1})d_R^0, \quad u_R^0 \to \Upsilon u_R^0, \quad \phi_2 \to \Upsilon\phi_2, \tag{D.88}$$

where $\Upsilon^2 \neq 1$. Using (2.70), we obtain the Yukawa textures of these models,

$$\Gamma_1 = \begin{bmatrix} x & x & 0 \\ x & x & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \Gamma_2 = \begin{bmatrix} 0 & 0 & x \\ 0 & 0 & x \\ x & x & 0 \end{bmatrix}, \quad \Delta_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ x & x & x \end{bmatrix}, \quad \Delta_2 = \begin{bmatrix} x & x & x \\ x & x & x \\ 0 & 0 & 0 \end{bmatrix}, \tag{D.89}$$

from which we write their non-diagonal couplings,

$$N_d = [1-(1+t_\beta^{-2})V^\dagger P_3^{uL}V]D_d[t_\beta - (t_\beta + t_\beta^{-1})P_3^{dR}], \quad N_u = [-t_\beta^{-1} + (t_\beta + t_\beta^{-1})P_3^{uL}]D_u. \tag{D.90}$$

As such, eight new parameters arise in the Yukawa sector of a Type U25 model.

D.2 Special Models

In Chap. 4, we have shown that all MHDMs with UFV must satisfy (4.11), i.e., all scalars that couple to fermions must transform differently under the symmetry which implements the model. However, it is not true that all models which fulfill (4.11) have UFV. In this section, we list all 2HDMs that satisfy it without UFV.

D.2.1 Type S1*

The first of the special models, Type S1, is implemented by the following Abelian flavour symmetry,

$$q_L^0 \to \mathrm{diag}(1, \Upsilon^{-1}, \Upsilon) q_L^0, \quad d_R^0 \to \mathrm{diag}(1, \Upsilon^{-1}, \Upsilon) d_R^0, \quad u_R^0 \to \mathrm{diag}(1, 1, \Upsilon) u_R^0, \quad \phi_2 \to \Upsilon \phi_2, : \tag{D.91}$$

where $\Upsilon^3 = 1$ but $\Upsilon \neq 1$. Using (2.70), we obtain the Yukawa textures of these models in this WB,

$$\Gamma_1 = \begin{bmatrix} x & 0 & 0 \\ 0 & x & 0 \\ 0 & 0 & x \end{bmatrix}, \quad \Gamma_2 = \begin{bmatrix} 0 & x & 0 \\ 0 & 0 & x \\ x & 0 & 0 \end{bmatrix}, \quad \Delta_1 = \begin{bmatrix} x & x & 0 \\ 0 & 0 & 0 \\ 0 & 0 & x \end{bmatrix}, \quad \Delta_2 = \begin{bmatrix} 0 & 0 & x \\ x & x & 0 \\ 0 & 0 & 0 \end{bmatrix}. \tag{D.92}$$

While Type S1 models do not have UFV, their Yukawa couplings satisfy the following identities,

$$\begin{aligned} N_d^0 - \frac{1}{\sqrt{2}} v_2 (\Gamma_1)_{33} P_3 &= [P_1 - t_\beta^2 P_2 - t_\beta^{-2} P_3] \left[D_d^0 - \frac{1}{\sqrt{2}} v_1 (\Gamma_1)_{33} P_3 \right] [t_\beta P_1 - t_\beta^{-1} + t_\beta^{-3} P_3], \\ N_u^0 &= [P_1 - t_\beta^{-2} P_2 - t_\beta^2 P_3] D_u^0 [t_\beta P_1 + t_\beta P_2 - t_\beta^{-1} P_3]. \end{aligned} \tag{D.93}$$

