Application of nonlinear models to study discontinuity regions of structural concrete

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May 2018

Abstract

In the era of contemporary architecture many non-current shapes have been brought to the construction industry, therefore engineers must face these bigger structural challenges, knowledge on the structural concrete elements modelling is of extreme importance, namely in static and/or geometric discontinuities regions. It is presently recognised that tension field models and strut and tie models are a preferential tool to study, design and detailing of structural concrete structures. However, there is a lack of defined criteria to regulate the freedom the designer has when designing strut and tie models.

In this project simply supported deep beam with direct and suspended loads are studied. Many levels of internal stresses redistribution, or deviations from the elastic solution, which is greatly known, were considered to assess the scale of the greatness of these deviations without compromising the final capacity or the good serviceability behaviour. Also, many different reinforcement levels were studied to assess if this was a changing factor in the deviation capacity.

Based on strut and tie models design, the deep beams were detailed, and their structural performance was assessed based on the results provided by a finite element numerical non-linear analysis, both standard deep beams and deep beams with suspended load were studied. Many simplifications were considered, such as the behaviour of the steel and concrete mainly to establish a direct relation between the stress fields models and the finite element analysis.

This parametrical analysis helped to have a better understanding of the behaviour of these regions and to give some criteria to the designers on how far they can go from the elastic trajectories.

Keywords: strut and tie models, stress fields models, deep beam, discontinuity regions, finite element methods, non-linear analyses.

1 Introduction

The main goal is to assess principles and criteria to adopt while selecting deep beams strut and tie models, ensuring a suitable serviceability behaviour and good levels of ductility in the ultimate limit states. There is in any structural element some stress redistribution capacity, however deep beams have a higher redistribution capacity mainly in the ultimate limit state.

In the present day, the strut and tie models and the stress field models are based in the plasticity theory, with no compatibility is considered are mainly used to assess deep beams. By studying various models changing the lever arm in each one and by analysing the behaviour it can be seen which models have a good or bad serviceability or ultimate behaviour. By doing this it is possible to know which reinforcement levels result in a good structural behaviour. Knowing the reinforcement levels, the lever arms can be known as well and so it can be known how far from the elastic trajectories can the designer go without compromising the structural behaviour. There are some reference models studied and tested by Leonhardt e Walther 1966 based on the elastic trajectories and a great number of experimental results, (Schlaich & Schafer, 1991).

2 Strut and tie models

2.1 General procedures

It is acknowledged that strut and tie models are a useful tool while assessing, designing and detailing discontinuity regions. However, this methodology is not yet fully disseminated, doe to various reasons. One of the most spoken aspect is the fact that no compatibility is not accounted for, leading to a great freedom in the model choosing prosses, resulting in doubts related both with the structural capacity to adapt to the chosen model without a brittle failure and to guarantee a good serviceability behaviour. To avoid these problems, it is frequent do create models following the elastic trajectories. This solution guarantees a good serviceability behaviour; however, it lacks predicting some ultimate limit capacity that would be considered with the plasticity theory. (Muttoni, Schwartz, & Thurlimann, 1997) If the main goal is to calculate the ultimate limit capacity, the model can be changed to a higher capacity format, paying attention to the internal stress redistribution capacity.

2.2 Associating the load path method with stress diagrams

The strut and tie model choosing process can be somehow compered to choosing a static system to model some structure, both require some designing experience and are of extreme importance to the structure. There are some procedures that can help when it comes to create the load path method and consequently the strut and tie model. To do so it is usual to introduce stress diagrams taken from finite element programs to help orientating the paths. Then the struts are orientated according to the compression diagrams and its directions, the ties according to the tension diagrams. All the spoken diagrams must represent the elastic stresses as the load paths must represent the elastic trajectories of the load. (Schlaich, Schafer, & Jennewein, 1987)

2.3 Models selection process

Since strut and tie models must only verify the equilibrium equations, many different models are possible. While choosing the final models some doubts related with the good or bad approximation with the real structure might appear, which model would be the perfect one. It is difficult do foresee the exact behaviour of the structure when there are such complex materials involved as the concrete etc. In addition, for the same structures there are a big number of models which would work. Is then needed a tool to foresee which would be the best model for the structure. While selecting model it is very important to know that the structures always chose the shortest way to take the loads (Muttoni, Schwartz, & Thurlimann, 1997), these paths also lead to smaller deformations. While concrete breaks and opens cracks, the reinforcement steel stretches and take great deformations, it is why usually the models with less and shorter reinforcement bars are the best ones. This criterion can be formulated as following:

$$\sum F_i l_i e_{mi} = minimum$$

Where:

$F_i = force\ in\ the\ strut\ or\ tie\ i$

$l_i = \ lenght\ of\ the\ element\ i$

$e_{mi} = principal\ strin\ of\ the\ element\ i$

3 Models and methodology

3.1 Models simplifications

Since the results are being withdrawn from a FE a high computational processing power is required, as the deep beam is a symmetrical structure, the symmetry simplification is doubly beneficial, it means working with a smaller structure which leads to less finite elements and consequently to less processing time and computational power, also the result that are relevant are exactly in the symmetry axis.

