Optimization of hybrid fiber-reinforced composites

Aiming pseudo-ductility using meta-heuristics

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Abstract

Unexplored techniques in the field of composites, namely bringing together meta-heuristics and hybridization, provide a unique opportunity to achieve new optimal composite materials. Due to the complexity of micro-mechanical hybrid composites models, previous work on hybrid fibrous composites with more than two fibers is scarce. Therefore, it is a relevant research opportunity the search of solutions with the previously mentioned optimization techniques aiming to achieve new and better results.

The aim of this work is the adaptation of a model that reciprocates the failure mechanisms of composites, using known properties of fibers. The goal is to explore the potential of hybridization, adopting meta-heuristics, to open up a more extensive range of possible solutions.

This problem revealed itself as multi-objective, considering not only pseudo-ductility as a parameter but also specifications, such as, how high is the baseline achieved by the material, on a stress-strain curve, as well as an additional criterion known as pseudo-plastic work. To solve the optimization problem was developed a computational model using genetic algorithms with a weighted function approach and a multi-objective function with the purpose of identifying the Pareto Front.

The achieved results show evidence of a significant improvement on the project specifications when compared to the previous research on this particular subject, using both in-house and commercial algorithms.
Keywords: Genetic Algorithms, Hybridization, Fibrous Composites, Pseudo-ductility

1. Introduction

It is widely accepted that composite materials will have a paramount influence in the future. No single high-performance engineering sector has not embraced them yet. Composites are an innovative design choice for many applications. With their ability to be formed in any shape, combining properties not usually found together in a single material, for example, high strength and low weight, these materials play a huge role in structural applications, such as in the aerospace and automotive industries. In summary, these materials are cost-effective substitutes for many higher-cost and higher maintenance materials.

There are still many challenges for the composites' industry [17]:

- Overall price reduction to compete against steel or aluminum.
- Still ineffective supply chain for the demand.
- Inefficient repairing and recycling chain.
- Lack of development in manufacturing technologies such as hybridization.
- Improvement needed in simulation and prediction techniques.

Hybridization provides an opportunity to achieve a better product design, by mixing ingredients until reaching the finest output. For this reason, it is vital to understand how regular composites behave and, in particular, how hybridization works. Apart from having multiple possibilities, it is crucial to learn how failure mechanisms function and how they lead to more (pseudo-)ductile materials, allowing thus, further alternatives to their traditional use.

Due to the growing interest in hybridization in composites, it is critical to understand whether if it is worthwhile to use more than two fibers in the same material, regarding its pseudo-ductility characteristics. Therefore, the thesis statement of this dissertation is:

To quantitatively assess the technical viability of combine than two fibers in hybrid composites for pseudo-ductility purposes.

Given the objective stated, in this dissertation it is aimed to answer the following main research question:

- Do hybrid composites with more than two different fibers perform better than traditional hybrid composites?

To answer the previous question secondary inquiries must be considered:

1. Is Genetic Algorithm suitable for optimizing hybrid composite materials?
2. How much improvement is attained using hybrid composites with more than two different fibers, when compared to classic hybrid composites?
3. Regarding those improvements, is it fair to assume that composites' industry is closer to an "optimal" hybrid composite in terms of pseudo-ductility?

2. Literature Review

Even though there is no universal definition of composites materials, a general definition can be obtained from a dictionary: *"made up of different parts or materials"* [12]. In this thesis context, composites can be described as combined materials by the synthetic assembly of two or more components or constituents, to achieve specific characteristics and/or properties [24]. If well designed, these materials exhibit the best features of their constituents and, often, other properties that neither possesses. Surely not all of these attributes are upgraded at the same time. The goal is to create a material that performs as needed by the designed task [13].

Fiber-reinforced composites consist of fibers bonded to or embedded in a matrix, meaning that these materials exhibit a large length-to-diameter ratio. Both chemical and physical identities are maintained, yet they produce a material with a combination of properties that cannot be accomplished when either of the constituents acts alone. They are considered the most significant composites in today's technology, due to their potential to compete with metals where other types of composites fail [3].

Depending on the fibers arrangement, these composites can be classified as continuous or discontinuous. The continuous type consists of aligned fibers, contrasting with the discontinuous type, composed of short fibers that can be randomly oriented or aligned. Studying unidirectional (UD) composites permits a better comprehension of these materials overall properties due to anisotropy - properties dependent on their fibers orientation [3, 12].

The hybrid composites concept is a natural extension of the composites' principle of combining materials to enhance the design performance. Mixing two or more fibers, layers or particles, grants an even closer adjustment of materials properties to satisfy requirements that cannot be obtained with a single fiber species [9]. In this case, they attempt to produce an alternative material with enhanced mechanical properties. Different structures lead to particular properties and distinct mechanisms of failure. In the literature, hybrids are used with two different fibers. Usually, these are referred as low elongation (LE) and high elongation (HE). HE fibers do not necessarily need to possess a more substantial failure strain value, but it must always be more significant than the LE ones [25].

