

# Control of a wave energy converter based on the oscillating water column principle

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**Abstract-** The search for ways of sustainable energy conversion has nowadays made alternative technologies of electric power generation gain more visibility and academic support for their development. With this, this project has as its scope the conversion of energy from the waves of the sea, using more specifically the principle of an oscillating water column. The thesis is focused on the development of a control action for the turbine of Instituto Superior Técnico installed in the station of Mutriku, Spain. This control refers to the manipulation of the airflow inlet valve in the turbine in order to maximize the output power.

The controller was designed from the concepts of Optimal Control by the Linear Quadratic Regulator method, thus generating a Latching control for the parameters of a dynamic linearized system similar to the station.

The simulation of the applied controller showed a significant increase of the power of the turbine, as well as verified the dynamics of the linearized system as true to the real nonlinear model. In this way, it was concluded that, even if the control action is not optimal, the application of the optimal control techniques in linearized systems is feasible to the type of system present in this project.

**Keywords-** Optimal Control, Oscillating Water Column, Mutriku, LQR, Latching

## 1. INTRODUCTION

Wave energy is one of the most notable ways of obtaining electricity today, mainly due to the increasing need for renewable and alternative sources of energy. The conversion of energy from the sea waves is considered a particular branch of the scope of the solar energy, since the flow of ocean mass comes predominantly from the action of air currents formed by the convection due to differences of temperatures.

The aforementioned notoriety comes not only from the complexity of the methods and equipment used in the projects of this theme, but also due to the irregular and destructive effect of the waves.

The first patent relating to wave energy converters was made in France in 1799 by the researcher named Girard [1]; however, the subject gained strength only in the 1940s when Yoshio Masuda (1925 - 2009), a researcher considered the pioneer and father of modern technology in the field, began studies and practical applications in Japan. Masuda developed the first and most famous boat with wave energy converters called Kaimei, equipped with air turbines in the shape of oscillating water columns, OWC, in 1976 [2].

In the 1970s, with the oil crisis (1973) as a major event, the study and development of clean energy sources gained international prominence in the academic world, as an alternative way to replace, or at least complement, the energy by natural extraction resource. As a result, wave energy has gained more visibility than ever before, resulting in the creation of the first wave development program in England in 1975 and the first conference aiming the subject in 1979 [3].

From the 1970s to the 1990s, activities were basically summarized in academic studies [4], mainly due to difficulties in modeling the aspects of energy absorption, diffraction and radiation of the waves, numerical tests and prototypes of systems that made it impossible to construct of functional equipment with operation in a viable regime. Many models were constructed based on linear theories of the waves, disregarding aspects such as loss of point loads, viscosities, compressibility, turbulence, nonlinearities, among others, which in the end results end up compromising the project.

Other aspects, not known at the time, were not considered as well because the use of wave energy involves a very complex chain of conversion processes, each of which has different efficiencies and constraints to be introduced, and must be controlled. For example, initial studies of absorbers show that the frequency of the oscillating object must be equal to the incident wave, and since this was not considered previously, many projects failed by not consider this aspect.

Turning back to the historical view of wave energy, in 1991, the European Commission drastically influenced the realization of the projects by including this branch in the renewable energy incentive program, and in 2001 the first specific wave energy agreement was created. Since then, wave energy conversion has

been researching not only in Europe, but also in America and Asia.

With the development of the research, it encountered other obstacles, increasingly complex. Currently, one of the major problems in the subject is the need to obtain a prediction of the incident waves to the device in the long term, an information which is extremely relevant for the design of optimal controller; besides, as the knowledge of the wave state for the efficiency of the model is something essential, it ends up making it difficult to execute numerous processes. In order to solve this, European institutes have carried out studies of wave situations, both on the high seas and in the coasts, for a better temporal characterization of the waves in several places of the continent.

Even with all the hardships, numerous wave energy conversion technologies have been developed over the last few years, each dependent on one form of absorption, the characteristics of the installation site and the conversion devices. Among the similarities between them, it is known that the construction process of a power conversion station starts in theoretical modeling, numerical modeling, computational simulation, construction and prototype testing in small scale, so that it can later be built in size and actual operation.

Several methods of classification of wave energy systems have been proposed, based on location, operating principles, size, etc. The most famous, presented below, was based on the operating principle of each of the systems.

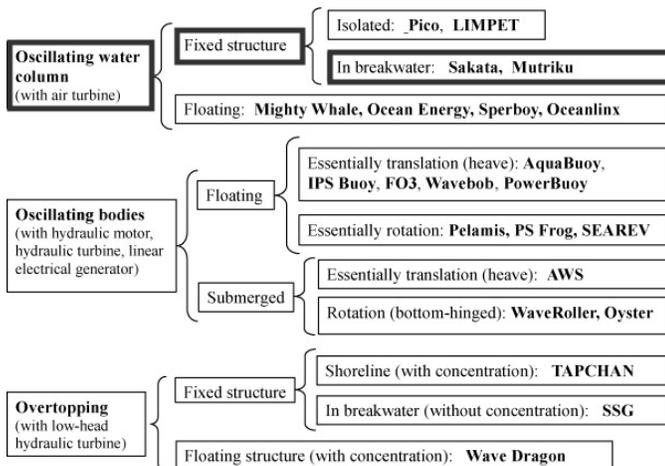


Fig. 1 – Classification of Wave Energy Converters [8]

Most of the projects built are either coastal or deployed near the coast, considered this first generation of these equipment's. These projects have the advantage of lower costs and maintenance and installation facilities, but less energy potential, since coastal waves have less energy than those at deep sea [5].

