TÉCNICO LISBOA

Health Care and Game Theory - An application to liver transplantation

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Resumo

Embora hoje em dia a medicina esteja dependente da tecnologia, é usada para servir pessoas. De facto, uma consulta médica pode ser um exemplo extremamente complexo de interação humana. É na modelação desse tipo de interações que a Teoria dos Jogos pode desempenhar um papel fundamental na melhoria dos resultados médicos. A teoria dos jogos é amplamente usada em estudos de diversas áreas. No entanto, existem poucos exemplos de estudos onde é a aplicada a questões de saúde, nomeadamente à relação entre o médico e o paciente em tratamentos médicos mais complicados e prolongados. Este estudo usa teoria de jogos para modelar a consulta médica de transplante hepático para pacientes que sofrem de doença hepática alcoólica (DHA). A DHA é uma doença que deixa os pacientes em situações muito delicadas, especialmente fase terminal. O processo de transplante hepático envolve negociação com o paciente, e é comum este tentar enganar os médicos para ser beneficiado e receber o transplante. Tendo em conta que os fígados disponíveis para transplante são um recurso escasso, é necessário que o candidato cumpra rigorosamente uma série de requisitos. Foram analisados jogos nas formas normal e extensiva, e os respetivos equilíbrios foram calculados. Os resultados sugerem algumas implicações de política em matéria de saúde, que devem ser usadas para obter melhores resultados. A análise do jogo na forma normal mostra quais os parâmetros que influenciam o nível de cooperação, tanto para o médico como para o paciente. A análise do jogo na forma extensiva mostra como alterar o jogador (paciente ou médico) que decide primeiro pode levar a equilíbrios mais cooperativos.

Palavras-chave: Teoria dos Jogos, Transplante Hepático, Doença Hepática Alcoólica, Consulta Médica

Abstract

Even though nowadays medicine is necessarily linked with technology, it is still a service involving a lot of human interaction. Indeed, a medical consultation can be an extremely complex example of human interaction. This is where game theory (GT) may play a key role in helping to improve the results of medical processes. GT is widely used in an immense variety of study fields, but there is little application to healthcare issues, namely the doctor-patient relationship. This study uses GT to model the liver transplantation consultation for patients suffering from Alcoholic Liver Disease (ALD). ALD is a very delicate disease, and patients at its end-stages require special dedication. They may try to deceive doctors which may lead to bad outcomes. Strategic and extensive form games are analysed, and the equilibrium solutions computed. Results show some health policy implications on the parameters that should be managed to achieve better outcomes. The analysis of the strategic form game shows which parameters influence the cooperation rates, both for the doctor and the patient. The analysis of the extensive form game shows how to obtain a more cooperative outcome, depending on the player (patient or doctor) that moves first.

Keywords: Game Theory, Liver Transplantation, Alcoholic Liver Disease, Medical Consultation

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Chapter 1

Introduction

This chapter briefly introduces the subject of the current dissertation and its motivation, as well as the goals set to achieve. Section 1.1 provides a general explanation of the problems that motivated this study. Section 1.2 contains an overview of our particular subject, backed with some factual data. Section 1.3 enlightens the reader about the main objectives of this dissertation. Lastly, section 1.4 provides a broad vision over the dissertation layout.

1.1 Motivation

Medical practice is evolving extremely rapid, and breakthroughs occur on a daily basis. Advances in technology are one of the main contributors to these exceptionally fast and frame-breaking changes. However, health care is still a service for people offered by people, and the role of physicians has changed substantially (Henry, 2006). As stated in Elwyn (2004, page 415), “Medicine is a service delivered by a mix of episodic and repeated interactions between humans, medicated by the use of technologies such as tests, drugs and procedures”. However, it is widely recognized that there exists an obvious communication gap between the actors in a medical consultation, which usually leads to a 40-50% less than optimal adherence to physicians’ advice and treatment. Successful attempts and efforts to address this problem are rare (Hockstra and Miller, 1976).

Physicians are decision makers under extreme pressure, not only to offer the right treatment but also to make the best allocation of resources. Furthermore, technology has enabled physicians and patients to access a massive amount of information. Therefore, patients are nowadays more informed (for better or for worse) and are more aware of the options they have. This led to a new scenario where empowered patients also assume the role of decision makers, being able to negotiate with physicians in order to defend their own interests. In an interaction like this, it is common to have conflict of interests of both players - the patient and the doctor - and the outcomes of the situation will be influenced by the decisions taken by them. This is where Game Theory appears as a suitable tool to study this interaction (*e.g.* Djulbegovic et al., 2015). Game theory is a mathematical programming tool with many applications in solving complex human (or animal) interaction situations. It models decision making when players’ outcomes are interdependent. It is widely applied to firm competition, but is powerful to find the equilibrium outcomes in many other social contexts. The relationship between doctor and patient, or decisions concerning health services provision may be modeled with this tool. One example of a possible game theory application can be the choice between eutocia or caesarean delivery, taking into account the incentives given to reducing the caesaren rate. Game theory is considered to offer medicine a potential study method, yet the complexity of the consultation must not be underestimated. Designing a model that portraits correctly the consultation is a considerable challenge. Despite that, if it is done, it may bring over improvements in the quality of health care and benefits for all interested parties (Elwyn, 2004; Hockstra and Miller, 1976).

This dissertation focuses on the alcoholic liver diseased treatment process, namely in its end stage where patients usually require a liver transplantation. The major problem is that there are not enough liver grafts for all the patients in need (demand exceeds supply), so a very prudent management of each situation must be made, in order to best allocate the existent liver grafts. Consequently, the entire transplantation process is very complex, and requires doctors to take exceptionally difficult decisions. It also requires patients to change their lifestyle - especially to become alcohol abstinent - in order to be able to receive the much needed transplant. However, not always everything goes perfect, and there may be times where one of the players is not willing to cooperate, due to several possible reasons. The whole process engages a tremendous negotiation between physicians and patients, which will be the object of the current dissertation, using game theory. Simulating the consultation using game theory may lead to interesting results that can be used to come up with health policy implications targeted to increase patients' adherence to doctors' recommendations and promote a better allocation of a very scarce resource - liver grafts.