Regarding these composites, it is essential to introduce the definition of hybrid effect. A general interpretation is provided elsewhere [16]: *"a deviation from the linear rule of mixtures"*. Even though it is a general explanation, it grants an application to several mechanical properties and permits the occurrence of positive and negative hybrid effect, which corresponds to an enhancement or degradation of the characteristic in question, respectively. Presently, three main hypotheses attempt to describe this phenomenon: residual thermal stresses, changes in the damage development and dynamic stress concentrations. Unfortunately, neither can, separately, fully clarify the causes of hybrid effect [25].

In recent years, fiber hybridization has captivated increasing attention, due to gain a more gradual failure in brittle composites, a behavior analogous to the one exhibited by metals and known as pseudo-ductility. Conventional fiber-reinforced composites show an abrupt and catastrophic failure occurring without any previous signal. A conventional hybrid composite has a distinctive load drop when the brittle fibers break. Controlling damage mechanisms, it is possible to attain a more progressive failure [25, 29].

There are several models for stress rupture in UD continuous fiber composites. It is well known that failure in these cases is a progressive one and that fibers fracture gradually, forming clusters that grow until they reach a critical size, leading to a volatile propagation and ultimately to failure. It is a mechanism governed by fiber strength statistics and micromechanical stress redistribution, which is affected by several parameters. The most forceful ones are size effects, stress redistribution after fiber failure, critical cluster size, effects of the matrix and fiber-matrix interface [25].

Modelling the tensile behavior of hybrid composites has shown to be a difficult task. Even though models have since been presented, there is still no model that can adequately predict the hybrid composites behavior up to failure [27].

Tavares et al. [28] designed three relevant models with increasing complexity to study hybridization in composites. The most straightforward model uses a dry bundle of fibers, that does not consider the presence of a matrix, and it is mainly used to understand the effects of fiber strength distribution in a composite. It is based on the works of Calard and Lamon [2]. Tavares et al. concludes that to obtain a progressive tow failure, strength distributions in both fibers must be continuous. Conde et al. [4] used the most straightforward model described above to formulate an optimization problem for a commercial software to solve, using Genetic Algorithm (GA), for pseudo-ductile purposes. For the parameterization of the stress-strain curve, some criteria were selected to quantify the pseudo-ductility and some other defined objectives. In his work, he used a weighted objective function and a classical multi-objective approach.

Marler and Arora [15] define a multi-objective optimization problem as follows:

$$\begin{aligned} \underset{\boldsymbol{x}}{\text{Minimize}} \quad & \boldsymbol{F}(\boldsymbol{x}) = [F_1(\boldsymbol{x}), F_2(\boldsymbol{x}), \ldots,\ F_n(\boldsymbol{x})]^T \\ \text{subject to:} \quad & g_j(\boldsymbol{x}) \leq 0,\ j = 1, 2, \ldots, m. \\ & h_l(\boldsymbol{x}) = 0,\ l = 1, 2, \ldots, e. \end{aligned} \tag{1}$$

The vector $\boldsymbol{x}$ denotes the decision/design variables and $\boldsymbol{F}(\boldsymbol{x})$ is a vector of objective functions. The parameter n represents the number of objective functions, m the number of inequality constraints, and e the number of equality constraints. The vector that minimizes $\boldsymbol{F}(\boldsymbol{x})$ is denoted by $\boldsymbol{x}^*$, and is known as the optimal solution. Other two important concepts are: feasible decision space, $\boldsymbol{X}$, which refers to the constraint set, and feasible criterion space $\boldsymbol{Z}$, that specifies the attainable set.

Pareto [21] defined the main concept of an optimal point in a multi-objective optimization as Pareto optimality:

Definition 1. *Pareto Optimal: A point, $\boldsymbol{x}^*$, is Pareto optimal iff there does not exist another point, $\boldsymbol{x} \in \boldsymbol{X}$, such that $\boldsymbol{F}(\boldsymbol{x}) \leq \boldsymbol{F}(\boldsymbol{x}^*)$, and $F_i(\boldsymbol{x}) < F_i(\boldsymbol{x}^*)$ for at least one function.*

There are a considerable number of approaches to deal with multi-objective problems. Marler and Arora presents a survey of the most relevant ones, categorizing them as methods with a posteriori articulation of preferences, methods with a prior articulation of preferences and method with no articulation of preferences [15].