As such, we can write them in the following way after diagonalizing the mass matrices,

$$\begin{aligned} N_d &= [P_1^{dL} - t_\beta^2 P_2^{dL} - t_\beta^{-2} P_3^{dL}] D_d [t_\beta P_1^{dR} - t_\beta^{-1} P_2^{dR} + t_\beta^{-3} P_3^{dR}] + q (U_{dL}^\dagger P_3 U_{dR}), \\ N_u &= [V P_1^{dL} V^\dagger - t_\beta^{-2} V P_2^{dL} V^\dagger - t_\beta^2 V P_3^{dL} V^\dagger] D_u [t_\beta - (t_\beta + t_\beta^{-1}) P_3^{uR}], \end{aligned} \tag{D.94}$$

where $q = \frac{1}{\sqrt{2}} \frac{v_1^6 + v_2^6}{v_2^5} (\Gamma_1)_{33}$. The presence of the term $U_{dL}^\dagger P_3 U_{dR}$ implies that we can only remove three phasis from one of those unitary matrices. Thus, we conclude that type S1 models have twenty new parameters in their Yukawa sector (four in P_3^{uR}, six in U_{dR}, nine in U_{dL} and q). This includes one parameter in their down sector, q, whose magnitude is not controlled by the symmetry.

D.2.2 Type S2*

The second of the special models, Type S2, is obtained after applying the following symmetry,

$$q_L^0 \to \mathrm{diag}(1, \Upsilon^{-1}, \Upsilon) q_L^0, \quad d_R^0 \to \mathrm{diag}(1, \Upsilon^{-1}, \Upsilon) d_R^0, \quad u_R^0 \to \mathrm{diag}(1, \Upsilon^{-1}, \Upsilon) u_R^0, \quad \phi_2 \to \Upsilon \phi_2, \tag{D.95}$$

where $\Upsilon^3 = 1$ but $\Upsilon \neq 1$. Applying (2.70) results in the following Yukawa textures for these models,

$$\Gamma_1 = \begin{bmatrix} x & 0 & 0 \\ 0 & x & 0 \\ 0 & 0 & x \end{bmatrix}, \quad \Gamma_2 = \begin{bmatrix} 0 & x & 0 \\ 0 & 0 & x \\ x & 0 & 0 \end{bmatrix}, \quad \Delta_1 = \begin{bmatrix} x & 0 & 0 \\ 0 & x & 0 \\ 0 & 0 & x \end{bmatrix}, \quad \Delta_2 = \begin{bmatrix} 0 & 0 & x \\ x & 0 & 0 \\ 0 & x & 0 \end{bmatrix}. \tag{D.96}$$

It is straightforward to compare Type S2 models to Type S1 models in order to find that the non-diagonal couplings of Type S2 models can be written as

$$\begin{aligned} N_d &= [P_1^{dL} - t_\beta^2 P_2^{dL} - t_\beta^{-2} P_3^{dL}] D_d [t_\beta P_1^{dR} - t_\beta^{-1} P_2^{dR} + t_\beta^{-3} P_3^{dR}] + q_d (U_{dL}^\dagger P_3 U_{dR}), \\ N_u &= [V P_1^{dL} V^\dagger - t_\beta^{-2} V P_2^{dL} V^\dagger + t_\beta^{-4} V P_3^{dL} V^\dagger] D_u [t_\beta P_1^{uR} - t_\beta^3 P_2^{uR} - t_\beta^{-1} P_3^{uR}] + q_u (V U_{dL}^\dagger P_3 U_{uR}) \end{aligned} \tag{D.97}$$

where $q_d = \frac{v_1^6 + v_2^6}{\sqrt{2} v_2^5} (\Gamma_1)_{33}$ and $q_u = \frac{v_1^6 + v_2^6}{\sqrt{2} v_2^5} (\Delta_1)_{33}$. As such, Type S2 models have 23 new parameters in their Yukawa sector, including one with an arbitrarily large magnitude in each sector.

D.2.3 Type S3*

The third of the special models, Type S3, is implemented by the following symmetry,

$$q_L^0 \to \mathrm{diag}(1,1,-1) q_L^0, \quad d_R^0 \to \mathrm{diag}(1,1,-1) d_R^0, \quad u_R^0 \to \mathrm{diag}(1,1,-1) u_R^0, \quad \phi_2 \to -\phi_2. \tag{D.98}$$

After using (2.70), we obtain the Yukawa textures of these models expressed in this WB,

$$\Gamma_1 = \begin{bmatrix} x & x & 0 \\ x & x & 0 \\ 0 & 0 & x \end{bmatrix}, \quad \Gamma_2 = \begin{bmatrix} 0 & 0 & x \\ 0 & 0 & x \\ x & x & 0 \end{bmatrix}, \quad \Delta_1 = \begin{bmatrix} x & x & 0 \\ x & x & 0 \\ 0 & 0 & x \end{bmatrix}, \quad \Delta_2 = \begin{bmatrix} 0 & 0 & x \\ 0 & 0 & x \\ x & x & 0 \end{bmatrix}. \tag{D.99}$$