1.2 Topic Overview

It is common knowledge that alcohol consumption is harmful for peoples' health, harming especially one vital organ: the liver. In fact, alcohol consumption is considered to be the main cause of hepatic disease in Europe, though it is often combined with other risk behaviors (APEF, 2014; Marinho, 2010). Alcohol abuse led to 3.8% of all deaths in the world in 2004 and 4.5% of the global disease burden which is measured in disability-adjusted life years lost. This is why alcohol abuse is often considered a public-health problem (Marinho et al., 2015). Besides causing alcoholic liver disease, alcohol consumption also contributes to diseases in the pancreas, heart, digestive track, among others.

Alcohol has a specific mechanism of action when striking the liver. It leads to steatosis, which consists of deposing fat in the liver, followed by cellular lesion and fibrosis. Lastly, if it reaches the end stages, hepatic alcoholic cirrhosis can be reached. Once at this stage, there is no treatment that will cure the disease. Naturally, the larger the doses consumed the more likely the development of the illness. However, there is no minimum quantity defined to avoid the emergence of alcoholic liver disease, and not even non-excessive drinkers can be considered risk-free. There are other factors that have been demonstrated to influence the evolution of this disease, such as gender, race, and obesity (Matos, 2006; APEF, 2014).

As mentioned before, hepatic cirrhosis is the end-stage of alcoholic liver disease and it is irreversible.

Additionally, it is common to observe the development of hepatic cancer in cirrhotic patients. Alcohol consumption at this stage assumes a critical role. It is of tremendous importance that patients stay abstinent, or the cirrhosis can aggravate and eventually lead to death. Alcohol abstinence can bring noticeable benefits for patients' health, and many patients are advised to seek counselling in order to remain abstinent. The last resort treatment for alcoholic liver disease is liver transplantation, with this disease being one of the main causes for liver transplantation referral in Europe (APEF, 2014). The main issue with liver transplantation is that there are not enough liver grafts to cover the demand, thus requiring a very complex and strictly ruled candidate selection process (Carrion et al., 2013). The selection criteria comprises requisites such as alcohol abstinence (minimum six months), assessment of the severity of patient's disease and existence of any other clinical conditions, given by scores such as Model for End-Stage Liver Disease (MELD), as well as verification of the post-transplant prognostic. This last criterion is of utmost importance since 20%-25% of the patients lapse or relapse to heavy drinking post-operatively (Telles-Correia and Mega, 2015).

The selection process engages a tricky negotiation between the multidisciplinary team of practitioners and the patients and their families. The physician must check if the patient has been following the recommendations, and at the same time assess the prognostic for the patient's liver transplantation. If the patient relapses into drinking after receiving a new healthy liver, it will be a big loss for all interested parties. At the same time, the patient may or may not comply with the doctor - since complying entails several sacrifices - and she/he might try to deceive the doctor in order to not be excluded from the transplantation waiting list. Using game theory one may find interesting equilibria that will help defining new ways to encourage patients to cooperate.

1.3 Objectives

This dissertation aims at developing a game theory model to study the interaction between doctors and patients in consultations, particularly for the liver transplantation process of patients suffering from alcoholic liver disease. The game is solved to find the equilibrium or equilibria behaviors.

This model is expected to support medical decision and to present health policy implications in order to better collaborate with such patients. A more collaborative process, where patients feel the doctor is there to help instead of to punish them, will improve outcomes both for the doctor and the patient, or society in general. The ultimate goal is to assure that liver grafts are transplanted to patients with the best possible prognostic. Actually, more than a single model were developed, and the solutions compared.

1.4 Dissertation Outline

This dissertation is organized in six chapters, as explained below.

Chapter 2 introduces the disease in question, explaining how it develops and including factual data that demonstrates the impact it may have in a society. Moreover, the entire transplantation process is thoroughly detailed. Chapter 3 gives an overview of game theory models and a literature review on applications to a wide variety of study fields. Two applications of game theory to health care issues are presented in a detailed way. The model construction and analysis is made in chapter 4. The results achieved are presented as the analysis proceeds, with the necessary comparisons being made. Chapter 5 discusses and interprets the results obtained and draws some possible health policy implications. Finally, chapter 6 outlines the main conclusions and presents possible developments for future work.

Chapter 2

Background

This chapter describes the Alcoholic Liver Disease (ALD), which is one of the main contributing diseases to liver transplants in Portugal, and also goes through all the transplantation requirements and process. Section 2.1 addresses the causes, characteristics, and related health issues of this disease. Section 2.1.1 presents some relevant statistical evidence on its incidence in Europe and in Portugal, and also gives some specialist insight into the Portuguese situation. Section 2.2 deals with the transplantation process, which is thoroughly explained since it serves as baseline for the application of the model. Diagnostic is only roughly exploited whereas it can be a very complex process and does not have a substantial contribution to the results of this study. It is important to notice that this work will focus on alcoholic chronic liver disease and not on acute liver disease, since the former one allows for a better application of the model of interest. Section 2.3 wraps up the context introduction made along the chapter, outlining the most important aspects that serve as motivation for improvement efforts.

2.1 Alcoholic Liver Disease and Alcoholism

Chronic Liver Disease is characterized by the progressive destruction of the liver parenchyma and its replacement by fibrous tissue (Hammer and McPhee, 2014). Cirrhosis is an example of long-term chronic liver disease where scar tissue replaces healthy tissue, after the latter one suffered damage for a long period of time (Matos, 2006).

The liver is an essential organ with a wide variety of functions, including expelling toxins from the body, bile production for food digestion, manufacturing triglycerides (fat) used for energy, and proteins production (Tortora and Derrickson, 2014). These are vital functions that will be compromised when liver disease reaches its end-stage, cirrhosis. This failure might lead to death if not taken care of in good time. Cirrhosis may also entail portal hypertension and increase the risk of development of hepatocellular carcinoma (Kumar et al., 2015).

Alcohol consumption has been a reality for a long time in Humanity history. Despite that, its connection with ALD is much more recent, starting to appear in the 19^{th} century. Though alcohol consumption does not necessarily lead to cirrhosis, it definitely increases the risk of (irreversibly) damaging the liver, especially if consumed regularly (Matos, 2006). Although other disorders or risky behaviours may lead to hepatic cirrhosis, approximately 80% of the cases derive from alcoholic consumption (Marinho, 2010).

But alcohol consumption repercussions do not end here, it may also lead to diseases in other organs such as the pancreas, the heart, or the digestive tract. Besides, alcohol abuse is also strongly correlated with car accidents and domestic violence, sometimes leading to homicides or suicides (APEF, 2014; Marinho et al., 2015). This is why excessive alcohol consumption can be seen as a public-health problem. Alcohol abuse led to 3.8% of all deaths in the world in 2004 and 4.5% of the global disease burden which is measured in disability-adjusted life years lost (Marinho et al., 2015).