Evolutionary algorithms (EA) are non-classic, unorthodox and stochastic optimization algorithms that are a type of meta-heuristics. These can replicate nature's principles in a search for an optimal solution. One of the great novelties regarding these algorithms is the usage of populations of solutions in each iteration, that can be very useful either for single solution problems, where all the individuals converge for it, and for multi-objective, where it can obtain multiple optimal solutions. This type of programming is also much more efficient in a discrete search space [7]. One of the most used EA are the so-called Genetic Algorithms (GA). Their extensive applicability, ease of use and general perception are the main reasons for their success [11]. There is no correct version of the algorithm because the user must adapt it to each problem.

3. Proposed Solution

The analytical model for UD bundles previously introduced was extended to accommodate four different types of fibers. A flowchart of this modification is presented in Figure 1. Similar to those studies, the required properties of each fiber are the same. Summarizing:

- E – Young modulus.
- R_f – Fiber radius.
- σ_0 – Weibull scale.
- m – Weibull shape.
- L_0 – Characteristic length for the Weibull parameters.

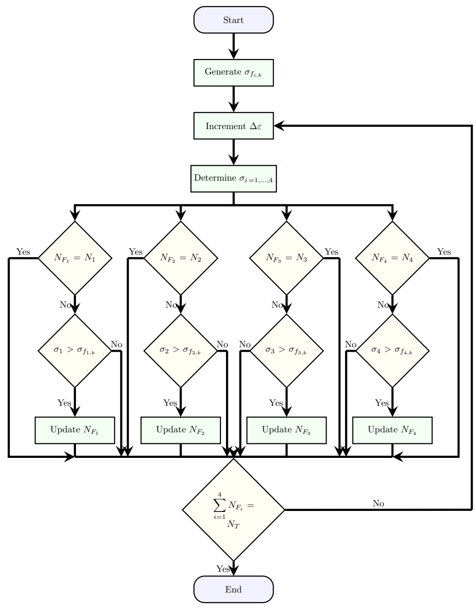


Figure 1: Flowchart of the model for dry bundle failure for hybridization with 4 fibers.

One of the fundamentals of this model is that fiber strength is considered to be described by a Weibull distribution:

$$P(\sigma) = 1 - \exp\left[-\frac{L}{L_0}\left(\frac{\sigma}{\sigma_0}\right)^m\right] \tag{2}$$

where $P(\sigma)$ is the probability of failure of a fiber with length L when subjected to a tensile loading σ. Weibull scale, σ_0, and shape, m, parameters, are calculated at a reference length L_0 from a bundle of N_T parallel fibers with radius R_f and length L. To determine the strength of each fiber the following expression is used:

$$\sigma_f = \sigma_0 \left[-\frac{L_0}{L} \ln{(1-X)}^{\frac{1}{m}} \right] \tag{3}$$

in which the probability $P(\sigma)$ from Equation 2 is replaced by a real random number X between 0 and 1.

For simulating a tensile test, one must firstly select the total of fibers, N_T, a standard length for them, L, and the desired increment of strain $\Delta\varepsilon$ at each iteration. The number of fibers in the bunch of each type is calculated according to the following equation:

$$N_i = \frac{N_T \, \upsilon_i \prod_{\substack{j=1 \\ j\neq i}}^{4} R_{f_j^2}}{\sum_{k=1}^{4} \upsilon_k \prod_{\substack{l=1 \\ l\neq k}}^{4} R_{fl}^2} \tag{4}$$

knowing that N_i represents the number of fibers of type i, V_i the volume occupied by N_i, and υ_i the percentage of fiber of type i in the composite.

The model starts by generating the strength of each fiber, $\sigma_{f_{i,k}}$, where k denotes the fiber number of type i, applying the Weibull distribution (Equation 3). Then, strain is incremented, and the current stress applied to each sort of fiber using the Elastic Modulus of one another is evaluated:

$$\sigma_i = E_i \, \varepsilon \tag{5}$$

Assessing each fiber individually, if the imposed stress, σ_i, is larger than the generated rupture stress $\sigma_{f_{i,k}}$ the fiber breaks; thus the variable that stores the number of broken fibers of each kind, N_{F_i}, is incremented. Assuming a global-load-sharing, the load is uniformly redistributed by the unbroken ones. The overall stress in the composite is determined by:

$$\overline{\sigma} = \frac{F}{N_T \sum_{i=1}^{4} S_i \, \upsilon_i} \tag{6}$$

with the sectional area occupied for each fiber i, S_i given by:

$$S_i = \upsilon_i \pi \, R_{f_i}^2 \tag{7}$$

The strain value is continuously augmented until the sum of all broken fibers is equal to the total number of fibers, N_T, defined initially. The overall stress value at each increment of strain is cached, allowing, at the end of the computation described, to plot a stress-strain curve of the combination assessed. Every test uses a settled number of fibers, N_T, and a standard length, L, that is 500 and 75 mm, correspondingly. The value used for the increment $\Delta\varepsilon$ is 10^{-5}. These values will remain the same throughout the remaining work.

Prior to the formulation of an optimization problem, information of known fibers was gathered. For the sake of comparison with previous works [4, 28], an identical database was used and is disclosed in Table 1.