The fixed structure OWC devices have a partially submerged concrete structure open at the bottom where the air is internally compressed and forced to operate an air turbine, as shown in the diagram below [6].

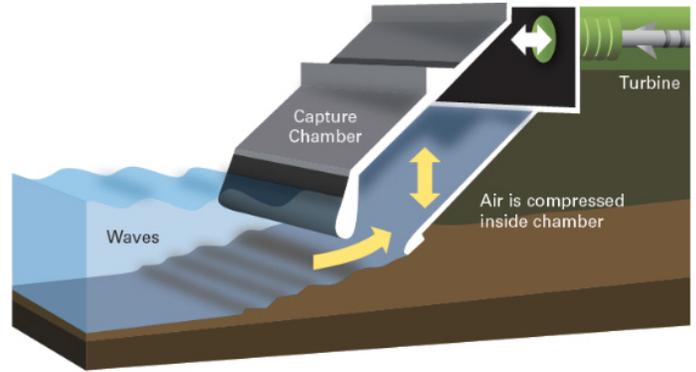


Fig. 2 – Fixed OWC illustration [10]

The design and construction of the structure, with the exception of the air turbine, are the most critical parts of the design of the fixed OWC device, and are the factors that most influence the economic part of the project. For this reason, civil construction companies, currently, dominate the construction of the plant of these projects.



Fig. 3 – Mutriku Station Picture [9]

The Mutriku installation is a fixed structure OWC device, located in Mutriku, Spain, with 18 chambers and 18 Wells turbines with about 18.5 kW of power each [7].

The proposal of this thesis is to elaborate a control action for the valve that manipulates the mass flow of air in the turbine, installed in Mutriku, from concepts of optimum control, in order to maximize the power of the turbine in question.

The concept of optimal control that will be approached in this project is more specifically that of a Linear Quadratic Regulator, LQR, based upon the Minimum Principle of Pontryaguin, to obtain an optimal Latching controller for a dynamic linearized system of the Mutriku model with incidence of irregular waves. And from the obtained controller, analyze its impacts and characteristics with respect to external and internal parameters to the project.

The structure of the thesis will be the following: Section 2 will approach some preliminary theoretical concepts referring to the themes used in the realization of the computational models; in Section 3 we have the modeling of the Mutriku Project, both mathematical and computationally, and the controllers; in Section 4 the results and final analysis of the thesis will be shown and Section 5 presents the conclusion and suggestions of future work to the proposed work.

## 2. THEORY

In order to understand the analysis of the project, it is necessary to precede a theoretical basis, from which the basis for the concepts addressed during the modeling and solution of the following problems is obtained.

This preliminary theory has the following topics: Wave Energy Concepts, Turbines, Linearization of Nonlinear Systems, Latching and Optimal Control.

### 2.1 - Wave Energy Concepts

To better fit the model presented in this work, we chose to simulate an irregular wave, instead of regular waves, since real waves, besides being irregular, are quite random.

For this, we will use the concept of a wave spectrum, that is, we will characterize the wave as a stochastic process in order to obtain the approximate function more similar to an actual irregular wave behavior. And so, get something that behaves similarly to the following figure.

For this, we can express an irregular wave as an infinite sum of harmonic waves, based on a Fourier Series, the wave elevation being expressed by:

$$\zeta(t) = \sum_{i=1}^N a_i \cos(2\pi f_i t + \alpha_i) \quad (1)$$

with  $a_i$  and  $\alpha_i$  the amplitude and phase of the waves, respectively. In this way, we can also obtain the average amplitude of the Spectrum for all the frequencies, being:

$$\bar{a}_i = \frac{1}{M} \sum_{m=1}^M a_{i,m} \quad (2)$$

Such concepts will be approached in the elaboration of the excitation force for our model, which is represented as its input; where in the real model would be the wave that reaches the dam ([11] and [12]).

### 2.2 - Turbines

In order to allow the application of air turbines in oscillating water column systems, the concepts of dimensional analysis are used, especially in two situations: transporting prototype results to real-scale models and studying the influences of the variables under the system, in particular the effect of changes in rotational speed under turbine performance.

We must consider that the numbers of Mach, Reynolds and Pressure Differential must have the same dimensionless value and must be considered if air compressibility is to be computed, otherwise the variation of Mach and Reynolds numbers, leaving only the pressure differential.

Thus, for the air turbines, we have the following dimensionless numbers applied:

$$\Psi = \frac{p}{\rho_{atm} \Omega^2 D^2} \quad (3)$$

$$\Phi = \frac{w}{\rho_{atm} \Omega D^3} \quad (4)$$

$$\Pi = \frac{P_t}{\rho_{atm} \Omega^3 D^5} \quad (5)$$

with  $w$  representing the air flow in the turbine,  $\rho_{atm}$  the atmospheric air density,  $\Omega$  the rotation speed,  $D$  the rotor diameter of the turbine and  $P_t$  the output power of the turbine. We can also call  $\Psi$  as the dimensionless of pressure,  $\Phi$  as the dimensionless of the flow, and  $\Pi$  as the dimensionless of torque or power.