A tall deep beam was adopted so there was no problem related with setting a top boundary to how much the structural lever arm could rise, it means, the Z/L relation is able to increase freely so that the compressed strut does not gets close to the top boundary. If instead shorter deep beams were considered the Z/L relation would be limited by the deep beam height conditioning the stress redistribution capacity. See Figure 3.2 and Figure 3.1. The same way the supports dimensions and the concrete's strength were defined so that the failure would not happen due to lack of concrete capacity in the supports but the reinforcement yielding.

Related with the reinforced concrete's tie behaviour it is acknowledged that it is related with the effective width of the concrete around the reinforcing bars. This area was assessed from the idea of the minimum reinforcement quantity, proceeding the following way: an elastic analysis was made so that the height of the tensioned concrete when it reaches its highest value fct is known. Integrating the stresses in this tension zone and the maximum traction force before concrete cracks is obtained, from that force it is possible to define the area of reinforcement, which leads to the minimum reinforcement. The area of concrete around the reinforcement bars is obtained by assuming that all the area is stressed with fct and the resultant force is equal to the reinforcement yielding force.

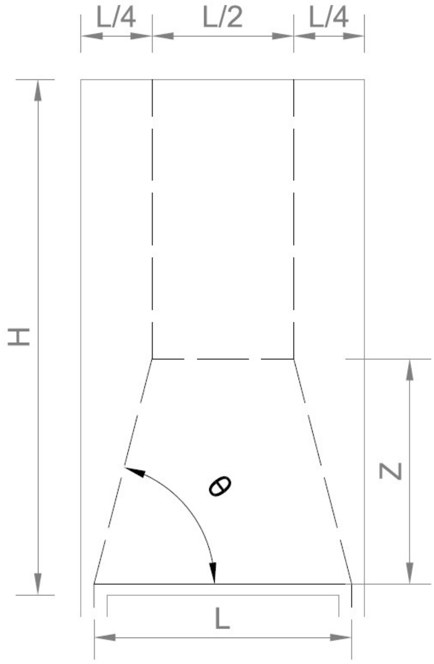

Figure 3.2 –Model and dimensioning geometrical parameters for the deep beam directly loaded

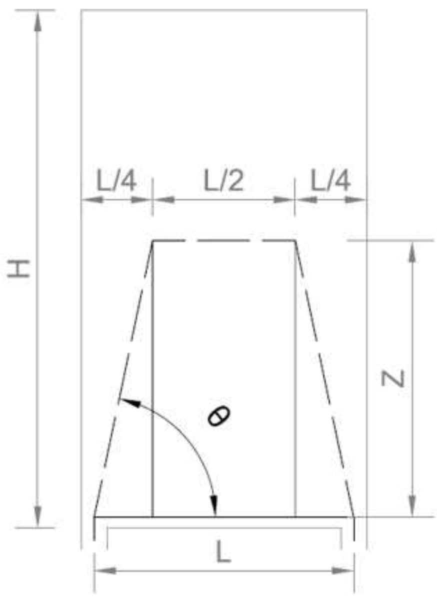

Figure 3.1 – Model and dimensioning geometrical parameters for the deep beam with suspended load

.3.2 Parametrical analyses

A parametrical analysis was done, the lever arm of the structure (represented by Z in the Figure 3.2 and Figure 3.1) was the changing parameter and consequently the θ angle is also changing as shown in equation (1). It is important to notice that small changes in the θ angle leads to the reduction of considerable amounts of reinforcement (T_s) as it is possible to see by equation (2), the amount of reinforcement varies with the cotangent of θ. Various models with different reinforcement were studied making use of a FE program, also various levels of loads were considered.

$$T_s = \frac{q_{ed}L}{2}\cot\theta \qquad \text{eq (1)}$$

$$\cot\theta = \frac{L/_4}{Z} \qquad \text{eq (2)}$$

The Table 3.1 and

Table 3.2 ahead shows the normal deep beams parameters taken into account in the FE programs, it can be seen in the tables that only the load and the correspondent reinforcement is changing, meaning, the Z/L relation remains unchanged for the same percentage of reinforcement in the different tables. It is important to know that the following tables represent equivalent beep beams but with different reinforcement levels, one is highly reinforced and the other has less reinforcement. The same thing appends in deep beams with suspended load also studied in this project, with the difference of the constant existing suspended reinforcement that is $A_{susp} = \frac{q_{ed}}{f_{ysd}} = \frac{1000}{43.5} \approx 23\ cm^2/m$ for the beam loaded with 1000KN/m and $A_{susp} = \frac{q_{ed}}{f_{ysd}} = \frac{3000}{43.5} \approx 69\ cm^2/m$ for the beam loaded with 3000KN/m.