#	**Material**	**Fiber**	E_0 **(GPa)**	R_f **(μm)**	σ_0 **(MPa)**	m	L_0 **(mm)**
1	Carbon	HTS [1]	230.0	3.500	4493.0	4.80	19.0
2	Carbon	X5 [19]	520.0	5.050	2500.0	6.10	25.0
3	Carbon	AS4 [6]	234.0	3.500	4275.0	10.70	12.7
4	Carbon	T300 (1) [6]	232.0	3.500	3170.0	5.10	25.0
5	Carbon	T300 (2) [23]	232.0	3.500	3200.0	5.50	30.0
6	Carbon	T300-B4C [23]	232.0	3.500	3150.0	5.40	30.0
7	Carbon	700°C [26]	55.0	3.300	1400.0	11.00	10.0
8	Carbon	100°C [26]	240.0	2.900	4500.0	4.50	10.0
9	Carbon	T800G [26]	295.0	2.750	6800.0	4.80	10.0
10	Carbon	M30S [26]	295.0	2.800	6400.0	4.60	10.0
11	Carbon	M40S [26]	380.0	2.700	4900.0	5.20	10.0
12	Carbon	M50S [26]	480.0	2.650	4600.0	9.00	10.0
13	Glass	E-Glass (1) [20]	76.0	6.500	1550.0	6.34	24.0
14	Glass	E-Glass (2) [8]	66.9	7.800	1649.0	3.09	20.0
15	Glass	E-Glass (3) [22]	70.0	5.000	2300.0	3.60	10.0
16	Glass	AR-HP [10]	70.0	7.000	1363.0	9.60	60.0
17	Glass	AR-HD [10]	70.0	7.000	876.0	4.80	60.0
18	Kevlar	29 [18]	85.3	6.895	3445.8	11.80	25.0
19	Kevlar	49 [18]	149.1	5.135	4083.3	8.20	25.0
20	Kevlar	119 [18]	61.4	5.460	3101.2	11.80	25.0
21	Kevlar	129 [18]	99.0	5.790	3433.0	10.30	25.0

Table 1: Mechanical properties for different fibers.

It is previously stated that the probability of failure of a fiber, $P(\sigma)$, is randomly substituted by a real number X, between 0 and 1, when the strength of each fiber is estimated. Neglecting this factor may lead to an incorrect evaluation of fibers' combinations. A plausible solution to work around this problem is to determine an averaged stress-strain curve of a defined number of trials, n_{mean}. However, a trade-off between the computational time to estimate these essays and the exactness of the mean stress-strain curve must be defined. To measure the exactness of mean stress-strain curves, an estimator must be determined. For this, a set of one million trials, an unrealistic array, is averaged. It is impractical due to its computational time but allows to establish a close enough optimal value for this situation, $\hat{\sigma}$. Then, for each mean stress-strain with a correspondent n_{mean}, it is evaluated the mean-squared error (MSE), which measures the average of the squares of the deviations to the "optimal" value. However, due to the randomness applied in this design, there are no two equal mean stress-strain curves. For that reason, a converged mean of MSEs is considered. The computational time for each number of tests for the average stress-strain curve are also displayed in 2

The chosen value of n_{mean} for the rest of this research work is 60. Running 60 essays for a mean stress-strain curve takes an average time of 0.4 s in a personal computer and exhibits a mean mean-squared error of 50, which can be accepted.

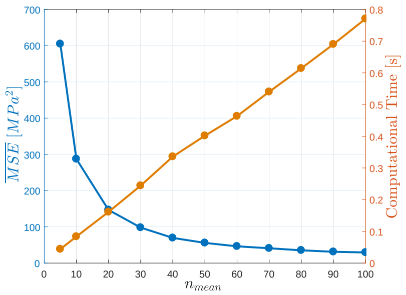


Figure 2: n_{mean} vs. Computational Time and $\overline{MSE}$. Approximated curves for $0 < n_{mean} < 100$.

It is already known that maximizing the pseudo-ductility is the main objective for this optimization. However, optimizing only that parameter results in a residual value of failure tension and with great amplitudes between the fracture stress and the minimum stress value in the plastic region. These type of results are not desirable, since it may lead to unstable behavior of the composite. To reach a good solution, it is crucial to obtain a stable baseline as high as possible. For that reason, this is a multi-objective problem. To obtain a useful solution, these criteria must be quantified. In Figure 3 (a) a set of parameters to calibrate the stress-strain curve are proposed. Figure 3 (b) displays an ideal solution following these guidelines.

