ZFC Superconductor Magnetic Levitation System
”Maglev-Tuga”

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Abstract—This paper makes the comparison between two types of geometries for a magnetic levitating vehicle using YBCO zero field cooled (ZFC) high temperature superconductors (HTS) under: a rectangular geometry, using rectangular bulks of permanent magnets to make the tracks, and a car with superconductors, levitating above the magnetic track; and a cylindrical geometry, consisting of a cylindrical magnetic track, with equally spaced permanent magnets, and superconductors with the geometry of an half ring, levitating on the cylindrical track, with a constant radial air gap. The metric for comparison is the lateral stability of both geometries.

The analysis made in this paper is the following: for the rectangular geometry, the prototype is used to make an analysis to estimate its response to lateral displacements. For the cylindrical geometry, 3D simulations are made using a finite element method (FEM) software. The simulations made are for the dimensioning of the new cylindrical geometry, regarding the thickness and depth of the YBCO superconductor half rings, the spacing between the permanent magnets, and the air gap between the YBCO superconductor half ring and the permanent magnets. These studies are evaluated and chosen considering if their levitating forces are close to the gravitational force of the superconductor, to maintain a constant radial air gap, the two geometries are then compared, regarding their response to lateral displacements.

It was found that, the rectangular geometry presents a nonlinear response for a lateral displacement and the maximum displacement for which the system is still stable is around 21 mm. For the cylindrical geometry, the response is very close to linear and it has an inherent stability. However, the maximum displacements for the possible geometries can be lower than 21 mm, since the geometries that guarantee a stable levitation have a small air gap, and the air gap is closely related to the possible lateral movement, due to the nature of the geometry.

Index Terms—High-temperature superconductor (HTS), Magnetic Levitation (Maglev), Guidance Forces, Lateral Stability

I. INTRODUCTION

The studies presented in this paper are a continuity of previous research made, regarding the use of ZFC HTS for the design of a new magnetic levitating system [1] One main aspect was the comparison between field cooled (FC) and ZFC HTS. To prove such concept, the prototype of the rectangular geometry was developed, and studies regarding the levitating forces and lateral forces were made, as well as a study comparing the power losses of both cooling methods. The main conclusions of this research were that, although FC systems have a better stability than ZFC systems, their levitating forces are lower, and the guidance forces of FC systems are dependent on the initial cooling height. Lower levitating forces makes FC systems having more superconductors and magnets than ZFC systems, for the same application. This has implications on the costs between FC and ZFC systems, making FC systems more expensive. Regarding the guidance forces of FC systems, they are dependent on the initial cooling height, and are less strong for increased cooling heights, making the stability height dependent for FC systems, a situation that in ZFC systems does not happen. Also, the power losses are higher in FC systems, and can compromise their superconductivity state. One can conclude that ZFC systems present to be a better candidate for vehicle applications, since the objective for vehicle applications are to increase efficiency, to achieve higher speeds and have the capability of higher loads. In this paper, all the studies will focus on vehicle applications using ZFC HTS, being a continuation of the previous work stated.

This paper follows a proposal of Bruno Painhos future work [2] about the proposal of a new cylindrical geometry, and indicates that the lateral stability is better than the rectangular geometry.

II. SUPERCONDUCTIVITY

Superconductivity is a state that some metals obtain when cooled down below a certain temperature. There are two types of superconductors, type 1 and type 2. The main difference between them is the temperature at which they become superconductor. Type 1 superconductores become superconductors for temperatures lower than 10 K, while type 2 superconductors become superconductors at around 100 K.

A. Critical Region and Superconductive States

Superconductivity is characterized by three parameters: the critical magnetic field \( H_C \), the critical current density \( J_C \) and the critical temperature \( T_C \) [3]. If the material surpasses any of this values, it loses its superconductive state. Figure 1 shows the volume defining the superconductive region. Any of these parameters can be exceeded only locally, turning some parts of the superconductor to a normal state. These parts will turn superconductive again once the quantities in question fall back bellow the respective critical values [4].

Regarding the magnetic field density in which the HTS is present, there can be two possible superconductive states, which are represented in Figure 2. Those are the diamagnetic state and the mixed state.
B. Diamagnetic State and Meissner Effect

Regarding Figure 2, if the magnetic field density \( B \) is lower than \( B_{C1} \), the superconductor is in its diamagnetic state, and the superconductor repels the magnetic field that tries to penetrate it. This effect is called the Meissner Effect [3] [6] [4] and it is represented in Figure 3.