Table 3.1 - strut and tie parameter's resume table to a deep beam loaded with 1000KN/m

	8 ϕ 20	6 ϕ 20	4 ϕ 20	3 ϕ 20
Z/L	0.7	0.915	1.37	1.83
$A_s\ [cm^2]$	25.12	18.84	12.56	9.42
θ [degrees]	70.35	74.72	79.66	82.22
% Reinforcement	100%	75%	50%	37.5%

Table 3.2 - strut and tie parameter's resume table to a deep beam loaded with 3000KN/m

	24 ϕ 20	18 ϕ 20	12 ϕ 20	9 ϕ 20
Z/L	0.7	0.915	1.37	1.83
$A_s\ [cm^2]$	75.36	56.52	37.68	28.26
θ [degrees]	70.35	74.72	79.66	82.22
% Reinforcement	100%	75%	50%	37.5%

3.3 Finite element method used

The behaviour for the concrete is accessed according to a given a displacement field over the continuum, its corresponding strain field can be determined. In this work, the principal stress directions are assumed parallel to the principal strain directions and their values are obtained from them (this hypothesis is classical in the development of stress fields). The principal stresses are then computed from the principal strains. The concrete stress-strain response is considered elastic-perfectly plastic in compression and the tensile strength of concrete is neglected. (Fernández Ruiz M., 2007). The concrete is modelled using Constant Strain Triangle elements – 3 node triangles with 6 freedom degrees.

The behaviour of the reinforcing steel is modelled by a uniaxial response with a bilinear elasto-plastic law with strain hardening. It is implemented in the FE programs using 1D bar elements with an axial behaviour with 4 freedom degrees. (Fernández Ruiz M., 2007). Tension stiffening is considered by the constitutive relations.

To the nonlinear solving problem, a modified Newton Raphson method was used, modified because the number of elements was very big, so the calculation of the new stiffness matrix and its inverse was too long. The option was to maintain the same stiffness matrix through all the iteration process.

4 Results and Discussion

4.1 Deep beams loaded on the top

All the models show a similar behaviour:

- Before cracking, the stresses in the steel reinforcement bars are low, and the force lever arm Z/L is mainly constant. See Figure 4.2 and Figure 4.4.

- After Cracking of the bottom tie a quick increase in the reinforcement bars stresses are verified, which is as bigger as smaller is que quantity of reinforcement adopted. See Figure 4.1 and Figure 4.3.
- Step by step while the load is increasing, the stresses in the bottom reinforcement is practically constant during some steps, this behaviour is only possible due to a significant increase in the force's lever arm, it means, an important internal stress redistribution occurs ant this redistribution is as bigger as smaller is que quantity of reinforcement adopted.
- To end, to different load levels according to the different steel reinforcement, an increase in the force's lever arm associated with a increase in the reinforcement stresses is observed.