The definition of pseudo-ductility strain was already presented, but analytically it can be assessed as the difference of strains in the pseudo-plastic regime. In other words, the percentage of strain where the sample loses its structural integrity.

$$\varepsilon_d = \varepsilon_{max} - \varepsilon_{E_0} \tag{8}$$

The virtual point ε_{E_0}, which indicates the starting point of the plastic region, is given by:

$$\varepsilon_{E_0} = \frac{\overline{\sigma}}{E} \tag{9}$$

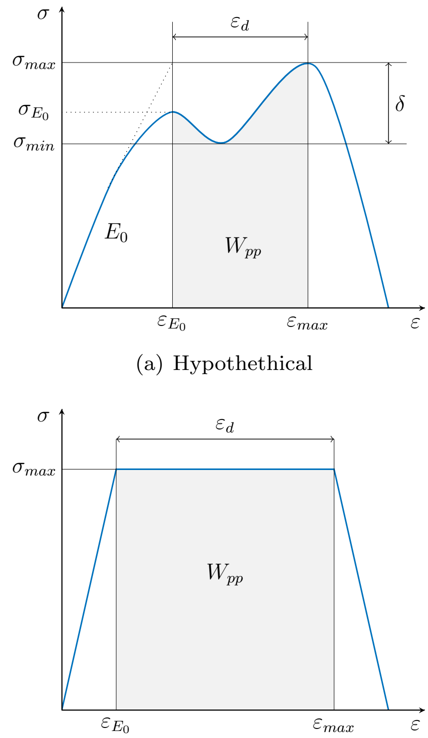
with the Young Modulus calculated through the linear rule of mixtures:

$$E = \sum_{i=1}^{4} \varepsilon_i \, E_i \tag{10}$$

The difference between the maximum and minimum stress in the pseudo-plastic domain, δ, was already defined by Conde [5] as a parameter.

$$\delta = \sigma_{max} - \sigma_{min}, \, \forall \, \varepsilon \in [\varepsilon_{E_0}, \varepsilon_{max}] \tag{11}$$

A novelty concept was also formulated to substitute some parameters used by Conde et al.. It can be defined by the pseudo-plastic work and represents the dissipated energy in that pseudo-plastic



(a) Hypothethical

(b) Ideal

Figure 3: Hypothetical and ideal response of a hybrid composite to tensile test and definition of parameters for optimization

region. The definite integral can be approximated using, for example, the trapezoidal rule, partitioning the integration interval:

$$W_{pp} \approx \sum_{k=1}^{N} \frac{\sigma(k-1) - \sigma(k)}{2} \, \Delta\varepsilon \tag{12}$$

with

$$N = \frac{\varepsilon_{max} - \varepsilon_{E_0}}{\Delta\varepsilon} \tag{13}$$

For a combination to be defined as optimal, a stress-strain curve must be as similar as possible to Figure 3 (b), which means:

- Maximum pseudo-plastic work: high as possible values of rupture stress and pseudo-ductility strain.
- A stable baseline, i.e. the value of the amplitude δ to be equal to zero.

To solve this optimization problem three different approaches were exploited. First, using a single objective function. Then, a weighted approach using two objective functions and finally, a standard multi-objective calculation. The different mathematical formulations are presented in the subsequent sections. The vector of variables, $\boldsymbol{x}$, used in these problems is given by:

$$\boldsymbol{x} = (f_1, f_2, f_3, f_4, \upsilon_1, \upsilon_2, \upsilon_3, \upsilon_4) \tag{14}$$

where f_i represent the fibers' index mentioned in Table 1, and υ_i recalls to the fiber's i fraction in the composite.

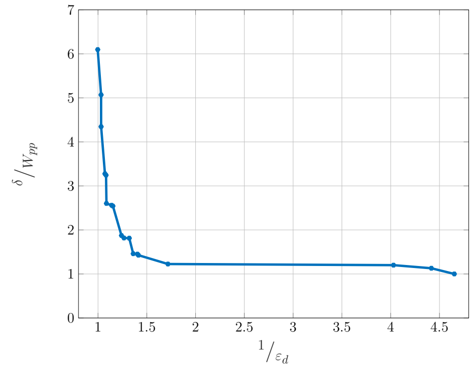
$$\begin{aligned} \underset{\boldsymbol{x}}{\text{Minimize}} \quad & \boldsymbol{F}(\boldsymbol{x}) \\ \text{subject to:} \quad & 1 \le x_i \le 21,\ i \in \{1,2,3,4\} \quad (1) \\ & 0 \le x_j \le 100,\ j \in \{5,6,7,8\} \quad (2) \\ & \sum x_j - 100 = 0 \quad (3) \\ & x_i \in \mathbb{N} \\ & x_j \in \mathbb{N}_0 \end{aligned} \tag{15}$$

The objective function uses the parameters earlier specified: the pseudo-plastic work, W_{pp}, which has to be maximized, and the amplitude between maximum and minimum stress, δ, which has to be minimized. W_{pp} is a specification that indirectly recalls ε_d. The amplitude criteria prevent a high difference of stress in the pseudo-plastic region.