Some comorbidities associated with ALD can be seen in table 2.1.

Table 2.1: Comorbidities associated with alcohol related liver disease. Adapted from Varma et al. (2010).

Comorbidities associated with alcohol related liver disease	
Cardiovascular	Alcoholic cardiomyopathy Cirrhotic cardiomyopathy Coronary artery disease
Musculoskeletal	Myopathy Osteopenia
Neurologic	Wernicke-Korsakoff psychosis Alcoholic dementia Alcoholic cerebellar degeneration Peripheral neuropathy
Malnutrition	
Chronic pancreatitis	
Hepatocellular carcinoma	
Hepatitis B or C infection	
Other malignancy	Upper aerodigestive tract malignancy
Psychiatric	Depression or mood disorders Personality disorders Anxiety disorders Psychosis

It is important to briefly mention some mechanisms through which alcohol can harm the liver. It is fully absorbed throughout the digestive tract, and this absorption can take from minutes to hours depending on whether the individual was fasting (Matos, 2006). After being absorbed, the alcohol must be metabolized and the resulting metabolites will be the harmful entities (APEF, 2014). Acetaldehyde is a toxic compound which may cause protein denaturation and suppress some DNA repair mechanisms. Alcohol metabolization may also enable some bacterial endotoxins to enter the liver and lead to an immune response that can harm the tissue. Nonetheless, it has been proved that not every individual has the same likelihood of having alcoholic cirrhosis. Gender, age, and other genetic factors may influence the susceptibility an individual has to develop cirrhosis. For example, women or African people are more prone to develop such a disease. Lastly, alcohol consumption can deepen other diseases effects, for example hepatitis C (Marinho, 2010; Matos, 2006).

Physicians have good understanding of the reality of these two diseases (ALD and Alcoholism), albeit the context has seen some changes like the age at which alcohol consumption begins. Portuguese teenagers start drinking at the age of 13, which is incredibly early and is associated with a risk of future addiction around 50% (Marinho, 2010).

Alcoholism is a disease strongly related with social and familial issues. Despite the tremendous burdens and problems alcoholism and alcoholic cirrhosis may bring to society, they still are not hot topics amongst medical community due to the lack of therapeutic and technological innovations in this area (Beresford, 1997; Marinho, 2010).

2.1.1 ALD in Europe and in Portugal

This subsection gives some insights and statistical facts about how alcohol consumption is deeply rooted in human society and on some other consequences it may bring.

According to a report conducted by the Portuguese National Health System (Serviço Nacional de Saúde - SNS) chronic liver disease is positioned in the top ten causes of death in Europe and also in Portugal (Pedroto et al., 2016; Marinho, 2010).

As previously mentioned, and despite all the health problems it may be associated with, alcohol is the most accepted drug. Thus, it naturally is also the most consumed one.

Not surprisingly, "Alcoholic liver disease (ALD) is the second most common diagnosis among patients undergoing liver transplantation (LT) in the United States and Europe" (Burke and Lucey, 1998). It is only exceeded by viral hepatitis (Varma et al., 2010). Between 1988 and 2004, 31% of liver transplants performed in Europe were due to ALD (17,2% in the USA) (Telles-Correia, 2011). This goes in accordance with the fact that alcohol is consumed in high quantities. Indeed, around 15% of the European citizens drink alcohol in an excessive and hazardous way (APEF, 2014). The World Health Organization (WHO) puts Portugal in the 11th position of the top alcohol consuming countries in Europe (2010 data) (Pedroto et al., 2016).

Alcoholic liver cirrhosis is one of the main contributors to the substantial number of deaths due to hepatobiliary diseases. In Portugal there were 18,279 deaths (25.1 deaths *per* 100,000 inhabitants) caused by hepatobiliary disease reported from 2006 to 2012, with more than 5,200 (7.1 deaths *per* 100,000 inhabitants) having ALD as main cause. In addition, this same study concluded that from all the deaths caused by hepatobiliary disease in Portugal 72% were from male patients, thus underlining the influence of other factors such as gender. Though women are more prone to develop alcoholic liver disease, Portuguese men drink more alcohol than Portuguese women, which contributes to the previous statistic (da Rocha et al., 2017). The number of deaths from hepatobiliary disease by sex in Portugal between 2006 and 2012 is depicted in figure 2.1.

Another curious aspect of mortality from liver cirrhosis is its regional distribution in Portugal. It reaches its highest values in the archipelago of Madeira, which can be linked to historical elevated levels of alcohol consumption and intravenous drug abuse. In continental Portugal, there is a north-south gradient that can be explained by the location of the reference centres (north centre in Porto and Coimbra and Lisbon are the central centres). Besides this, this result is in accordance with the different levels of alcohol consumption along the country (da Rocha et al., 2017).

Furthermore, it is estimated that around 11% of individuals in Western Europe die as a result of alcohol consumption (Marinho, 2010).