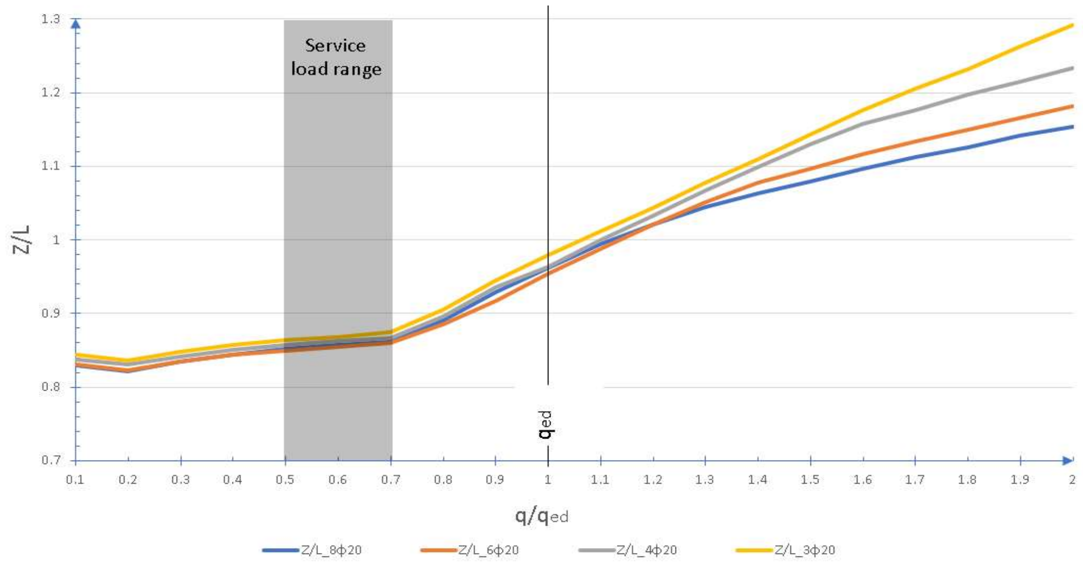

Figure 4.1 - Z/L variation with the applied load; qed = 1000KN/m; (Blue) elastic model 100% of reinforcement; (orange) 75% of reinforcement; (grey) 50% of reinforcement; (yellow) 37.5% of reinforcement.

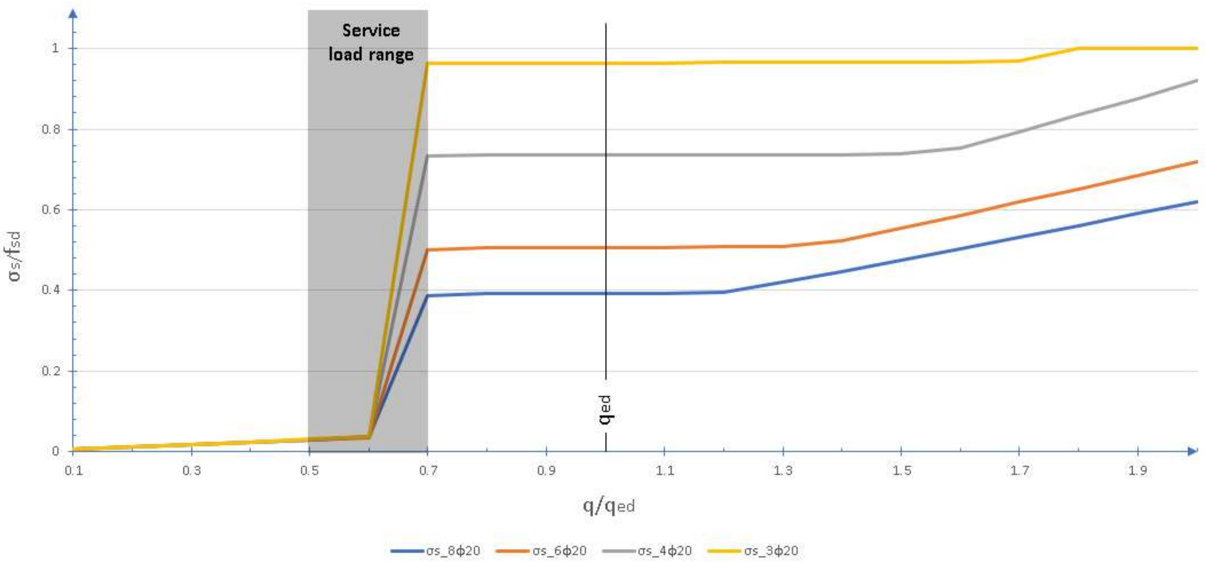

Figure 4.2 – Stress in the reinforcement bars variation with the applied load; qed = 1000KN/m; (Blue) elastic model 100% of reinforcement; (orange) 75% of reinforcement; (grey) 50% of reinforcement; (yellow) 37.5% of reinforcement.

It is important to notice that due to the assumed simplifications in these deep beams models, mainly, the established fact that the failure would occur due to reinforcement yielding, the failure load is higher that the design load.