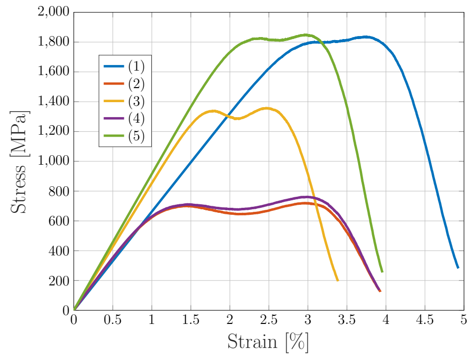
All constraints recall Equation 14 variables. The first one recalls what fibers index are available to evaluate, that varies from one to twenty-one, using only natural numbers. There is no constraint about having different fibers. This means that this formulation considers solutions with one, two, three or four distinct fibers. Next, the fraction of each fiber must be selected. These values may be natural numbers from zero to one hundred, bearing in mind that the sum of all fiber fractions must be equal to 100%, as it is explicit in Constraint (3) of Equation 15.

Using the analytical model explained and objective function defined above, a subproblem was tested: bundles of dry tows with two different types of fibers, as performed elsewhere [5]. This problem has exactly 20,790 solutions, plus the 21 results with one type of fiber. For this case, the mean stress-strain curves are an average of ten tests, $n_{mean} = 10$ since the goal is obtaining an idea of the possible results.

The main difference comparing with the formulations previously presented is that two fiber fractions value is equal to zero since are only bundles with at least two distinct type of fibers are considered. Figure 4 presents the results regarding this verification: (a) displays the Pareto Frontier for the problem above and (b) the most satisfactory results. Table 2 discloses the parameter values for the stress-strain curves exhibited in Figure 4 (b). Comparing to the results that were reached by Conde et al. [4], shows that the formulation has been correctly implemented and the parameters are accurately chosen.



(a) Normalized Pareto Frontier.



(b) Best Overall Results.

Figure 4: Model and formulation validation results.

#	Hybridization	υ_i [%]	W_{pp}/δ [−]	ε_d [%]
1	Kevlar 119 & 129	87–13	0.1543	0.9040
2	E-Glass (2) & Kevlar 29	69–31	0.1259	2.1577
3	HP-AR & Kevlar 129	37–63	0.1194	1.0664
4	Kevlar 29 & 49	83–17	0.1099	1.1628
5	E-Glass (3) & Kevlar 119	61–39	0.1082	2.5739

Table 2: Parameters value regarding combinations with two different fibers.

A first analysis of the non-dominated combinations, shows that results that are on the right side of the graphic are more satisfactory than the ones that prioritize pseudo-ductility, since fibers of type 1 have less strength comparing with fibers of type 2, and most of them break before fiber of type 2 begin to fracture. It is an expected event, considering the fibers parameters (Table 1) and their strength (Equation 3). Taking into account this type of situations, the right side of the Pareto Frontier will have more prevalence during the optimization algorithms computation.

The algorithm used for optimization is the Genetic Algorithm briefly earlier. In this "in-house" algorithm, there are three variables predefined before each run: number maximum of generations/iterations, g_{max}, population size, p_{size} and fraction of a new population that is recombined by two individuals from the earlier population, $C_{\%}$. The generation/iteration in which the GA is in is denoted by g. The algorithm ends each run when the maximum number of generations is reached. This means there is only one stoppage criterion. Each step of the method is disclosed next.

here are two methods for the genesis of the initial population. The main approach loads a set of individuals that were previously evaluated in an exhaustive computation of possible combinations of fibers. Solutions were stored in a cache file and will provide the non-dominated points so far, in case of the multi-objective approach and the best fitness individuals for the single objective goal.

In the other procedure, a population is randomly conceived, and it is assessed how close the individuals will get to the non-dominated ones.

To evaluate an individual, a simple algorithm was designed. First, the number of different fibers is determined. The algorithm can generate combinations where there are equal values of f_i or when there is, for example, a fiber fraction value equal to 0. For each number of different fibers, there is a cache variable, where all the evaluated combinations and respective parameters are stored. The program searches for the individual in that file, and if it does not find the specific hybridization, it calculates the required parameters and stores them in that cache. Otherwise, the algorithm calls up the stored values.

The selection operator is one of the leading features that distinguishes Genetic Algorithms from other meta-heuristics. Multi-objective selection operators are more complicated than single objective ones. For this case, a cell-based density approach is used, since the more valuable results appear on the right side of the Pareto Front. In this method, the solutions space is divided into even cells, where the most valuable ones get a higher score [14]. For this particular problem, solutions with more preponderance of ${}^{\delta}/_{W_{pp}}$ receive a higher rate, and are thus selected for the next generation. For the single-objective approach, the fittest individuals are selected for the next generation, alluding to a term known in this field as Elitism. In both cases, neither population accepts a repeated solution, helping the set of results to be as diverse as possible.