C. Mixed State and Flux Pinning

If the field is between \( B_{C1} \) and \( B_{C2} \), with \( B_{C1} < B_{C2} \), the superconductor loses its property of magnetic field expulsion and enters the mixed state. In this mixed state, the magnetic field lines penetrate the superconductor in non-superconductive regions, where impurities exist, and microscopic tubes are created, allowing the concentration of a magnetic flux. This causes an electric current to flow, and surrounds those tubes of magnetic flux, creating what is called as Abrikosov vortexes.

Due to the vortexes, in this mixed state, a magnetic field can be trapped inside the superconductor, making the superconductor being in a region of space where the net force is null. This effect is called flux pinning and it is represented in Figure 4.

III. LATERAL STABILITY OF THE RECTANGULAR GEOMETRY PROTOTYPE

A. System Description

The rectangular geometry prototype is shown in Figure 5. It is composed by two separated parts: i) one fixed part containing an array of 2x2 superconductor bulks and ii) one movable part formed by an array of 3x4 neodymium permanent magnets. The middle row of magnets is shifted and with an opposite magnetic polarization from the two other rows, as it is shown in Figure 5.

B. Description of the Lateral Movement

Regarding the lateral movement, there is a stable position for the superconductors to be with respect to the magnetic track. When the middle line of the rows of the superconductors are aligned with the middle line of the gaps between the rows of permanent magnets, the resultant lateral magnetic force is null, and the superconductors are laterally stable [8]. To see this better, Figure 6 shows the magnetic flux density distribution when the car is center aligned with the magnetic track, which is its stable position (Figure 6a) and when it is off-centered to the left or to the right (Figure 6b and 6c, respectively). When the HTS car is center aligned with the...
magnetic track, the resultant force is null, and the car is in a stable position. When they are off center, the net force will not be null, and it will have the opposite direction of the direction of the movement. This means that there will be an attractive force to the center position if they are laterally displaced. This attraction force will occur if the displacements are no further than 21.5 mm from the center position [8]. If the displacement is higher, the forces will be in the direction of the movement, making the system unstable.

C. System Lateral Dynamics

Since the system has an attractive force towards the center, one can make a relationship between the magnetic force $F$ and the displacement from the center $x$:

$$\begin{align*}
F_{\text{mag}} &> 0, \quad x < 0 \\
F_{\text{mag}} & = 0, \quad x = 0 \\
F_{\text{mag}} &< 0, \quad x > 0
\end{align*}$$

(1)

This relationship expresses that the force is symmetric to the displacement

$$F_{\text{mag}} = -f(x),$$

(2)

being $f(x)$ a function with respect to the displacement. With (2), and considering that the friction force is proportional to the velocity, one can express the system’s dynamic:

$$m\ddot{x} + b\dot{x} - f(x) = 0.$$  

(3)

Equation (3) is the one used to estimate the lateral dynamics of the system using the experimental data.

D. Experimental Setup

An experimental rig was made to test the conditions for lateral stability shown in Figure 7, with the overall configuration presented in Figure 7(a). The rig consists of a base with the mechanical rails and threaded rods (Figure 7(b)), the magnetic rail (Figure 7(c)) and the car holding the YBCO HTS bulks (Figure 7(d)).

For all experiments made, there are necessary steps that are needed to make. First, the YBCO bulks are Zero-Field Cooled. To do that, they are placed in the car, and cooled with liquid nitrogen, without the presence of a magnetic field. When the YBCO Bulk plus liquid nitrogen reach the thermal equilibrium, the YBCO bulks are superconductive. Second, the magnetic rail is positioned in the base with the mechanical rails. Last, the car is placed in the experimental rig, and tightened with nuts in the threaded rods, and with a height off 15 mm with respect to the magnetic track, as previously shown in Figure 5. This will make the magnetic forces align the magnetic rails (Figure 7(c)) with the superconductors, and have the reference position of Figure 6a.

The experiments made in this setup are to determine the dynamics and the response of the car to lateral displacements. In this sense, the transient response to initial conditions of displacement was determined, and a model was developed to estimate the experimental results.