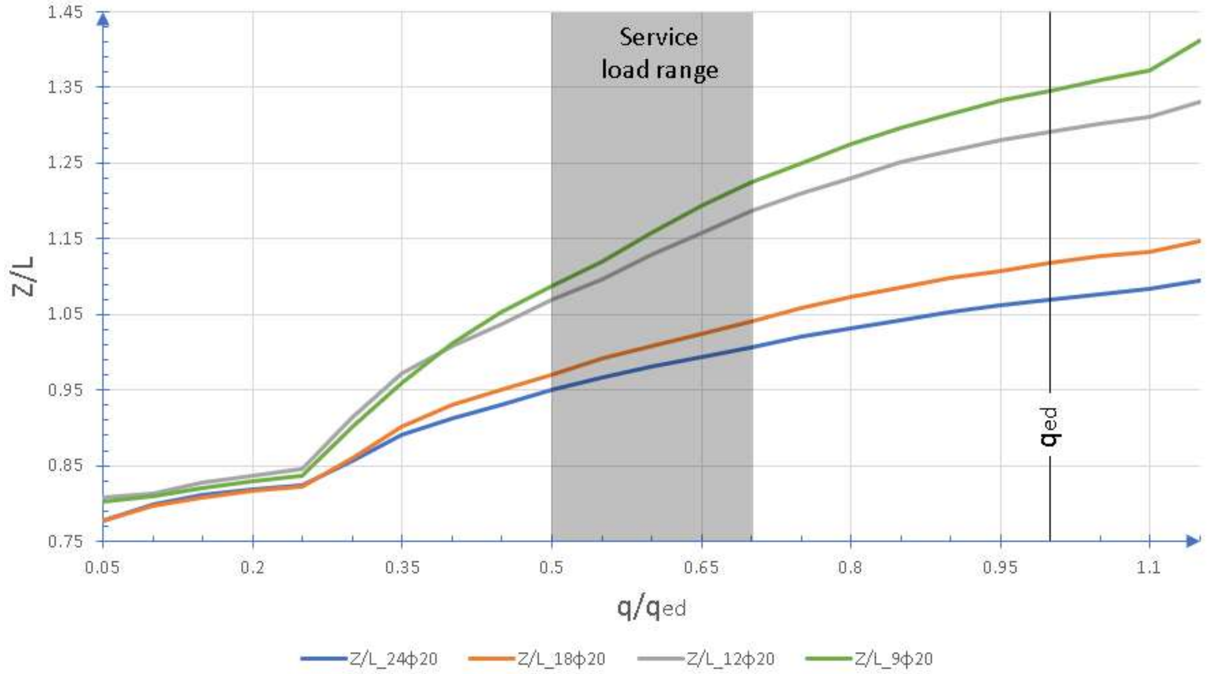

Figure 4.3 - Z/L variation with the applied load; qed = 3000KN/m; (Blue) elastic model 100% of reinforcement; (orange) 75% of reinforcement; (grey) 50% of reinforcement; (green) 37.5% of reinforcement.

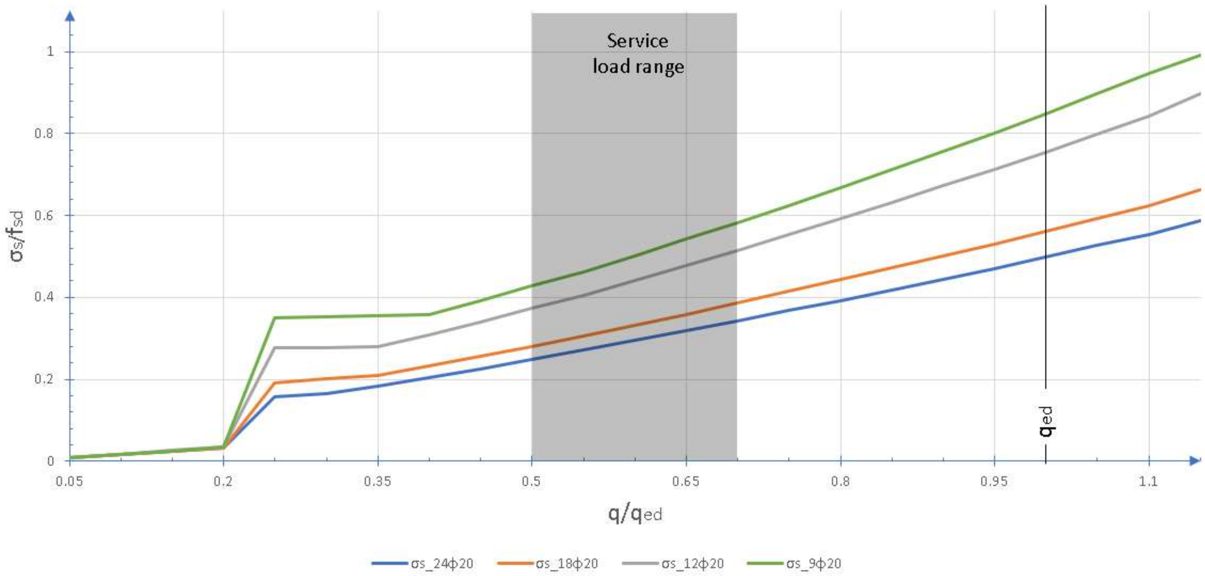

Figure 4.4 - Stress in the reinforcement bars variation with the applied load; qed = 3000KN/m; (Blue) elastic model 100% of reinforcement; (orange) 75% of reinforcement; (grey) 50% of reinforcement; (green) 37.5% of reinforcement.