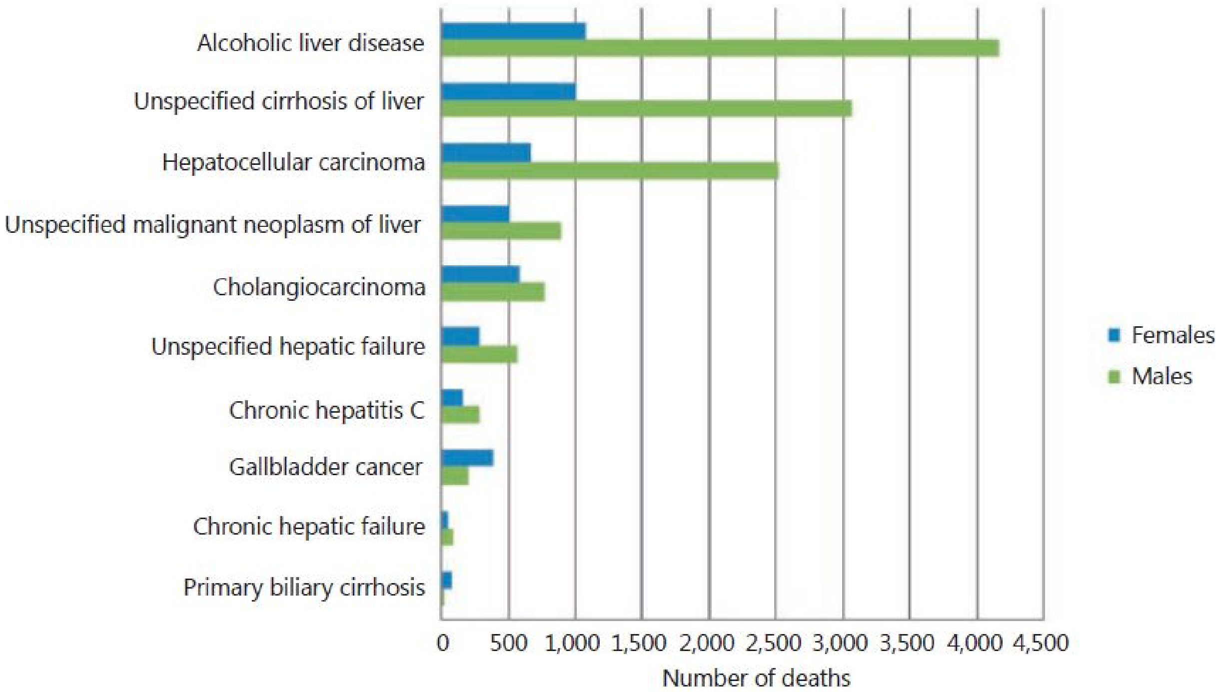

Figure 2.1: Number of deaths from hepatobiliary disease by sex in Portugal, 2006-2012. Extracted from da Rocha et al. (2017)

Looking into the National Statistical Institute (INE) data it is possible to validate the previous estimation and verify that Portugal follows the Western European trend, with the mortality rate due to chronic liver disease and cirrhosis *per* 100,000 inhabitants being approximately 10, in other words, in every 100,000 individuals 10 die from chronic liver disease (INE). This outnumbers the previously mentioned mortality ratio for ALD (7.1/100,000) because it includes other chronic liver diseases.

The impact range of ALD is much wider than the addressed previously. A clear example of the enormous burden of this disease is the number of hospital admissions. Approximately 5,500 admissions are registered *per* year in Portugal and 84% of the cases are related with alcohol (data from 2008). Approximately twenty years ago this percentage was 63% (Pedroto et al., 2016; Marinho et al., 2015). These admissions are commonly longer than the usual and the mortality rate is three times the national average. Thus, hospitals and the SNS incur high costs with ALD (Marinho, 2010; Vitor et al., 2016). Additionally, patients are being admitted at earlier ages than in the past. In the mid-nineties the mode would fall between 60 and 64 years old whereas between 2004 and 2008 it was confirmed to fall within 50 and 54 years old, thus 10 years younger (Marinho et al., 2015). A study published in 2016 stated that by 2008 liver disease was the third most expensive disease for the Portuguese Public Health Service when considering only hospital admissions, with a cost of €62,950,631. 42,6% of this cost came from admissions where patients suffered from alcoholic cirrhosis (Vitor et al., 2016). Along with those costs, transplantation costs have increased and can nowadays reach €100,000 *per* transplant. Even so, Portugal is one of the top ranked countries regarding the amount of liver transplants executed, with the number being around 200 transplants *per* year (Marinho, 2010).

An indirect economic burden also important to consider derives from the fact that the major share of patients admitted is aged between 20 and 60 years, *i.e.*, the most socioeconomically active age group. Between 2006 and 2012, it was evidenced that ALD mortality rate may reach 25-30/100,000 in males aged amid 50 and 65 years, therefore possibly having severe productivity repercussions (da Rocha et al., 2017). Besides the accountability costs associated to hospital admissions and treatments to patients, there is also an opportunity cost associated with the lost production. Thus, considering just the number of hospital admissions is an underestimating approach since it does not account for the lost of productivity and other factors of the previously socioeconomically active admitted patients. It is then very complicated to truly quantify the economic burden of ALD. However, it already shows how problematic this disease can be for a society (Vitor et al., 2016).

Portuguese teenagers are consuming larger quantities of alcohol at younger ages, especially following the pattern of binge drinking. This basically consists on drinking many alcoholic beverages in order to become intoxicated over a short period of time. These kind of behaviours can be very harmful for teenagers and they end up being taken to the intensive care unit very often. In the future, the number of admissions for ALD tends to increase and patients will be even younger, which is subject of big concern. Governments should address these problems and work to find more effective regulation on alcohol consumption in order to revert the situation (Marinho et al., 2015).

2.2 Liver Transplantation

This section focuses on the liver transplantation process, defining and describing each fundamental portion of it. Section 2.2.1 explains the details and particularities of the diagnosis of the disease in question giving an overview of the complexity it may encompass. A clear and accurate diagnosis is a key point for the outcome of the treatment. Following after, indications for transplant and prognosis are discussed in section 2.2.2. Knowing that liver grafts are a scarce resource for the large number of patients in need of a transplantation, this task is fundamental for selecting the candidates who will receive the transplant. Lastly, section 2.2.3 covers the aspects that might exclude patients from the waiting list. Besides the clinical parameters, psychosocial aspects take a decisive role in this process and are also detailed.

2.2.1 Diagnosis

The patient arrives to the hepatology consultation usually redirected from specialties like gastroenterology, infectiology, or internal medicine. If the patient has developed hepatocellular carcinoma then the consultation will be with a surgeon. In this first pre-transplant consultation a thorough diagnosis is made, and the steps will be briefly explained in this subsection.

First of all it is important to differentiate two, sometimes mistakenly mixed, concepts: alcohol dependence and alcohol abuse, being the former one more serious than the latter. Alcohol dependence involves the urge of alcohol to avoid some sort deprivation or satisfy some physical or mental need. Alcohol abuse can be defined as drinking behaviours that have recurring negative impacts on an individuals health, relationships and work. A third but not least important concept is tolerance: the more tolerance to alcohol an individual has, the more she/he has to drink to get the same level of satisfaction (Matos, 2006; Beresford, 1997).