E. Force Field and Mass Measurement

Before conducting the experiments, measurements of the mass and the lateral forces experienced by the magnetic track were made. For the measurement of the mass, a simple scale was used, and the mass of the car is $m = 1.8 \text{ kg}$. For the lateral forces, a SCAIME K-12 force sensor is used to measure the lateral forces. The experiment to make the measurements for the force were as following: first, the magnetic track is first displaced from the center. Second, the force sensor is placed in the opposite direction of the displacement, for it to measure the force experienced by the magnetic track. The results of this experiment and the fitting curve are shown in Figure 8.
The fitting used for the measurements of the magnetic force in Figure 8 is a sinusoidal function

\[ F_{mag} = -F \sin \left( \frac{\pi}{x_{\text{max}}} x \right), \quad (4) \]

with a spatial period of 43 mm which corresponds to null magnetic forces in the maximum lateral displacements \( |x_{\text{max}}| = 21.5 \text{ mm} \), and an amplitude of \( F = 2.6 \text{ N} \). Equation (3) with (4) becomes, then:

\[ m\ddot{x} + b\dot{x} + F \sin \left( \frac{\pi}{x_{\text{max}}} x \right) = 0. \quad (5) \]

Using (6), the data is then transformed to displacement values, shown in Figure 11.

To determine the system’s dynamic function, first it is needed to estimate the friction coefficient \( b \). Since (5) is a non-linear function, analytical methods based on the solution of the equation are not possible, and computation methods are required. The approach used was to sweep values for the friction coefficient \( b \), solve equation (5) computationally
Fig. 10. Oscillograms of the transient response to initial conditions with initial position (orange curves) of: a) 20 mm and b) 15 mm. The displacement reference is the center position. The blue curve is the trigger switch to pinpoint the start of the movement of the magnetic track.

Fig. 11. Response of the system due to initial condition of $x_0 = 20$ mm and $x_0 = 15$ mm.

and check the root mean square error of the between the experimental response and the solution of (5):

$$RMSE = \sqrt{\frac{1}{N} \sum (y_i - s_i)^2},$$

where $y_i$ is a sample point of the experimental response at time $t_i$, $s_i$ is a sample point of the solution of (5) at the same time $t_i$ and $N$ is the number of samples. This approach will be used for the response shown in Figure 10(a) and it will be validated using the response shown in Figure 10(b). Figure 12 and Figure 13 show the value of the RMSE with respect to $b$ and the minimum value achieved, and the estimation and validation of the solution of (5), respectively. The results of the validation are very similar, making this model valid to use in the dynamics of this system. Using the results of the experiment, the dynamic equation is:

$$1.88\ddot{x} + 6.1\dot{x} + 2.6 \sin \left( \frac{\pi}{0.021} x \right) = 0.$$  (8)

Fig. 12. root mean square error as a function of the friction coefficient $b$.

Fig. 13. Dynamic estimated for the $x_0 = 20$ mm response and the validation for the $x_0 = 15$ mm response.

IV. NEW HTS CYLINDRICAL GEOMETRY AND SIMULATION ANALYSIS

The approach for a new ZFC-Maglev geometry is to guarantee a better lateral stability and thus better guidance for the vehicle. A proposed geometry from [2] is a cylindrical geometry. In [2], it is stated that a cylindrical geometry will behave better than the rectangular geometry of Section III, regarding the lateral stability, while also using less material. In this sense, a cylindrical geometry is developed and using a finite element analysis, its magnetic field distribution and
current densities are simulated to evaluate the levitation and guidance forces experienced by the HTS. The choice for a cylindrical geometry is to guarantee a more symmetrical and linear magnetic field than the one obtained in the rectangular geometry prototype. Figure 14 shows the conceptual design of the new magnetic circuit topology. It represents the final design of the ZFC-Maglev rail. Two, or more, HTS half rings are levitating under the magnetic rail, which is made up with permanent magnets, equally distanced apart.

FEM simulations are made to determine the levitation and guidance forces of the cylindrical YBCO bulk. Since there are geometric variables that are subject to change, a study was made regarding the levitation and guidance forces with respect to the geometric variables. The levitation forces are also compared to the weight of the YBCO and the liquid nitrogen required for the cooling. This comparison is made to check what are the geometries that can guarantee levitation forces higher than the weight of the system, to guarantee they can be under additional load.

Fig. 14. Conceptual design of the magnetic rails of the new cylindrical geometry for the ZFC-Maglev vehicle.