4.2 Beep Beams with suspended load

These models are quite like the previous ones, however there are some differences, but in all the models it is possible to identify the following behaviour:

- Before cracking, the stresses in the steel reinforcement bars are low, and the force lever arm Z/L is mainly constant.
- After the cracking of the lower tie a fast increase in the reinforcement bars takes place and this increase is bigger when the reinforcement is lower.
- As the load increases the stress in the lower reinforcement is practically constant in some load steps, this behaviour is possible due to an increase of the force's lever arm, it means, a redistribution of internal forces takes place inside the deep beam. It is to be noticed that this internal redistribution is lower as the reinforcement is bigger. This redistribution is also lower when compared with the deep beams with direct load. See Figure 4.6 and Figure 4.8.
- Globally for higher load steps and accordingly with the different reinforcement levels, un increase of the force lever arm takes place in addition with an increase of the reinforcement stresses right after the concrete cracking. However differently from the direct loaded deep beams for higher loading a high increase in the reinforcement occurs while the force lever arm remains constant or even slightly decreasing. See Figure 4.5 and Figure 4.7.

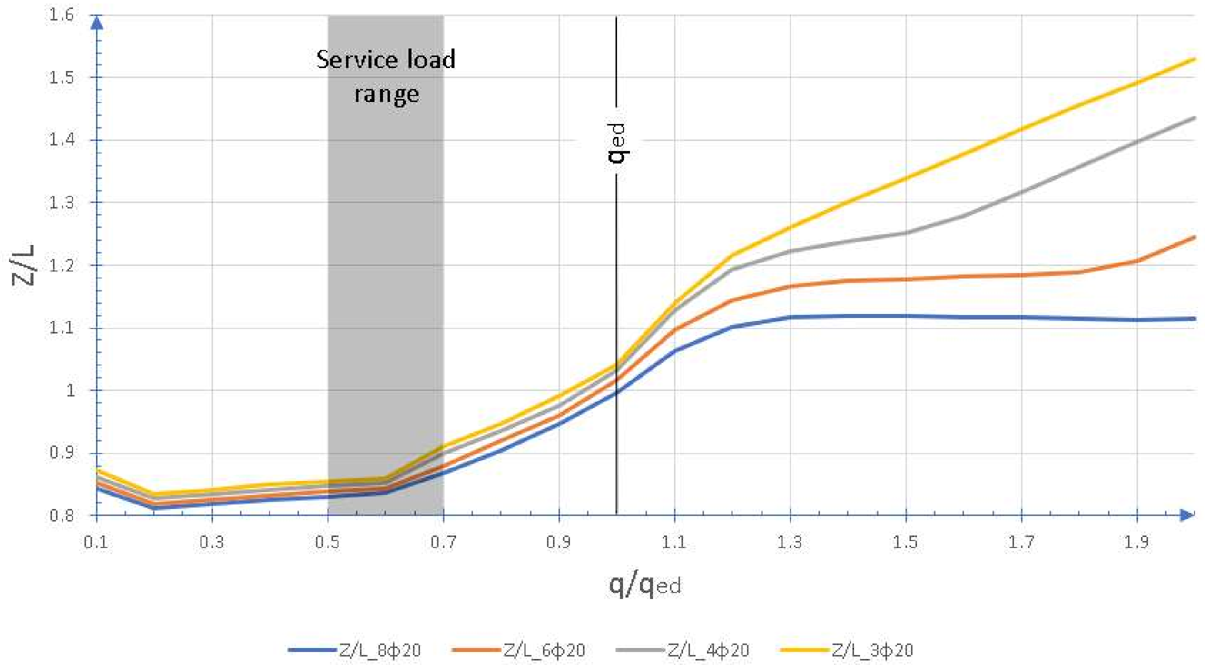

Figure 4.5 - Z/L variation with the applied load; deep beam with suspended load; qed = 1000KN/m; (Blue) elastic model 100% of reinforcement; (orange) 75% of reinforcement; (grey) 50% of reinforcement; (yellow) 37.5% of reinforcement.