The relation between the three dimensionless ones gives the efficiency of the turbine, expressed by:

$$\eta = \frac{\Pi}{\Psi \Phi} \quad (6)$$

As stated earlier, one of the aspects that define the turbine is its control valve. This valve will be the most relevant point regarding the turbine for this project, since the control of the system will be carried out acting thereupon.

Such a valve manipulates the air inlet in the turbine: if the valve is open, we have the maximum airflow in the turbine; if it is closed, there is no airflow in the turbine, and its movement is based only on its inertia.

Thus, we wish to control it so that the power of the turbine is as large as possible in the system's operating time while avoiding functioning regimes that could damage the equipment. [13].

### 2.3 - Linearization of Nonlinear Systems

The linearization of systems in control projects occurs due to the complexity of the nonlinear control techniques, in addition to that the majority of the theories of control of dynamic systems are based on linear (or linearized) systems. Other factors, such as non-performance assurance, need to use adaptive control, among others, also influence the decision to linearize the model instead of manipulating non-linearities.

One of the most famous linearization methods is by Taylor Series Expansion, that assumes linearizing the system around an operating point, thus enabling the use of linear control tools for this neighborhood. This makes it impracticable to apply ownership of this linearization to the global case of the problem [14].

Considering therefore a nonlinear system  $\dot{x} = f(x(t))$ , where  $f()$  is a known function and an operating point  $x_0$ , which is not necessarily a point of equilibrium, we have that the Taylor Series Expansion function is:

$$f(x) = f(x_0) + f'(x_0)(x - x_0) + \frac{1}{2}f''(x_0)(x - x_0)^2 + O((x - x_0)^3) \quad (7)$$

In this way, we have a linear function around the chosen point of interest.

For a system with multiple variables, we can apply the partial derivatives for each unknown variable, and obtain a linearized system of first order, or superior, with respect to each one of them. The following is the first-order Taylor series linearization expression for each variable of a random function f.

$$f(A, B) = f(A_0, \dots, Z_0) + \frac{\partial f}{\partial A}(A_0, \dots, Z_0)(A - A_0) + \dots + \frac{\partial f}{\partial Z}(A_0, \dots, Z_0)(Z - Z_0) \quad (8)$$

#### 2.4 – Latching

One sector of the study of electric power conversion that has gained increasing visibility is the control of the output device; it is the part of the project that aims to manipulate certain specific equipment in order to maximize energy production and efficiency from the project.

Historically, the application of control theory to wave energy was initially proposed using a phase control [15], later adapted to a Latching [16], to prevent the flow of air in the turbine from the limitation of the air input to the turbine in frequency with the excitation force of the system, thus taking advantage of the moments of greater energy potential [17], and more recently analyzes using the concepts of optimum control and Pontryaguin Maximum Principle gained importance in the numerical resolution of the controller, for enabling the elaboration of a perfectly control action theoretically. However, an optimal control has not been presented and applied until now.

Thus, we can summarize Latching as a controller that aims to take advantage of moments with greater energy potential, limiting the system to work only in these moments, while in situations of low potential the controller inhibits the dynamics of the model, as shown in the following figure.

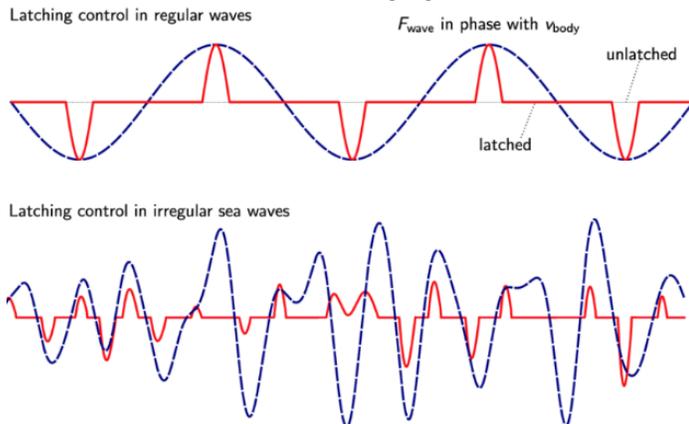


Fig.4 – Latching Control [18]

#### 2.4 - Optimal Control

Optimal Control theories propose methods to calculate the controller that provides the optimum operation of the system with reference to a given cost function. For this project, we chose to perform the control from a Linear Quadratic Regulator, LQR.

In order to obtain an optimal controller, we have to quantify the cost function. This cost function can be expressed by:

$$J(t) = \int_0^t (x^T Q x + u^T R u) dt \quad (9)$$

where Q is the weight matrix of the States and R is the weight of the control action u.

Thus, we have that, by the LQR equation, also known as the Riccati equation:

$$A^T P + P A - P B R^{-1} B^T P + Q = 0 \quad (10)$$

we get a controller of the form:

$$u = -Kx = -R^{-1} B^T P x \quad (11)$$

It should be remembered that the parameters A and B are the matrices that represent the linearized model of the system in question, and the matrix P is the solution of the equation itself [19].

The LQR to be implemented, we must put it into a feedback system, whose gain is decreased, or updated, from the initial reference value.