A. Geometric Description of the System

Figure 15 shows a representation of a HTS cylindrical block in a magnetic rail in an overall view (Fig. 15(a)) and a front view (Fig. 15(b)). There are six geometric parameters: Radius of the permanent magnet $R_{PM}$, diameter of the PM $D_{PM}$, radius of the air gap $R_{air}$, thickness of the superconductor $T$, depth of the HTS $D$ and spacing distance between permanent magnets $d_{PM}$. These are the geometric parameters used as variables in the FEM analysis, to determine the levitation and guidance forces.

B. H Formulation and Finite Element Method Model

This model is based on the H-formulation model used in [4] and [9]. The model uses the magnetic field $\mathbf{H}$ as an independent variable, and all the constitutive relations can relate to $\mathbf{H}$.

For the HTS, it is used:

$$\begin{align*}
\nabla \times \mathbf{E} &= -\mu \frac{\partial \mathbf{H}}{\partial t} \\
\nabla \times \mathbf{H} &= \mathbf{J} \\
\mathbf{E} &= E_0 \left( \frac{J}{J_C} \right)^n \frac{1}{J} 
\end{align*}$$

(9)

where the third equation is the HTS constitutive relation between the electric field and the current density, and $J_C$ is described by:

$$J_C = \frac{B_0}{B_0 + B} J_{C0},$$

(10)

where the term $J_{C0}$ represents the maximum critical current density of the material (for $B = 0$) and $B_0$ represents the value of the applied magnetic flux density that reduces the value of the critical current density to half of its maximum value $J_C(B_0) = J_{C0}/2$.

The equations in (9) are going to be described coordinate wise. Starting with Ampere’s Law

$$\begin{align*}
J_x &= \frac{\partial H_y}{\partial y} - \frac{\partial H_z}{\partial z} \\
J_y &= \frac{\partial H_x}{\partial x} - \frac{\partial H_z}{\partial z} \\
J_z &= \frac{\partial H_x}{\partial x} - \frac{\partial H_y}{\partial y}
\end{align*}$$

(11)

and substituting in the E-J power law

$$\begin{align*}
E_x &= E_0 \left( \frac{J}{J_C} \right)^n \left( \frac{\partial H_y}{\partial x} - \frac{\partial H_z}{\partial y} \right) \\
E_y &= E_0 \left( \frac{J}{J_C} \right)^n \left( \frac{\partial H_x}{\partial y} - \frac{\partial H_z}{\partial x} \right) \\
E_z &= E_0 \left( \frac{J}{J_C} \right)^n \left( \frac{\partial H_y}{\partial z} - \frac{\partial H_x}{\partial y} \right)
\end{align*}$$

(12)
where $J$ is the norm of the current density, which is also a function of the magnetic field

$$ J = \sqrt{\nabla \times \mathbf{H}}. $$

(13)

For the Faraday’s law, one has, coordinate wise

$$ \begin{aligned}
\frac{\partial H_x}{\partial t} &= \mu \left( \frac{\partial E_y}{\partial y} - \frac{\partial E_z}{\partial z} \right), \\
\frac{\partial H_y}{\partial t} &= \mu \left( \frac{\partial E_z}{\partial z} - \frac{\partial E_x}{\partial x} \right), \\
\frac{\partial H_z}{\partial t} &= \mu \left( \frac{\partial E_x}{\partial x} - \frac{\partial E_y}{\partial y} \right),
\end{aligned} $$

(14)

and using (12) and (13), one has the complete description of the system in terms of the magnetic field, coordinate wise.

For the calculation of the levitation and guidance forces, the Maxwell stress tensor is used

$$ T_{mn} = \mu H_n H_m - \frac{\mu}{2} \delta_{mn} H_k H_k. $$

(15)

and the force density is calculated as:

$$ f_m = \frac{\partial T_{mn}}{\partial x_n}. $$

(16)

To find the $n$th component of the total force $\mathbf{F}$, the volume integration of the force density is made:

$$ F_n = \iiint_V f_m dV = \iiint_V \frac{\partial T_{mn}}{\partial x_n} dV. $$

(17)

If the components of a vector $\mathbf{A}$ are defined as

$$ A_1 = T_{m1}, \quad A_2 = T_{m2}, \quad A_3 = T_{m3}, $$

(18)