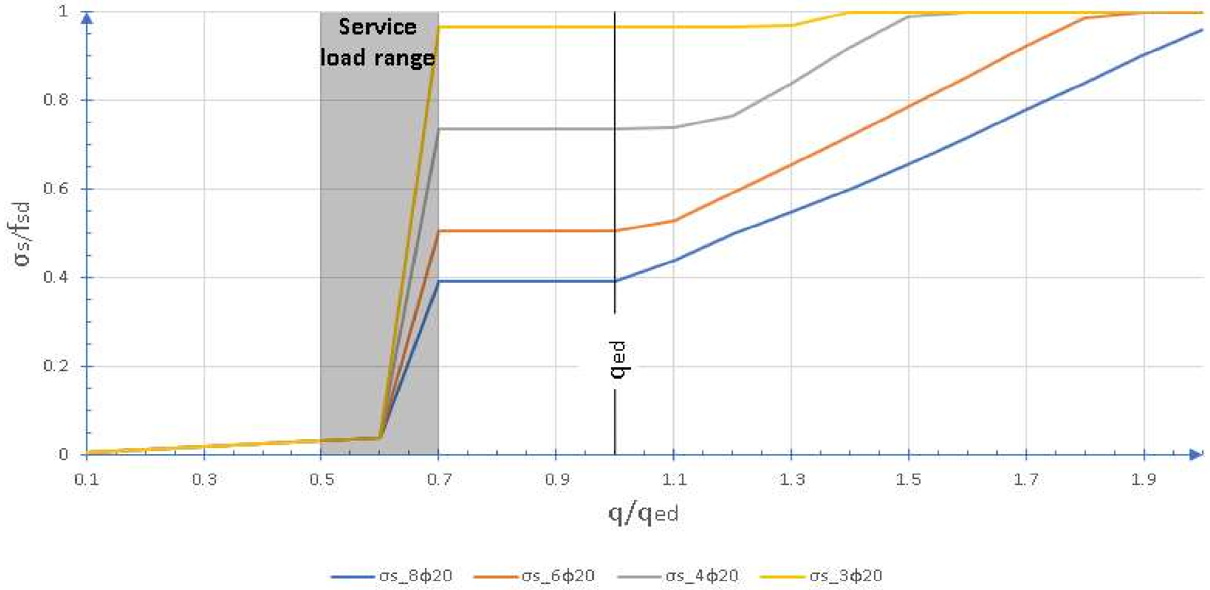

Figure 4.6 - Stress in the reinforcement bars variation with the applied load; deep beam with suspended load; qed = 1000KN/m; (Blue) elastic model 100% of reinforcement; (orange) 75% of reinforcement; (grey) 50% of reinforcement; (green) 37.5% of reinforcement.

This highly loaded deep beam with suspended load has a different behaviour of the top loaded deep beam because as seen before and more evident now in Figure 4.7 after the concrete cracking for 600KN/m the lever arm of the structure increases a bit but then it remains unchanged giving place to a big increasing in the reinforcing bars stresses (Figure 4.8), as a consequence the service behaviour is not so good for the less reinforced structures as the reinforcing stresses rise to 85% of the total capacity.