In either case, the recombination used is known as arithmetic/average crossover and can be defined as a linear combination of two parents, previously selected. Parents are no more than two individuals selected from the prior population. The selection of parents is performed using the roulette wheel selection method. To either individual, a cumulative probability of being chosen is attributed, accordingly with their fitness.

In every generation, a fraction of the population is allocated for mutated individuals. These are calculated using a Gaussian distribution with two variables of control: scale, S, that specifies the standard deviation of the operator at the first generation and shrink, s, that controls the rate at which the average amount of mutation diminishes.

4. Results

An exhaustive calculation of combinations of fibers and volume fractions was collect, before the optimization was undertaken. The original problem has exactly 6,458,012 and 704,252,900 possible solutions with three and four different fibers, respectively. Recalling the computational effort evaluated, calculating the complete set of solutions would require roughly two years, if the number of trials used for the mean stress-strain curve, n_{mean}, was 10. This drawback encourages the employment of a subproblem of that, to collect an initial range of solutions.

The inital problem has 10 non-dominated points with three fibers and has 23 non-dominated points with four fibers. These points will be used in the first population of the multi-objective formulation.

The GA has many parameters which had to be analyzed for them to be appropriate to this problem. These variables are population number, a fraction of the population that is recombined, and therefore the portion that is mutated, and, finally, the number of generations. Each specification is evaluated individually with the others settled. Parameters were assessed using the single-objective approach, since it was simpler to determine the best combination of specs, knowing that the best solution will be unique, contrasting with the multi-objective, where there is more than one optimal solution. Parameters were evaluated for an selected initial population where the primary goal is to reach the optimal hybridization and from a random set of individuals to assess the quality of the algorithm.

The selected parameters for a selected population are:

$$C_{\%} = 60,\ p_{size} = 100,\ g_{max} = 75 \tag{16}$$

a random population the same parameters defined are:

$$C_{\%} = 10,\ p_{size} = 100,\ g_{max} = 200 \tag{17}$$

Figure 5 and Table 3 display the best results of the optimization using above parameters, for two, three and four different fibers and the respective parameters value.

#	**Hybridization**	υ_i [%]	W_{pp}/δ [−]	ε_d [%]
1	HP & Kevlar 49 & 119 & 129	19–11–61–9	0.2059	1.8217
2	Kevlar 49 & 119 & 129	14–75–11	0.1943	1.5752
3	Kevlar 119 & 129	88–12	0.1399	0.8389

Table 3: Parameters values for the single objective approach (see Figure 5).

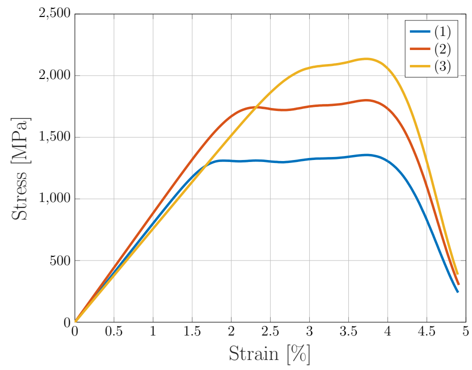


Figure 5: Best results for a single objective approach with different number of fibers.

As in the preliminary tests, Kevlar fibers dominate the results, mainly due to their high modulus and strength. Furthermore there is a dominant fiber which is Kevlar 119, that can be considered the HE fiber, when compared to the traditional hybrid composites.

Since the Pareto Front for this formulations is relatively convex, the solutions from the weighted approach are similar to the ones presented in the next subsection, depending on which zones are pretended in the frontier.

The multi-objective algorithm was ran with the same parameters as the single objective approach. After some runs, it was possible to obtain the following non-dominated frontier solutions normalized (Figure 6).

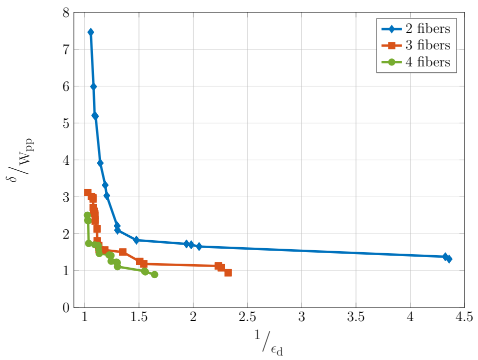
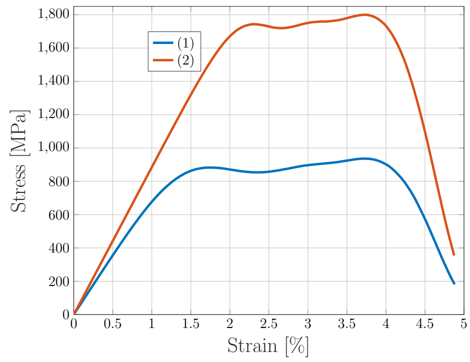
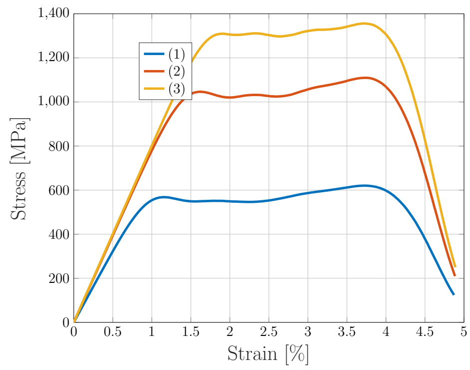