### 3. MUTRIKU PROJECT: MODELS AND CONTROL

The CAO fixed plant model was designed based on the theory of a rigid oscillating piston. Thus, we have as dynamic model:

$$(m + A^\infty)\ddot{z} + \rho_w g S z + p_{atm} S p^* = F_d - R \quad (12)$$

being  $m$  the mass of the piston,  $A^\infty$  the added mass of water,  $z$  the height of the inner water of the chamber,  $\rho_w$  the density of the water,  $g$  the acceleration of gravity,  $S$  the cross-sectional area of the chamber,  $p_{atm}$  at atmospheric pressure.

The term  $p^*$  refers to a dimensionless term of pressure oscillation within the chamber, expressed as:

$$p^* = \frac{p}{p_{atm}} - 1 \quad (13)$$

where  $p$  is the instantaneous pressure in the chamber.

In the right side of the equation, we have the terms of the diffraction force and radiation, expressed respectively by:

$$F_d = \sum_{m=1}^N A(\omega_m) \Gamma_1(\omega_m) \cos(\omega_m t + \phi_1(\omega_m) + \phi_r(\omega_m)) \quad (14)$$

$$R = \int_0^t K_1(t-s) \dot{z}_1(s) ds \quad (15)$$

where  $\Gamma_1$  is the excitation force per unit wavelength,  $\phi_1$  is the frequency response of the piston,  $\phi_r$  is the random frequency,  $A$  is the wave amplitude, all as a function of the angular frequency  $\omega_m$  and  $K_1$  is the Kernel term.

In addition to the hydrodynamic model, there are also the models of the chamber and the turbine.

The model of the air chamber can be expressed as a function of the airflow in it, being:

$$m_{turb} \dot{p} = \rho \dot{V}_c + V_c \dot{\rho} \quad (16)$$

where  $\rho$  is the air density and  $V_c$  is the instantaneous volume of the chamber, expressed as:

$$V_c = V_0 - S z \quad (17)$$

with  $V_0$  representing the volume of the chamber under hydrostatic conditions.

Considering the processes in the chambers as polytropic, we can assume that:

$$\frac{p}{\rho^\gamma} = \frac{p_{atm}}{\rho_{atm}^\gamma} \quad (18)$$

with  $\gamma$  being the specific heat ratio, we can obtain the model of the chamber as:

$$\dot{p}^* = -\gamma(p^* + 1) \frac{\dot{V}_c}{V_c} - \gamma(p^* + 1)^\beta \frac{m_{turb}}{\rho_{atm} V_c} \quad (19)$$

with

$$\beta = \frac{\gamma - 1}{\gamma} \quad (20)$$

The turbine model can be expressed by:

$$I \dot{\Omega} = T_t - T_{gen}^{em} \quad (21)$$

where  $I$  is the moment of inertia of the turbine,  $\Omega$  is the angular velocity of the rotor,  $T_{gen}$  is the instantaneous torque of the electromagnetic generator and  $T_t$  is the instantaneous torque of the turbine, which can be represented by:

$$T_t = \rho_{in} \Omega^2 d^5 n_{turb} \Phi \Psi \quad (22)$$

where  $\rho_{in}$  the functional density between the chamber and the atmosphere,  $\Psi$  is the dimensionless pressure,  $\Phi$  is the dimensionless of the air flow,  $n_{turb}$  the turbine efficiency and  $d$  the turbine diameter, these being represented by (3), (4), (5) and (6).

Considering all the models and aspects mentioned above, one could obtain a set of differential equations, such that in a single model we have:

$$\dot{x} = F(t, x) \quad (23)$$

$$\begin{pmatrix} \dot{v} \\ \dot{z} \\ \dot{p}^* \\ \dot{\Omega} \\ \dot{\bar{P}}_t \\ \dot{\bar{P}}_e \\ \dot{I}_r \end{pmatrix} = \begin{pmatrix} (m_1 + A_1^\infty)^{-1}(-\rho_w g S z - S p_{atm} p^* + F_d - R) \\ v \\ -\gamma(p^* + 1) \frac{\dot{V}_c}{V_c} - \gamma(p^* + 1)^\beta \frac{\dot{m}_{turb}}{\rho_{at} V_c} \\ \Gamma^{-1}(T_t - T_{gen}^{em}) \\ P_t T_f^{-1} \\ P_e T_f^{-1} \\ \beta_r I_r + \alpha_r v_1 \end{pmatrix} \quad (24)$$

where  $T_f$  is the time interval of the simulation. In order to simulate the model, we can assume as initial condition:

$$x_0 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ \Omega_0 \\ 0 \\ 0 \\ 0 \\ 0_p \end{pmatrix} \quad (25)$$

with  $0_p$  being a vector of zeros of dimension  $p$ .

### 3.1. Model Linearization

The linearization of the model comes from the application of expanding the differential equations (29) in Taylor Series, in order to linearize each of them with respect to the system variables.