ALD may be divided in three histological stages: hepatic steatosis, alcoholic hepatitis, and cirrhosis. It is common to find at least two stages coexisting. This division is advantageous when considering the progressive character of ALD and its continuous spectrum of varying severity, being steatosis less severe and cirrhosis the most severe. One important feature of this classification is the possibility to ascertain whether the disease is reversible (steatosis) or not (alcoholic cirrhosis) (Matos, 2006; APEF, 2014). It is very important that the diagnosis of ALD is done at an early stage. This diagnosis is based on the patient's clinical history, physical examination, and complementary diagnostic exams such as laboratory and imaging exams (APEF, 2014; Matos, 2006). The complexity of this diagnosis is vast and the practitioner must bear in mind that several alcohol dependent patients may not develop ALD and also that there might be symptoms that mimic ALD but do not derive necessarily from it or from alcohol consumption (e.g. non-alcoholic steato-hepatitis, which would be neglected in favour of a non-existent ALD). It is also imperative that the diagnostic is purely unbiased by social stereotypes or prejudices. However, patient history and family precedents should never be disregarded (Telles-Correia, 2011; Matos, 2006).The physician may need to cross-check the patient and the family testimony (APEF, 2014).

At the early stages, ALD is usually asymptomatic. When the disease reaches more severe stages the symptoms start to appear. However, these symptoms are very often unspecific and are common for most types of hepatic disease. Some examples of symptoms are jaundice, ascites, ginecomasty, and testicular atrophy (Matos, 2006; APEF, 2014). The laboratory tests may target ALD at the early stages by detecting a high volume of erythrocytes and hepatic enzymes. At more advanced stages albumin levels decrease while bilirubin levels increase (APEF, 2014). Amongst the imagiolic exams, abdominal ecography, computerized tomography, and magnetic resonance are quite common for diagnosing ALD. While the former two are helpful detecting steatosis, the latter one is handy to detect slight hepatic changes, such as the emergence of tumors (Matos, 2006).

Despite all these diagnostic tools, hepatic biopsy is the gold standard of ALD diagnosis even though it might have some risks associated (Matos, 2006; APEF, 2014). If a physician suspects that the patient is not being honest about alcohol consumption in the past, he may confirm this consumption if the biopsy reveals the AST/ALT ratio to be higher than 2. This is the ratio between the concentrations of the enzymes aspartate transaminase (AST) and alanine transaminase (ALT) and is commonly verified in medical diagnosis of liver disease. Most causes of liver injury are associated with a greater increase in ALT than AST; however, an AST/ALT ratio of 2:1 or higher is suggestive of alcoholic liver disease (Lucey et al., 1994; Telles-Correia, 2011).

2.2.2 Prognosis and Indication for Transplant

After the diagnosis is completed, it is necessary to stage the disease and evaluate how it can progress. This evaluation also includes the assessment of the psychosocial status of the patient. The prognosis of the disease is a very complex task and whether the patient should obtain a transplant or not is still a motif of discordance. The fact that this task can be subjective and the predictors used have still not proven to be totally flawless shows there is still a lot of work to do in order to improve this process and create consensus amongst the specialists.

It is common knowledge that alcoholic patients quickly improve their health status when they stop drinking alcohol. At times, this improve can be so significant that the patient might not need to go through transplantation anymore. The plasticity of the ALD can really hinder the prognosis of the liver injury and that is why a thorough assessment is crucial. Every physician would prefer to avoid liver transplantation in cases where abstinence and other therapies would be effective in recovering the liver functions (Lucey, 2015).

In order to establish standard indications to liver transplantation the European Association for the Study of the Liver stated the following: "Liver Transplantation (LT) should be considered in any patient with end-stage liver disease, in whom the LT would extend life expectancy beyond what the natural history of underlying liver disease would predict or in whom LT is likely to improve the quality of life (QoL)" (Burra et al., 2015). Following this definition, patients should be selected for transplantation if they are not expected to live more than one year if the transplant is not performed or if the disease has irreversibly decreased their QoL to unacceptable levels. However, the scarcity of liver grafts in comparison with the number of patients in need of a transplant complicates the entire situation, which seemed easy by the previous statements. In addition, there are huge costs associated with the transplantation process and one has to bear in mind that the ALD patient is an exceptional one. Therefore, it is mandatory to meticulously select the candidates for transplant (Telles-Correia and Mega, 2015).

With regard to assess whether patients cross-check all the requirements for becoming candidates for transplantation a multidisciplinary team gathers together on a weekly basis. They also discuss the place in which the candidates should enter the waiting list. Usually, this meeting team is composed by: surgeons, both the hepatologist and the psychologist that follow the candidate through the entire process, and two nurses, one of them being the chief-nurse. There are many parameters through which every candidate should be evaluated, from the clinical picture to psychosocial aspects and behaviour (Burra et al., 2015).

Regarding their clinical condition, patients can be classified according to several scores. Two relevant scores are the Child-Pugh Score (mainly for cirrhotic patients) and the Model for End-Stage Liver Disease (MELD). Though both of them are used nowadays, the latter one has gained relevance lately (Matos, 2006). The Child-Pugh Score assigns a value from 5 to 15 to the patient depending on the following five clinical criteria: total bilirubin, serum albumin, prothrombin time, ascites, and hepatic encephalopaty. Based on their punctuation patients are classified into class A (5-6), class B (7-9), and class C (10-15) (Burke and Lucey, 1998; Matos, 2006; Burra et al., 2015). Class A estimates a pre-transplant one year survival of 100%, class B a pre-transplant one year survival of 81%, and class C a pre-transplant one year survival of 45% (Cholongitas et al., 2005; Varma et al., 2010). Thus, class C patients urge the transplant earlier than patients from the other classes. Child-Pugh score criteria are outlined in table 2.2.

Table 2.2: Child-Pugh Score. Adapted from Burke and Lucey (1998)

Child-Pugh Score			
	Points		
Criteria	1	2	3
Encephalopathy (grade)	None	1-2	3-4
Ascites	None	Slight	Moderate-Severe
Bilirubin (mg/dL)	1-2	2-3	$>$3
Albumin (g/dl)	$>$3.5	2.8-3.5	$<$2.8
Prothrombin time (seconds prolonged)	1-4	4-10	$>$10

Due to the subjectivity of some parameters, such as the degree of ascites or encephalopathy, the Child-Pugh Score has been supplanted by the MELD score. This score was firstly used to determine the prognosis of a patient going through implantation of a transjugular intrahepatic portosystemic shunt (TIPS). It was modified and nowadays is used in most transplantation centers in the United States and in Europe, not just to assess whether the patient should get a liver transplant but also for organ allocation (Graziadei et al., 2016; Carrion et al., 2013). This mathematical predictive score is founded on three biochemical parameters: serum creatinine level, serum bilirubin level and international normalized ratio for prothrombin time (INR). The resulting value can be use to predict the 3-month mortality if the patient does not receive a liver graft. In a cohort study engaging 3437 adult liver transplant candidates, it was observed that mortality followed a pattern dependent on the MELD score assigned to the patient (Wiesner et al., 2003; Carrion et al., 2013):

– $<$ 9 – 1.9% mortality;

– 10 - 19 – 6.0% mortality;

– 20 - 29 – 19.6% mortality;

– 30 - 39 – 52.6% mortality;

– $>$ 40 – 71.3% mortality.