(17) becomes

$$ F_n = \iiint_V \frac{\partial A_n}{\partial x_n} dV = \iiint_V \nabla \cdot \mathbf{A} \ dV. $$

(19)

Using the divergence theorem, one arrives to the conclusion that the total force is a surface integral of the stress tensor:

$$ F_n = \int_S (\mathbf{A} \cdot \mathbf{n}_S) \ dS = \int_S A_n(n_n)_S \ dS, $$

(20)

where $n_n$ is the $n$th component of the outward-directed unit vector $\mathbf{n}$ normal to the surface $S$ and the surface $S$ encloses the volume $V$. Using (18) to substitute back to (20) yields

$$ F_n = \int_S T_{mn}(n_n)_S \ dS. $$

(21)

It is with (21) that the forces are calculated in the simulations.

**C. Material Characteristics**

For the simulation of the system, the electromagnetic, mechanical and geometric properties of the YBCO and the permanent magnets of the magnetic rail must be specified.

For the YBCO HTS, the electromagnetic parameters needed are the ones used in the E-J power law, shown in (9): $E_0$, $B_0$, $J_{c0}$ and $n$. The electromagnetic values used for the simulations are taken from [5]. For the mechanical values, the density is needed for the calculation of the mass and the weight of the YBCO. The density was measured using the YBCO bulks used in the system of Section III. These parameters are shown in Table I.

<table>
<thead>
<tr>
<th>Electromagnetic Par.</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_0$ (V/m$^{-1}$)</td>
<td>$1 \cdot 10^{-4}$</td>
</tr>
<tr>
<td>$B_0$ (T)</td>
<td>0.1</td>
</tr>
<tr>
<td>$J_{c0}$ (A/m$^{-2}$)</td>
<td>$1.82 \cdot 10^6$</td>
</tr>
<tr>
<td>$n$</td>
<td>30</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Mechanical Par.</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_{bulk}$  (kg m$^{-3}$)</td>
<td>$5.81 \cdot 10^3$</td>
</tr>
</tbody>
</table>

![Fig. 16. Front view of YBCO half ring bulk with the magnetic rail.](image)

The geometric parameters are needed to calculate the area, volume and, consequently, the weight of the YBCO. Figure 15(b) is shown again here, in Figure 16, for easier readability.

Using the variables in Figure 16, the inner and outer radius of the YBCO Bulk, $r_{in}$ and $r_{out}$, respectively, can be derived as:

$$ \begin{aligned}
    r_{in} &= D_{PM}/2 + R_{air}, \\
    r_{out} &= r_{in} + T = D_{PM}/2 + R_{air} + T.
\end{aligned} $$

(22)

The area of the YBCO bulk is, then:

$$ A_{bulk} = \frac{\pi}{2} (T + 2r_{in}) T. $$

(23)

For the volume of the half ring, (23) is multiplied by the depth $D$

$$ V_{bulk} = \frac{\pi}{2} (T + 2r_{in}) TD, $$

(24)

and using the density of the YBCO, its mass and weight are, then:

$$ \begin{aligned}
    m_{bulk} = V_{bulk}\rho_{bulk}, \\
    F_g = V_{bulk}\rho_{bulk} g.
\end{aligned} $$

(25)

For the permanent magnets, the magnetization curve is

$$ B_{PM} = \mu_0\mu_{r_{PM}} H_{PM} + B_r, $$

(26)

where $B_r$ is the remanent flux density and $\mu_{r_{PM}}$ is the relative permeability of the permanent magnets. Table II shows the electromagnetic and geometric parameters of the PM.
D. 3D FEM Model

For the 3D simulations, the model used is shown in Figure 17. For the simulations made, the magnetic track needs to be able to be made of a continuous cylindrical PM, or made with several PM, equally spaced apart. To address this property and maintain only on FEM model, the magnetic track was made with several PM, and when the spacing between them is null, it is regarded in the simulations as if it is a continuous cylindrical tube.

Figure 18 shows the initial conditions of the 3D simulations. The main simulation is made by making a step function of the magnetic field of the PM. This simulates the already ZFC YBCO being put near the permanent magnets. The remanent magnetic field is in the z direction.