For the equation of the velocity of the wave in the chamber, we know that it is already linear with respect to the system parameter, so it can be expressed in its original form:

$$\dot{v} = -\frac{\rho_w g S}{(m + A^\infty)} z - \frac{P_{at} S}{(m + A^\infty)} p^* + \frac{F_d - R}{(m + A^\infty)} \quad (26)$$

The same occurs with the wave height equation, expressed by:

$$\dot{z} = v \quad (27)$$

For the equation of the dimensionless pressure, we first have the expression:

$$\dot{p}^* = -\gamma(p^* + 1) \frac{\dot{V}_c}{V_c} - \gamma(p^* + 1)^\beta \frac{\dot{m}_t}{\rho_{atm} V_c} \quad (28)$$

Assuming the equations (16) and (17), we can get the complete equation as:

$$\dot{p}^* = \frac{\gamma S v}{V_0 - S z} [(p^* + 1) + (p^* + 1)^\beta u] \quad (29)$$

As we can see, the variable  $u$  is the position of the valve, where 1 would be the same opened, and 0, closed, which in the general scope of the design is our control action.

By expanding the previous equation in Taylor series, we have:

$$\begin{aligned} \dot{p}^* &= \frac{\gamma S}{V_0 - S z_0} [(p_0^* + 1) + (p_0^* + 1)^\beta u] \cdot v \\ &+ \frac{\gamma S^2 v_0}{(V_0 - S z_0)^2} [(p_0^* + 1) + (p_0^* + 1)^\beta u] \cdot z \\ &+ \frac{\gamma S v_0}{V_0 - S z_0} [1 + \beta (p_0^* + 1)^{\beta-1} u] \cdot p^* \\ &- \frac{\gamma S^2 v_0 z_0}{(V_0 - S z_0)^2} [(p_0^* + 1) + (p_0^* + 1)^\beta u] \\ &- \frac{\gamma S v_0 p_0^*}{V_0 - S z_0} [1 + \beta (p_0^* + 1)^{\beta-1} u] \end{aligned} \quad (30)$$

So that we can also obtain a mean power for the time interval of the simulation, we will use the expression of the electric power of the generator as:

$$\bar{P}_e = \frac{a \Omega_0^b}{T_f} \quad (31)$$

The same will be used for turbine power:

$$\bar{P}_t = \frac{\rho_{in} \Omega^3 d^5 \eta \Phi \Pi}{T_f} \quad (32)$$

Thus, we have the equations that will be the same in all the linearized models, the velocity, height, dimensionless pressure and the electric power in the generator.

To linearize the turbine speed and turbine power equation, we must first obtain their expressions. These expressions can be obtained from the dependence of the dimensionless turbine power, as shown above:

$$P_t = \rho_{in} \Omega^3 d^5 \Pi \quad (33)$$

These expressions are obtained from the manipulation of the non-dimensional terms (23), (24), (25) and (26) under the general turbine power equation (53).

### 3.2. Computacional Models

#### Model without control

The model without the performance of a control consists of the operation of the Mutriku Model in State-Space form, with the matrices being calculated at each instant of 0.1s, and assuming the valve always open. Thus we have the following scheme to represent its operation.

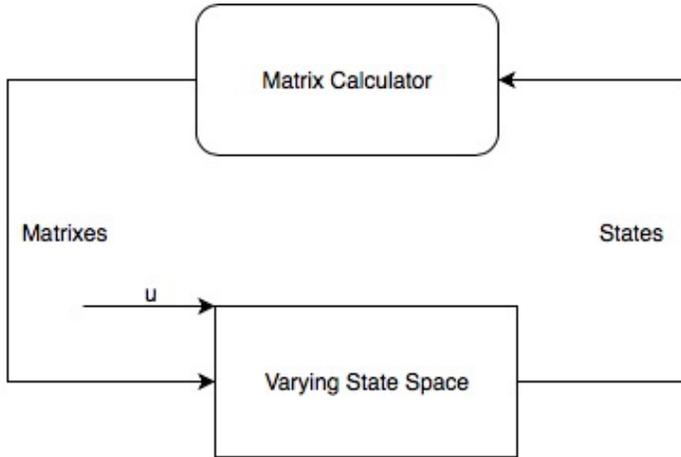


Fig. 5 – Model without control diagram

#### Model with simple LQR control

The model with a non-upgradeable LQR has the same structure as the non-control operation, but the valve operates with a predetermined position by the calculation of an LQR controller at each instant of the model without actuation of the controller. Therefore, the model was run without control and a LQR was calculated for this situation, after which the same model was simulated again, but with controlled valve input. This situation can be seen in the diagram below.

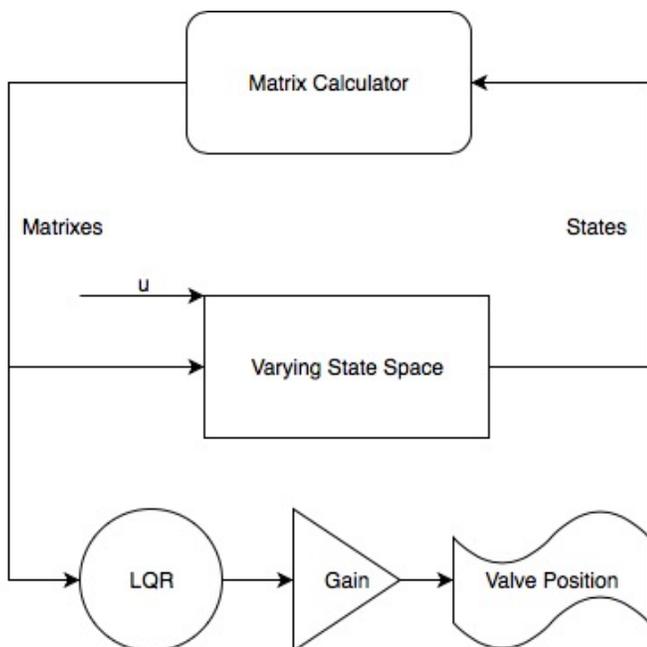


Fig. 6 – Model with simple LQR control diagram

In this model, as the control action is not applied in the system, we have the behavior of the states during the simulation and the same of the model without control; however, we have to obtain a controller for this model.