While Type S3 models do not have UFV, their Yukawa couplings satisfy the following identities,

$$\begin{aligned} N_d^0 - \frac{1}{\sqrt{2}} v_2 (\Gamma_1)_{33} P_3 &= [P_1 + P_2 - t_\beta^{-2} P_3] \left[D_d^0 - \frac{1}{\sqrt{2}} v_1 (\Gamma_1)_{33} P_3 \right] [t_\beta P_1 + t_\beta P_2 - t_\beta^{-1} P_3], \\ N_u^0 - \frac{1}{\sqrt{2}} v_2 (\Delta_1)_{33} P_3 &= [P_1 + P_2 - t_\beta^{-2} P_3] \left[D_u^0 - \frac{1}{\sqrt{2}} v_1 (\Delta_1)_{33} P_3 \right] [t_\beta P_1 + t_\beta P_2 - t_\beta^{-1} P_3]. \end{aligned} \tag{D.100}$$

Using these relations, we write the non-diagonal couplings of Type S3 models in the following way,

$$\begin{aligned} N_d &= [1 - (1 + t_\beta^{-2}) V^\dagger P_3^{uL} V] D_d [t_\beta - (t_\beta + t_\beta^{-1}) P_3^{dR}] + q_d (U_{uL}^\dagger V^\dagger P_3 U_{dR}), \\ N_u &= [1 - (1 + t_\beta^{-2}) P_3^{uL}] D_u [t_\beta - (t_\beta + t_\beta^{-1}) P_3^{uR}] + q_u (U_{uL}^\dagger P_3 U_{uR}), \end{aligned} \tag{D.101}$$

where $q_d = \frac{v_2^4 - v_1^4}{\sqrt{2} v_2^3} (\Gamma_1)_{33}$ and $q_u = \frac{v_2^4 - v_1^4}{\sqrt{2} v_2^3} (\Delta_1)_{33}$. Thus, Type S3 models have 23 new parameters in their Yukawa sector (six in U_{dR}, six in U_{uR}, nine in U_{uL} plus q_u and q_d), including one with an uncontrolled magnitude in each sector.

D.2.4 Type S4*

The final special model, Type S4, can be implemented through the following symmetry,

$$q_L^0 \to \mathrm{diag}(1,1,-1) q_L^0, \quad d_R^0 \to \mathrm{diag}(1,1,-1) d_R^0, \quad u_R^0 \to u_R^0, \quad \phi_2 \to -\phi_2. \tag{D.102}$$

Replacing this symmetry in (2.70) results in the following Yukawa textures valid in this WB,

$$\Gamma_1 = \begin{bmatrix} x & x & 0 \\ x & x & 0 \\ 0 & 0 & x \end{bmatrix}, \quad \Gamma_2 = \begin{bmatrix} 0 & 0 & x \\ 0 & 0 & x \\ x & x & 0 \end{bmatrix}, \quad \Delta_1 = \begin{bmatrix} x & x & x \\ x & x & x \\ 0 & 0 & 0 \end{bmatrix}, \quad \Delta_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ x & x & x \end{bmatrix}. \tag{D.103}$$

Comparing the Yukawa textures of Type S4 models with those of Type S3 and jBGL, it is straightforward to write the non-diagonal couplings of Type S4 models in the following way,

$$\begin{aligned} N_d &= [1 - (1 + t_\beta^{-2}) P_3^{dL}] D_d [t_\beta - (t_\beta + t_\beta^{-1}) P_3^{dR}] + q(U_{dL}^\dagger P_3 U_{dR}), \\ N_u &= [t_\beta - (t_\beta + t_\beta^{-1}) V P_3^{dL} V^\dagger] D_u, \end{aligned} \tag{D.104}$$

where $q = \frac{v_2^4 - v_1^4}{\sqrt{2} v_2^3} (\Gamma_1)_{33}$. As such, we conclude that Type S4 models have sixteen new parameters in their Yukawa sector (six in U_{dL}, nine in U_{dR} and q), including one which is arbitrarily large in their down sector.