![Schematic of the cylindrical ZFC-Maglev geometric model used in the 3D electromagnetic simulations.](image1)

![Initial conditions and representation of the magnetization direction in the PM used in the 3D simulations.](image2)

E. 3D FEM Simulations for the Computation of the Net Force Varying the YBCO Geometric Parameters

For the 3D simulations, the weight of the liquid nitrogen (LN2) and the container for the YBCO Bulk will be added to the analysis, since their weights cannot be neglected, especially the weight of the LN2.

For this simulation, a parametric sweep is made regarding the geometric parameters air gap $R_{air}$ and depth $D$. The values of the parametric sweep are shown in Table III.

![Weights of the YBCO, LN2 and the total weight for different air gaps and depths of the YBCO.](image3)
Having the total weight of the system and the YBCO Levitation Force computed, the net force \( F_{\text{net}} \) is calculated

\[
F_{\text{net}} = F_{\text{lev3D}} - F_{\text{YBCO}},
\]

and the results of the computation of the levitation and the net force are shown in Figure 20.

In Figure 20, it can be seen that the net forces decrease with an increasing air gap and, between the air gap values of 30 to 35 mm, the net forces do not vary significantly with respect to the depth of the YBCO. This indicates that between those two values of air gap, the system has a net force close to zero, and the system is inherently stable, without needing additional weight. To see that the net forces are considerably small, Table IV shows the values of the levitation and net force for the air gaps of 30 and 35 mm.

In Table IV it can be seen that the net force stays constant with respect to the changes in depth, for air gap values of 35 mm. Analysing these information, it means that with air gap values above 35 mm, this system can not support its own weight, and the system would lose its radial symmetry.

With lower values of air gap, all the geometries are possible geometries, since additional load can be added to the system, in order to have a null net force.

\[
\begin{align*}
\text{Dev}_{\text{PM10}} \% &= \frac{|F_{\text{lev10}} - F_{\text{lev15}}|}{F_{\text{lev10}}} \cdot 100, \\
\text{Dev}_{\text{PM15}} \% &= \frac{|F_{\text{lev15}} - F_{\text{lev10}}|}{F_{\text{lev10}}} \cdot 100.
\end{align*}
\]

This simulation is to determine the influence of the PM spacing \( d_{\text{PM}} \) regarding the levitation forces. In the simulations, the YBCO bulk is positioned on top of a PM, center aligned with it. The values of the parametric sweep are shown in Table V. The levitation forces drop considerably with a magnetic track with spaced PM, and so, the air gap values considered for this simulation were only up to 20 mm, since the levitation forces of the simulations shown in Section IV-E were close to zero for air gap values higher than 20 mm.

\[
\begin{array}{c|c|c|c|c|c|c|c|c}
\text{Air gap [mm]} & \text{T [mm]} & \text{D [mm]} & \text{PM10 [N]} & \text{PM15 [N]} & \text{Dev}_{\text{PM10}} \% & \text{Dev}_{\text{PM15}} \% \\
\hline
16 & 10 & 20 & 1.56 & 1.56 & -0.03 & -0.00 \\
20 & 10 & 20 & 2.13 & 2.13 & 0.15 & 0.15 \\
24 & 10 & 20 & 2.76 & 2.76 & 0.39 & 0.39 \\
28 & 10 & 20 & 3.45 & 3.45 & 0.69 & 0.69 \\
32 & 10 & 20 & 4.11 & 4.11 & 0.96 & 0.96 \\
36 & 10 & 20 & 4.81 & 4.81 & 1.26 & 1.26 \\
40 & 10 & 20 & 5.49 & 5.49 & 1.55 & 1.55 \\
30 & 16 & 10 & 1.03 & 1.03 & -0.75 & -0.75 \\
35 & 16 & 10 & 1.40 & 1.40 & -0.80 & -0.80 \\
35 & 24 & 10 & 1.83 & 1.83 & -0.82 & -0.82 \\
35 & 28 & 10 & 2.25 & 2.25 & -0.83 & -0.83 \\
35 & 32 & 10 & 2.71 & 2.71 & -0.80 & -0.80 \\
35 & 36 & 10 & 3.15 & 3.15 & -0.80 & -0.80 \\
35 & 40 & 10 & 3.57 & 3.57 & -0.81 & -0.81
\end{array}
\]

The results of this simulation are shown in Figure 21. In Figure 21, it can be seen that the net forces are lower in comparison to the net forces of Figure 20. This is due to the fact that the levitation forces are lower in this situation, which results in a lower net force, since the weight of the system for each geometric configuration is the same as it is on the simulations of Section IV-E.