Opposing to mortality, in the previous cohort study the rate of 3-month survival was also estimated and can be seen in figure 2.2.

Patients should be indicated for transplant if they belong to class C in Child-Pugh classification or if they have a MELD score equal or higher than 15 (Graziadei et al., 2016). However, patients with scores below 15 (MELD) or in class B (Child-Pugh) might also be advised about the role of transplantation, with the possibility of being referenced for it depending on an individual evaluation (for example if major complications have already occurred) (Graziadei et al., 2016; Carrion et al., 2013). Besides this, MELD can also be a good predictor of long-term outcomes of candidates who received a liver transplant, but only if the score is greater than 25 (Carrion et al., 2013). Lastly, it should be noted that patients with MELD scores above 40 have much higher risks of dying in the waiting period. Thus, these cases require special consideration (Sharma et al., 2012).

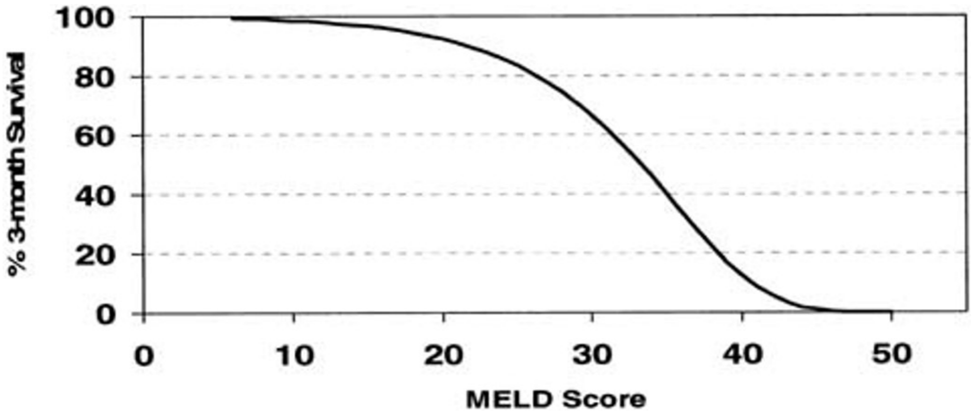

Figure 2.2: Estimated 3-month survival as a function of the MELD score. Extracted from Wiesner et al. (2003).

Nonetheless, MELD score has some limitations and it should not be used as the only criterion for indicating patients for liver transplantation. One of those limitations is that it does not consider some complications that might be attached to liver disease, e.g., portal hypertension. These complications may have a significant influence on the patient's prognosis (Graziadei et al., 2016).

The candidate's position on the waiting list, and consequently the liver grafts allocation, is conditioned only by the MELD score. Psychosocial aspects that will be addressed in 2.2.3 exclusively contribute to the entrance or withdrawal of the waiting list. Higher MELD scores mean top positions on the list, i.e., candidates in more severe states will get the transplant sooner. This score, and consequently the candidate's position, is dynamic in the sense that candidates are always under evaluation since they enter the candidacy list and their MELD score can be updated if they show improvement or deterioration of their health status.

There are some particularities in this process. One of them is when liver cirrhosis co-occurs with other complications, for example Hepatocellular carcinoma. When this is the case, patients followed in Hospital Curry Cabral (the reference centre for liver transplantation in Lisbon) get immediately a minimum MELD score of 22. Other singularity arises for very sick patients (MELD $>$ 30). For patients in these situations the risk of mortality and morbidity after transplantation should be taken into account individually, and in the worst case scenario they might not receive the transplant (Burra et al., 2015). In addition, a psychological assessment of the patient must be carefully and continuously carried out. This evaluation will be approached in the next subsection.

2.2.3 Contraindications and Psychosocial Aspects

An exhaustive evaluation of the patient is necessary to determine if ALD damaged more tissues than just the liver. It is frequent for the patient to show evidences of cardiac, renal, neural, or immune dysfunction due to alcohol dependence or abusive consumption. These previous systems should also be assessed apart from liver function and hepatocellular carcinoma (Lucey, 2015).

Since liver grafts for transplantation are a scarce resource when compared to the number of patients in need of a transplant, the candidate must be carefully selected. To back up physicians in this selection process several contraindications for liver transplantation have been defined. They are considered contraindications in the sense that they will have a negative impact on the outcome of the process, thus possibly excluding the candidates from the waiting list.

Contraindications to liver transplantation can be divided into absolute and relative contraindications (Farkas et al., 2014). Considering that liver transplantation is a last resort treatment, any patient who is not ill enough to go through this treatment should not be selected. Instead, alternative treatments should be assessed and followed (Lucey et al., 1992). Any patient who is excessively ill for transplantation should also be excluded. The most common ALD associated diseases that are considered absolute contraindications are uncontrolled systemic infections (e.g. sepsis), life-limiting medical conditions such as advanced cardiovascular, pulmonary, or neurologic disorders, and uncontrolled extrahepatic malignancy. Intrahepatic carcinoma with extrahepatic metastasis is also considered an absolute contraindication (Farkas et al., 2014; Graziadei et al., 2016). There is still the case of patients having hepatocellular carcinoma. To assess this medical condition the Milan criteria was created. This criteria defined the following set of parameters (Farkas et al., 2014; Ferreira et al., 2012; Graziadei et al., 2016):

– if there is a single nodule, it must not exceed 5 cm;

– there can be a maximum of three nodules, each one not exceeding 3 cm;

– there cannot be evidence of extrahepatic lesion nor vascular invasion.

If the heptocellular carcinoma is within the bounds of Milan criteria then the patient is eligible for transplantation. However, for the cases when the tumor does exceed the Milan criteria an individual evaluation should be performed and the physician will follow the national or local guidelines. Extended Milan criteria has been a topic of research and discussion (Farkas et al., 2014; Ferreira et al., 2012). Lastly, and probably one of the most important but yet most controversial contraindications, is the period of alcohol abstinence. It is generally accepted that patients should not be listed for transplantation without undergoing abstinence for a minimum period of six months (Farkas et al., 2014; Burra et al., 2015; Telles-Correia, 2011). It still raises debate and discordance, especially because many patients may not survive this period. One of the objectives of the required six months of abstinence is to diminish the alcoholic relapse after transplantation, also know as recidivism. Yet, there is no sufficient and meaningful data in the literature supporting the effectiveness of this six months preventing long-term recidivism (Lucey, 2015; Farkas et al., 2014). Actually, Vaillant (2003) advocates that proper soberness is only reached after 5 years of abstinence.