The control action is made of two methods: rotation speed limitation method and Linear Quadratic Regulator method. The method of limiting the speed of rotation of the turbine is to prevent such a rotation reaching its maximum value, i.e. if the turbine rotor angular velocity reaches its maximum value, the valve is automatically closed. Thus, we have that the control action is done by a relationship between the two methods.

In order to make it possible to calculate the controller, we have to obtain a variable to be minimized, and since we want to maximize our power in the turbine, we create a variable that represents the inverse of that power.

This variable is calculated by the following expression:

$$C = \frac{1}{P_t} \tag{34}$$

#### Model with feedback control

The project with real-time LQR operation consists of a feedback, that is, for each instant a new gain is calculated for the system, and consequently a new input, which is already used for the next instant. You can check this in the diagram below.

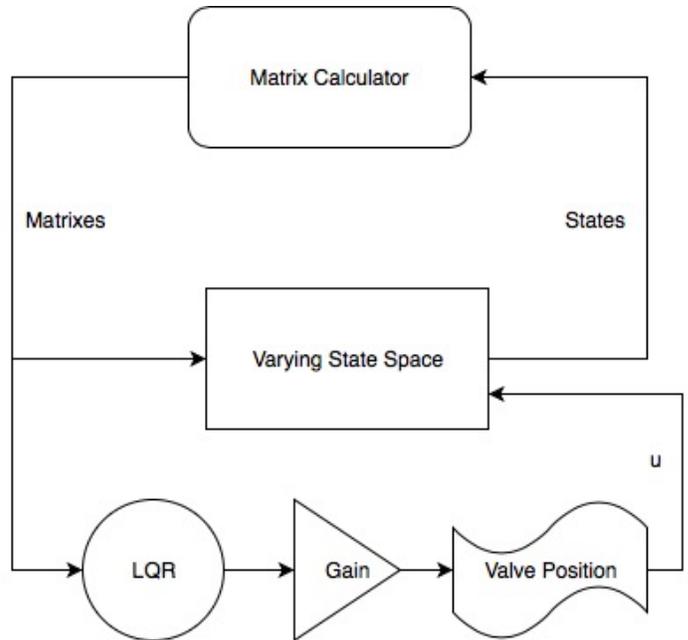


Fig. 7 – LQR feedback control diagram

#### 4. RESULTS AND ANALYSIS

Based on the computational models discussed above, we have two approaches: valve position for an LQR without feedback and valve position for an LQR with feedback.

To compare and obtain the optimal position of the valve we will compare the total power of the turbine for the two scenarios and see which one presents the highest value.

In order to understand the dynamics of the controller, we can see the evident relationship between its control action and the speed of rotation of the turbine, which is the factor that most influences its power.

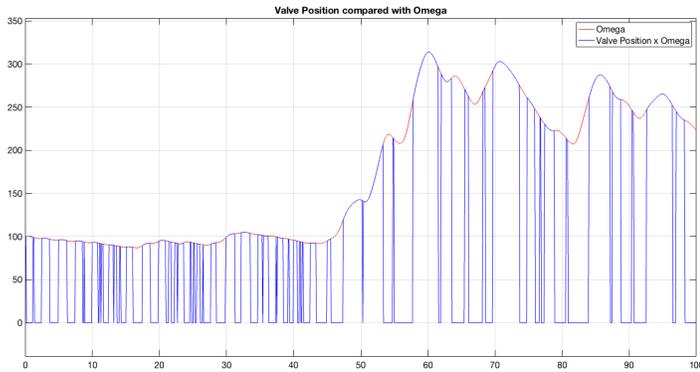


Fig. 8 – Relation between Omega and Valve position in Simple LQR model

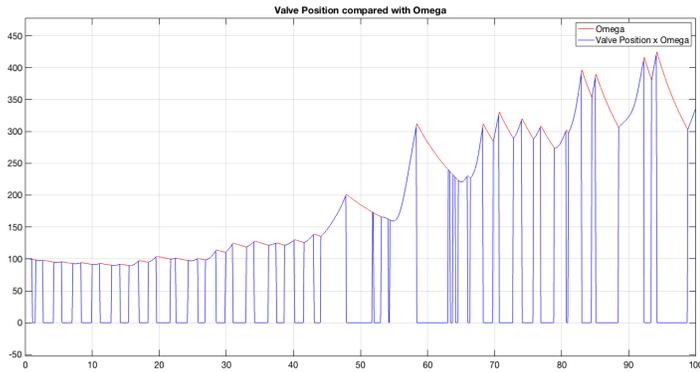


Fig. 9 – Relation between Omega and Valve position in Simple LQR model

This last two graphs also show the operation mode of each controller; while the non-feedback seeks to open the valve at peak times of the rotational speed, where the excitation force to the system is higher, the feedback one opens the valve at times when the turbine revolution is increasing, that is, in the turbine power is positive.