\[
\begin{align*}
\text{Dev}_{\text{PM10}} \% &= \frac{|F_{\text{lev10}} - F_{\text{lev15}}|}{F_{\text{lev10}}} \cdot 100, \\
\text{Dev}_{\text{PM15}} \% &= \frac{|F_{\text{lev15}} - F_{\text{lev10}}|}{F_{\text{lev10}}} \cdot 100.
\end{align*}
\]
where \( F_{lev} \) is the levitation force without PM spacing, and \( F_{lev}^{PM10} \) and \( F_{lev}^{PM15} \) are the levitation forces for a PM spacing of 10 and 15 mm, respectively. Figure 22 shows the results of the deviation in percentage of the levitation for the PM spacing of 10 mm and 15 mm.

Analyzing the deviations for both levitation forces with PM spacing, it can be seen that, as the depth of the YBCO increases, the value of the deviation for each air gap start to get similar. Also, For the air gap value of 20 mm, the deviations of the levitation forces remain almost constant, for different depths.

Regarding the computational and simulation aspect of the design of the new cylindrical geometry, the results shown in Figure 22 can provide us estimates for the levitation forces, without the need to make further simulations. With the results of Figure 20, if there is a need to interpolate the value of a levitation force for values of air gap, depth or PM spacing that were not simulated, the data shown in Figure 22 proves to be rather useful, since it is a faster way to estimate the levitation forces and the FEM analysis is too time consuming, reaching from hours to days of simulation time.

V. GUIDANCE Force of BOTH GEOMETRIES

Figure 23 shows the guidance forces measured of the rectangular geometry prototype, previously shown in Section III-E, and the guidance force of the cylindrical geometry. The computations of the guidance forces for the cylindrical geometry were only made for the 2D FEM model, for a thickness of 10 mm and an air gap value of 35 mm. It was not possible to make the computation for the guidance forces on the 3D FEM model, since it is too time consuming, and it was not possible to have the results in time. However, considering the error between the levitation forces of the 2D and 3D Simulations (Table ??), the error for a depth of 40 mm is of 8.55 %, and so, it is assumed that the error for the guidance forces is of similar value and the main aspects of the analysis using the values of the 2D simulations are the same for the 3D FEM model.

Analyzing the guidance force of both geometries shown in Figure 23, the cylindrical geometry presents a more linear force curve than the rectangular geometry. This indicates that the cylindrical geometry can be linearly controlled in the 21 mm displacement range, while in the rectangular geometry is not possible for the same displacement range.

Another difference between the two geometries is their range of motion. Although the maximum displacement for the rectangular geometry is of 21 mm, it is not due to a physical constraint, but because the system would be off centered from the magnetic track. In the Cylindrical Geometry, this does not happen, because the system cannot laterally displace more than its air gap value, since it will collide with the magnetic track. On top of that, as the Cylindrical YBCO gets closer to the magnetic track, the lateral forces increase in the opposite direction of the movement, which makes a stronger repulsive force in the YBCO.

A. LATERAL RESPONSE of BOTH GEOMETRIES

In this section, it will be compared the lateral response of the rectangular and the cylindrical geometry. Since there is no information about the friction forces of the cylindrical geometry, the comparison of both geometries is made using the undamped response. This way, the comparison will focus on the differences of the guidance forces of the two geometries. The undamped responses of both geometries are also compared with a linear response of the same characteristics as the undamped responses, to analyze the linearity of the undamped responses.

B. Dynamic Equations and Comparison Model

For the rectangular geometry, the expression of the guidance force \( F_{lat}^{Rect} \) is (8), and it is shown here again, for easier readability:

\[
F_{lat}^{Rect} = -2.6 \sin \left( \frac{\pi}{0.021} x \right). \tag{29}
\]

For the cylindrical geometry, a cubic fit is made for the curve of the guidance force in Figure 23 and the guidance force \( F_{lat}^{Cyl} \) is:

\[
F_{lat}^{Cyl} = -1.1 \cdot 10^6 x^3 - 3.8 x^2 - 500x + 0.006. \tag{30}
\]
With (29) and (30), the dynamic equations for the rectangular geometry and the cylindrical geometry are, respectively:

\[
\begin{align*}
    m_{\text{Rect}} \ddot{x} + 2.6 \sin \left( \frac{\pi}{0.002} x \right) &= 0 \\
    m_{\text{Cyl}} \ddot{x} + (1.1 \cdot 10^3) x^3 + 3.8 x^2 + 500 x - 0.006 &= 0
\end{align*}
\]  