Relative contraindications range from psychosocial aspects to the advanced age of the patient, comprising obesity or malnutrition, and other diseases such as hepatopulmonary or hepatorenal syndromes (Farkas et al., 2014). These parameters are to be assessed by the multidisciplinary team for each patient individually. Both absolute and relative contraindications to liver transplantations are shown in table 2.3.

One of the main concerns for physicians, psychologists, and psychiatrists is whether the patient will relapse drinking after receiving the liver transplant. Recidivism during the waiting period is equally worrying, and that is why a proactive and close to the patient monitoring is performed (Beresford, 1997).

Table 2.3: Contraindications to liver transplantation. Adapted from Graziadei et al. (2016)

	Contraindications to liver transplantation
Absolute	Severe cardiac or pulmonary diseases and severe pulmonary hypertension
	Alcohol addiction without motivation for alcohol abstinence and ongoing substance abuse
	Hepatocellular carcinoma with extrahepatic metastases
	Current extrahepatic malignancies (eventually reevaluation after successful therapy)
	Sepsis
Relative	Untreated alcohol abuse and other drug-related addiction
	Cholangiocellular carcinoma
	Hepatic metastatic neuroendocrine tumors (NET), metastatic hemangioendothelioma
	Morbid obesity
	Persistent non-adherence

There has been intense research to develop an applicable patient selection criteria or strategy, though this is a subjective evaluation (Telles-Correia and Mega, 2015). Some researchers came up with different scales that can be helpful in predicting the probability of recidivism of each patient. One example is the Michigan Alcoholism Prognosis Scale for Major Organ Transplant Candidates, which can be seen in table 2.4. It takes into account factors such as the acceptance of alcoholism, prognostic indices, and social stability, and in the end assigns points to each parameter (Burke and Lucey, 1998; Lucey et al., 1992).

Table 2.4: Michigan alcoholism prognosis scale. Adapted from Varma et al. (2010)

Michigan alcoholism prognosis scale	
Criterion	Points
Acceptance of Alcoholism	
Patient and family	4
Patient only	3
Family only	2
Neither	1
Prognostic indices	
Substitute activities	Yes 3, No 1
Behavioral consequences	Yes 3, No 1
Hope/self-esteem	Yes 3, No 1
Social relationship	Yes 3, No 1
Social stability	
Steady job	1
Stable residence	1
Does not live alone	1
Stable marriage	1

Notwithstanding, the usefulness of these scales on the daily tasks and consultations remains unclear. Due to the psychological vulnerability of this type of patients, psychologists will not spend the consultation doing questionnaires instead of trying to build a strong relationship with the candidates. Additionally, the score calculated is not binding and has always to go through medical appreciation and validation (Telles-Correia, 2011).

That is why some good and poor sobriety prognosis factors have been suggested: duration of abstinence, social support, family history of alcoholism, alcohol abuse in contrast with dependence, denial of the alcohol dependence diagnosis, presence of mental disorders, noncompliance with the physicians, and finally the number of years of alcoholism as well as the average daily alcohol consumption (Telles-Correia and Mega, 2015; Lucey, 2015).

Although a longer period of abstinence is considered a good prognosis factor, it must not be treated as an absolute indicator (Telles-Correia and Mega, 2015). Social support has been always considered a good prognosis factor. G. Vaillant and T. Beresford do enhance on their literature the importance of the patient having a stable partner and environment at home, as well as dedication to activities that deviate the patient from alcohol consumption. In contrast, keeping up with alcohol-related activities is considered a poor sobriety prognosis factor (Telles-Correia, 2011). Having a family history of alcoholism or a history of a mental disorder (such as schizophrenia) are considered poor prognosis factors. Additionally, noncompliance with the medical team as been related to worse results after liver transplantation, thus being also a poor sobriety prognosis factor. Lastly, but not least important, are the years of dependence and the average daily quantity of alcohol consumed (Telles-Correia and Mega, 2015). Researchers have tried to define a threshold for these two variables, and Yates et al. (1993) suggested that the risk of recidivism was greater for more than 25 years of alcoholism and more than 17 drinks per day.

The psychologist has an important role in the entire process described. But her/his role goes beyond trying to predict the probability of alcohol relapse, whether it is previous to the transplantation or afterwards. Identifying mental disorders that can occur before or after the transplantation, supporting, and guiding the patient throughout the whole process, which usually lasts until the patient's death, is also a task of upmost importance (Burke and Lucey, 1998). Therefore, the relationship between the patient and the medical team and its details have been subject of study. Albeit physicians may think their intervention is the most relevant during the perioperative period, they can really influence how the patient will behave in the long-term. A careful and caring surveillance (more intense after the surgery) can be a powerful tool in the recovery and rehabilitation of the patient (Beresford, 1997).

To finish this section it is important to note that the concept of relapse is sometimes misused. Abstinence should be the main goal of the medical team that follows the patient, but relapse can mean different things and not all relapses are worth the same concern. Relapse of drinking is used to assess the presence of abstinence or parse the quantified alcohol consumption. Some scientists consider a "slip" if the patient does not exceed the threshold of 5 daily drinks for 5 consecutive days (Kotlyar et al., 2008). Relapse of dependence is generally correlated with the psychiatric conditions of alcoholic dependence or abuse. It is straightforward to conclude that the former definition has a much better prognosis regarding survival rates when compared to the abusive drinking (Telles-Correia and Mega, 2015).

2.3 Overview

ALD is a disease with a vast range of impacting consequences for society. Whether it is for its severity or for its large extent, ALD is a major concern for government bodies. Alcohol is widely spread allover the world and its consumption is common practice. But this consumption has implications in people's health. The liver is a vital organ and its failure may lead to death, thus being essential to treat patients suffering from it, if possible in the early stages of the disease. Nonetheless, sometimes this is not possible and liver transplantation emerges as the last resort treatment.