We can thus compare the values of the powers with controller with the power without controller, which would give:

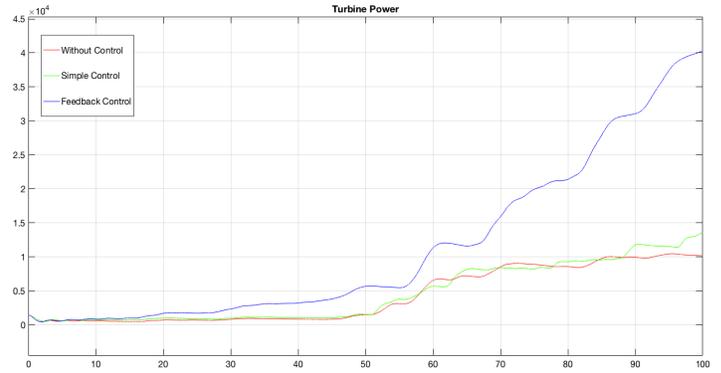


Fig. 10 – Comparison of turbine power in each model

It can be verified that the power of the turbine in the interval of 100s is maximized with the presence of the controllers in comparison with the system without control. Also, the controller obtained with the LQR with real-time feedback performs better than the simple LQR.

These results were obtained without limiting the speed of rotation; if we want to limit it, for example, to 200 rad / s, we have that the final control action must obey the moments that the turbine can be activated. Such moments are obtained by restrictive control action below.

Thus, we would have the following control action results and final powers:

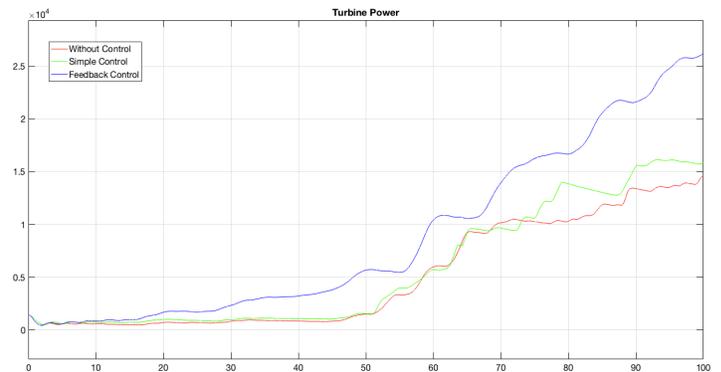


Fig. 11 – Comparison of turbine power in each model with rotational speed limitation

In this scenario, we observed the same behavior of the controllers, but with a less expressive result for the controller with feedback. This leads us to conclude that opting for an instant of greater excitation power is less advantageous and efficient than a moment with positive power variation.

#### Sensibility Analysis

In order to verify the implications of some key system variables, we performed a sensitivity analysis on their variations. Since we assume the air as incompressible initially, the first variable to receive the analysis was the air density inside the chamber. The other variable was the added mass, since part of the radiation force was neglected in the modeling of the system.

To perform the sensitivity analysis, we changed the value of these variables in order to obtain the system reaction for its variation. The value of the local air density is considered as 1.225, however for situations of pressure variation, it can assume values in the range of 1.1 to 1.3. It is noticed that the variation of the two states is not significant for the modeling of the system, therefore, we can even consider the incompressible air. For the same analysis, by varying the mass added at 10% of its value for more and for less, we saw that the change in mass added alters the behavior of water column velocity, water column height and initial value pressure with reduced excitation force, but in terms of absolute values with normal excitation force, this does not interfere with the dynamics or the quantification of states.

*Control Action in the Non-Linear Model*

The control action in the Nonlinear Model will be based on the elaboration of two algorithms that allow the understanding of the operation of the control projected in the thesis. Such algorithms refer to open-chain control and closed chain control, previously reported in the above sections. Both algorithms assume that the model has access to a wave prediction of at least an instant, that is, the excitation force of the next instant is known, which makes it possible to calculate the states at this future moment. The following illustration represents the algorithm for the first open-chain controller (Simple Control).

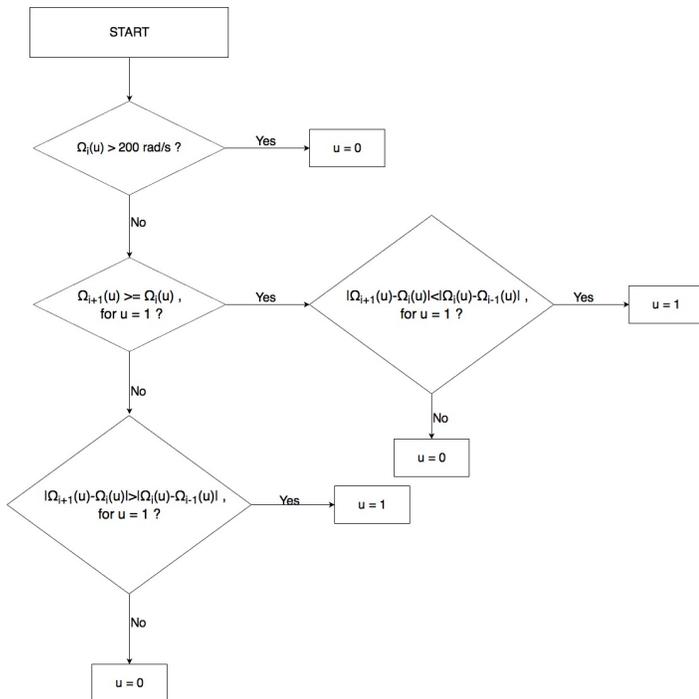


Fig. 12 – Open-Chain Control Algorithm

As can be seen in the scheme above, after the start of the model, it is assumed that there is access to the values of the rotational speed in the present, past and future step. Therefore, the first check to be made is the project boundary. In this case, a maximum value of 200 rad/s has been assumed, so if the