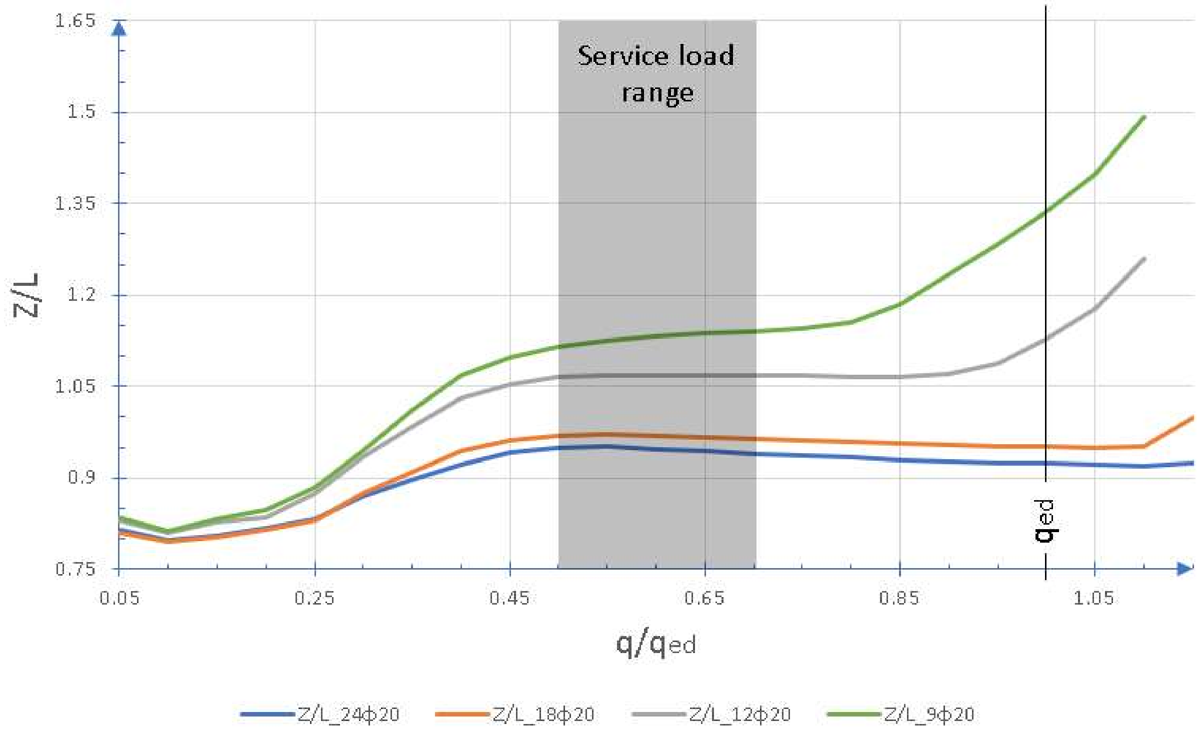

Figure 4.7 - Z/L variation with the applied load; deep beam with suspended load; qed = 3000KN/m; (Blue) elastic model 100% of reinforcement; (orange) 75% of reinforcement; (grey) 50% of reinforcement; (green) 37.5% of reinforcement.

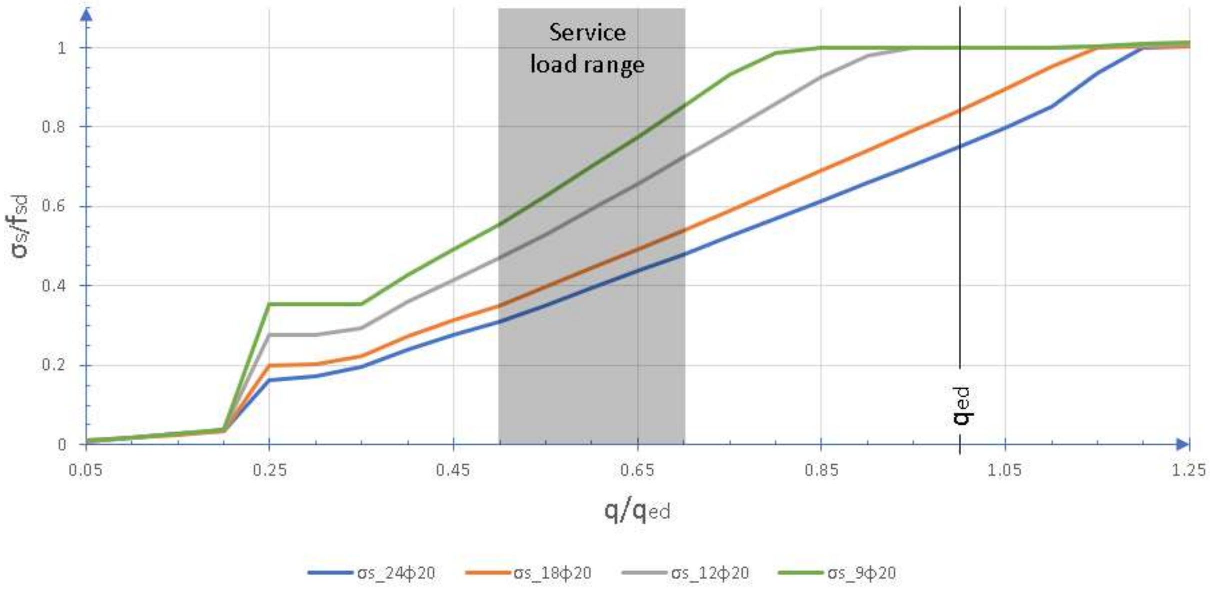

Figure 4.8 - Stress in the reinforcement bars variation with the applied load; deep beam with suspended load; qed = 3000KN/m; (Blue) elastic model 100% of reinforcement; (orange) 75% of reinforcement; (grey) 50% of reinforcement; (green) 37.5% of reinforcement.

5 Conclusions

- In all models a reduction of reinforcement was possible without compromising the service behaviour.
- This reinforcement reduction was higher in the highly loaded structures reaching up to 75% reinforcement reduction without affecting the serviceability behaviour and the load caring capacity
- In the deep beams with suspended load the possible reinforcement reduction was lower than in the beep beams directly loaded since the lever arm doesn't increase as much and the stresses in the reinforcement are higher. A smaller redistribution of internal stresses takes place.
- All sum up, the designer has some freedom while designing these models, this freedom is higher for the higher loaded models.

6 Bibliografy

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