Figure 6: Non-Dominated solutions for different number of fibers

Comparing with Pareto Frontier of hybridization with two fibers, it is possible to observe some progress relative to the non-dominated points for hybridization with three and four fibers. Not being sure that these points are optimal, it is preferable to call them non-dominated instead of referring to them as part of the Pareto Frontier. The most satisfactory results are on the right side of the graphic, being the solutions on the extreme right side, already presented in Figure 5. Other relevant solutions are shown in Figure 7.



(a) Three Fibers Optimization Results.



(b) Four Fibers Optimization Results.

Figure 7: Relevant outputs regarding the Non-Dominant Front (see Figure 6).

#	Hybridization	v_i [%]	W_{pp}/δ [−]	ε_d [%]
1 (a)	E-Glass (3) & Kevlar 119 & 129	53–42–5	0.1473	2.4266
1 (b)	E-Glass (3) & HD & Kevlar 119 & 129	34–32–31–3	0.1509	2.8131
2 (b)	E-Glass (1) & Kevlar 49 & 119 & 129	34–9–51–6	0.1758	2.1834

Table 4: Parameters values for the multi-objective approach results (see Figure 6)

The last stress-strain curve, in each Figure 7, is the extreme right side of the frontier, and was also displayed in Figure 5. The parameters of those can be recalled in Table 3. Furthermore, kevlar fibers are still the most dominant fibers, in both cases. It is also important to highlight that in the Figure 7 (a), the combination has similar fiber fractions of the three first fibers. It is fair to assume that kevlar fibers increase the strength and the modulus of the hybrid composite, and glass fibers improve their pseudo-ductility. Some solutions in the frontier are not displayed, since, they were combinations of the fibers presented but with slightly different fiber fraction values.

5. Conclusions

The primary goal of this research work was to understand if hybrid composites with more than two different fibers can perform better than traditional hybrid composites with two distinct fibers, regarding pseudo-ductility. The main findings from this research work are summarized and listed below:

1. To optimize discrete problems with a high number of possible solutions, a Genetic Algorithm reveals itself as a powerful tool. Although, if a set is too large, like in these situations, some initial information is required to achieve the optimal solution. Both results from a commercial software and from in-house algorithms reach similar outcomes, which answered research question 1.

2. The pseudo-plastic work, W_{pp}, is a robust parameter to evaluate combinations of hybrid composite materials, and using this criterion with the amplitude between the maximum and minimum stress in the pseudo-plastic region, δ, permits a reduction of complexity of the problem formulated in earlier works.

3. Using a single objective formulation with ${}^{\delta}/_{W_{pp}}$ is possible to obtain solutions from a Pareto Frontier of a multi-objective approach using this criterion and the pseudo-ductility. However, using a multi-objective is possible to achieve a different type of results, with more pseudo-ductility strain, but with a lower rupture stress.

4. Mixing three or four distinct fibers present solution with increasing values of pseudo-ductility with a stable baseline, when compared with with traditional hybrid composites. Results do not achieve a maximum stress as high as supported by the optimal solution with two distinct fibers, the answer to research question 2.

5. Kevlar fibers, in particular, Kevlar 119, play a bigger role in achieving pseudo-ductility with reasonable strength, due to their high modulus and stiffness.

6. Even though the model only assesses dry bundles of fibers, it is fair to assume that using a compatible matrix to bind these fibers it is possible to reach an optimal composite material regarding pseudo-ductility, which answered the reserach question 3.

With all the secondary inquiries answered, the main research question of this disstertation was answered: from the obtained results, it can be concluded that hybrid composites with more that two different fibers perform better than traditional hybrid composites.

Even though, the solutions drawn are fairly positive, there are still further research and improvements to be made in this topic. Some suggestions for future work are the following:

1. Adapt the model designed to a model with a matrix, to perceive how much improvement is attained of the usage of more than two distinct fibers.

2. Extend the list of fibers' possibilities to hybridize, to evaluate different combinations, regarding, of course, if they are compatible as Kevlar, Carbon and Glass are.

3. Usage of different meta-heuristics or machine learning algorithm to solve this problem.

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