rotational speed is greater than this value, the valve position is already assumed to be 0 (closed), and the algorithm ends for this iteration. Otherwise we have to check if omega will increase or decrease at the previous instant.

Knowing therefore whether the future value is greater or less than the present, one must check whether the difference of the future value with the present is greater or less than the present value with the past, since this will define if the dynamics of the rotational speed is a local maximum, as seen in Section 4, where the control action for the open chain was defined.

The algorithm for the closed chain (Feedback Control), different from the first one, is presented below.

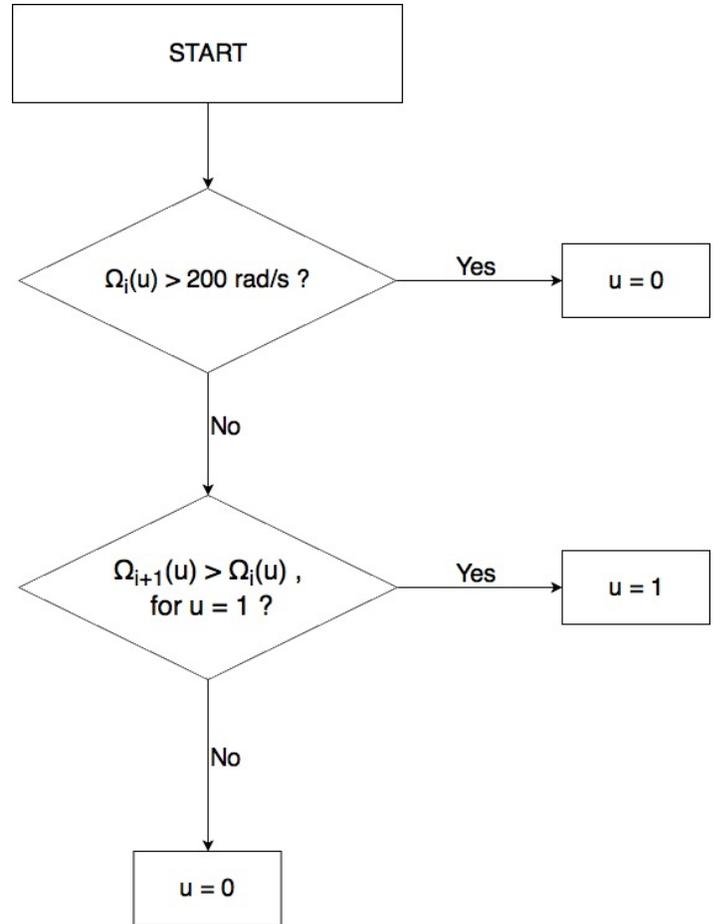


Fig. 13 – Closed-Chain Control Algorithm

It can be considered simpler, this control action starts with the same speed limit check, and then analyzes whether the future value of this state is greater than the present, and if so, imposes the valve opening.

It should be remembered that the calculation of the values of the rotational speed must always be done taking into account the valve opened, so that the dynamics of the system is in operation with the excitation force having implications in the turbine.

## 5. CONCLUSION AND FUTURE WORK

The initial proposal of the thesis had as its objective the design of an optimal controller for the energy conversion system in Mutriku, however, in order to enable its realization, it was necessary to reach a few steps first.

Firstly, because it was chosen to apply an LQR under the system, it would not be possible to use the nonlinear model of the converter, thus a dynamic linearized system was designed. Such linearization was done successfully, since it obtained a robust model, with dynamics similar to nonlinear and without significant singularity.

Subsequently, the controllers were applied under the system, and it was found that the goal of maximizing the average overall power of the installation was achieved; however, based on some analysis made on the behavior of the controller, optimally cannot be guaranteed. In this way, it was concluded that the obtained controller could be sub-optimal.

The test of the linearized system controller in the nonlinear model tends to show the feasibility of this type of application, since a promising result was obtained in both models, but a lower efficiency should be presented when adapting the linear controller in the model not linear. This would prove that the linear controller, which may be optimal in its origin system, would be sub-optimal in the real system.

Regarding the future work, there are two possible follow-up lines to improve the project: deepening the turbine behavior and the practical application in the nonlinear model.

Regarding the turbine, a controller can be realized that takes into account some more detailed parameters of the turbine, such as the response time of opening and closing of the valve, precise updating of the limits of the rotation of the turbine and equation of the real response of the turbine with the valve in an open position, thus allowing optimal control without being by Latching.

Regarding the application in the nonlinear model, it would be of great importance to know the behavior of the controller in the real model, in order to confirm the viability of the linearization made for real applications of this type of control.

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