(31)

The solutions of (31) considered in the comparison are made for the initial conditions of \(x_0 = 20\) mm and \(x_0 = 15\) mm. The solutions of the equations are to be compared with the response of a linear undamped system, which is

\[m \ddot{x} + k x = 0,\]

and its solution to a initial condition is

\[x = x_0 \cos \left( \sqrt{\frac{k}{m}} t \right).\]

(33)

Analyzing (33), and knowing that

\[\omega = \sqrt{\frac{k}{m}},\]

(34)

a linear response of the same frequency as the solutions for both geometries can be made. This way, a comparison between the linear and non-linear response is made.

C. Results of the Comparison of the Dynamic Response of the Two Geometries

Figure 24 and Figure 25 show the solution of the dynamic equations to a initial condition of the rectangular geometry and the cylindrical geometry, as well as the linear response, for \(x_0 = 20\) mm and \(x_0 = 15\) mm, respectively.

Regarding the rectangular geometry, it is noticeable that the frequency of oscillation is different for each response. In the response for the initial condition of \(x_0 = 20\) mm, the frequency of oscillation is of 0.9 Hz, and for the response for the initial condition of \(x_0 = 15\) mm, the frequency is of 1.57 Hz, which is an increase of 70%. This is another indication that the system is non-linear, which makes it a more complex system to control, since the frequency of oscillations is not constant for different initial conditions.

For the cylindrical geometry, the frequency of oscillation does not vary much from the different initial conditions, being the frequency 6.85 Hz and 6.21 Hz for the initial conditions of \(x_0 = 20\) mm and \(x_0 = 15\) mm, respectively, which is a variation of about 10%. Also, the response of the system for both initial conditions is very similar to the linear response. This strongly indicates that the cylindrical geometry can be more easily controlled, since it can be modeled by a linear system.

Comparing the two systems, the cylindrical geometry has a more linear and steady response, for different initial conditions. Also, the cylindrical geometry is inherently more stable than the rectangular geometry, since the range of lateral motion is limited due to the geometric aspect of the design and, if the YBCO bulks get closer to the magnetic track, they are repelled with a higher force. In the rectangular geometry, this does not happen, since the distance of the YBCO bulks to the magnetic track is constant, regarding the lateral movement, and the force distribution as an entirely different pattern than the cylindrical geometry has.

Two possible disadvantages of the cylindrical geometry are that the frequency of oscillations is higher, which needs to be accounted for the control aspect of the system, and also if the YBCO bulk gets to close to the magnetic track, flux pinning can occur, and the YBCO remains trapped in that position, making the system off centered from the magnetic track. This two situations need to be present when designing the control system of the maglev.
VI. CONCLUSIONS

This dissertation consisted of a two part study: experimental study regarding an HTS maglev prototype, with a rectangular geometry; and the design and simulation of a new HTS maglev with a cylindrical geometry.

Regarding the rectangular geometry, first, experimental measurements were made regarding the lateral forces experienced by the YBCO. Second, the lateral response to initial conditions was recorded. Last, a model was developed to estimate the lateral response of the rectangular geometry. The model can estimate with a high degree of accuracy how the system behaves regarding lateral disturbances, which is essential for any type of control, being it an active or a passive type.

For the new cylindrical geometry, 3D simulations were made, to study the influence that the geometric parameters have on the levitation and guidance forces. These studies revealed a number of possible geometric configurations for which the system is vertically stable, i.e., it has a null net force, with the possibility to have load.

A study regarding the lateral forces of the cylindrical geometry was made, as well as a model for the response to lateral disturbances. This model was then compared to the model of the existing rectangular geometry. The main conclusions were that the cylindrical geometry is inherently more stable than the rectangular geometry, since it is a more linear system than the system of the rectangular geometry, and it has lateral movement limitations, due to the geometry, i.e., it is more difficult to derail than the system of the rectangular geometry. A disadvantage of the cylindrical geometry could be that, if the HTS are too close to the magnetic track, it can occur a flux pinning effect, making the HTS off centered.

REFERENCES