The huge incidence of this disease makes it a pretty common diagnosis. In addition, it is almost asymptomatic at the earlier stages which leads to a high number of people suffering from chronic liver disease at final stages. These patients demand liver transplantation to survive. The big problem is that there are not enough liver grafts for all the patients in need. Therefore, candidates are meticulously assessed before being selected. This assessment is a very complex and exhaustive process, which may lead to conflicts sometimes. Clinical aspects evaluation and psychosocial assessment are examples of key areas for the appraisal of the candidates. It is important to mention that candidates are continuously being assessed, and they have strict rules to follow if they want to be selected or keep their place on the waiting list. One of the most important criteria is alcohol abstinence. This work is underpinned by alcohol abstinence, since it models the interaction between candidates and doctors regarding this specific criterion.

Chapter 3

Game Theory Essentials and Literature Review

This chapter describes some fundamental concepts of game theory (GT) and reviews previous work that handled health care issues relying on game theory. Section 3.1 provides a broad insight on the foundations of this mathematical tool, and some indispensable concepts that will allow for a better comprehension of the model applied are addressed in subsections 3.1.1 and 3.1.2. Several examples of the applicability of GT to social interactions in the medical field are presented in section 3.2. A few models of different applications of GT - in other domains than doctor-patient relationship - are briefly described in section 3.3. Lastly, section 3.4 gives a summary of all the work done on the field and how it launched the work done in this dissertation.

3.1 Game Theory

A wider definition of GT is given in Gibbons (1992): “Game theory is the study of multiperson decision problems”. It studies the strategic scenarios and decisions in a situation with several agents (two or more), whenever their decisions are interdependent. The range of agent types goes from individuals to firms, or combinations of both. It can be considered to be the formal study of conflict and cooperation (Turocy and von Stengel, 2002). Since a social interaction can be excessively complicated to simulate, considering it a mathematical game is an abstraction which simplifies this interaction. Therefore, it consists of three explicitly defined elements: a set of players, a set of actions for each individual player (strategies), and a preference profile over the set of action profiles (payoffs) usually represented in utility measures (Colman, 2003).

3.1.1 Games Representation

Games can be represented in normal form and extensive form. In the normal form representation, players simultaneously choose one of their possible strategies and the combination of the chosen strategies will lead to a specific payoff for each player (Turocy and von Stengel, 2002). The classic example of this representation of a game is called The Prisoners' Dilemma, where two prisoners in custody have to decide whether to confess the crime or remain silent. They are kept separate so each prisoner does not know what the other will do. There are four possible outcomes, depending on the strategy each prisoner chooses (Gibbons, 1992):

– If neither confesses, both are sentenced to one month in jail;

– If both confess, both are sentenced to six months in jail;

– If one confesses but the other does not, the confessor is liberated (best payoff) and the other is sentenced to nine months in jail (worst payoff).

These types of normal form games can be represented in a matrix (bi-matrix for the previous example since we have two players with two strategies each). The representation of the payoff matrix for this game can be seen in figure 3.1. Note that strategies are *Mum* (not confess) and *Fink* (confess).

		Prisoner 2	
		Mum	Fink
Prisoner 1	Mum	$-1, -1$	$-9, 0$
	Fink	$0, -9$	$-6, -6$

The Prisoners' Dilemma

Figure 3.1: Payoff matrix for Prisoners' Dilemma game. Extracted from Gibbons (1992).

Although both prisoners would be better off if they both cooperated (not confessing), this will not happen because joint cooperation is not an equilibrium (Nash equilibrium explained in subsection 3.1.2). If one prisoner decides to not confess, the other will be tempted to confess, thus being immediately set free while the non-confessor would have the worst payoff. This explains why they both will confess, even if they would both be better off cooperating.

The extensive form, or game tree, is a more exhaustive representation of the game and takes into consideration that decisions may not be taken simultaneously (Gibbons, 1992). By this, it allows to acknowledge when each player is playing, the information each player has by the time of the action, and the occasions when uncertainty in a situation is cleared up: "It is a complete description of how the game is played over time" (Turocy and von Stengel, 2002). A game tree representation must contain a root node at the top, branches, intermediate nodes with more branches coming out of them, and finally the leafs or terminal nodes at the bottom of the tree. The top node defines the first player to take action, and the branches coming out of it represent the set of possible actions for the first player. The intermediate nodes represent the other player or players and the branches depict their set of possible actions as well. Finally, the terminal nodes show the payoffs for each player, for the different strategies they may choose. In an extensive form representation, the complete plan of actions, one for each decision point of each player, represents that player's strategy. An example of extensive form representation of a game can be seen in figure 3.2.

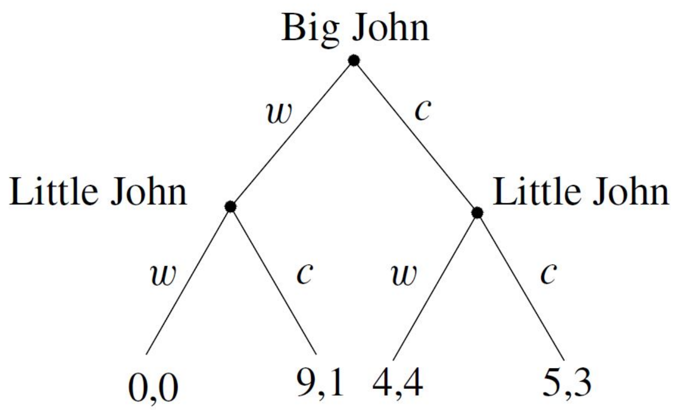

Figure 3.2: Extensive form representation of the Big John and Little John game. Extracted from Gintis (2009).

In Big John and Little John game, both players eat coconuts which are hanging at the top a palm tree. The coconut produces 10 Kc (kilocalories) of energy. Big John spends 2 Kc climbing the tree and running back down while Little John spends an insignificant amount of energy doing the same activities. Both players have two possible strategies: wait (*w*) or climb (*c*). The payoffs are depicted at the bottom of the tree and they represent the amount of energy each player will obtain from the coconut if that certain path is followed. The first number refers to the first decision taker (Big John in this case) and the second number to the following decision taker (Little John). For example, if both players wait (*w,w*) than both get 0 Kc of energy. In this case it was assumed Big John would decide first. So Big John has to inspect how Little John will react to both Big John's different choices. If Big John chooses to wait, then Little John will choose to climb since this will give him 1 Kc of energy instead of 0 Kc. On the other hand, if Big John climbs then Little John will choose to wait because this will get him 4 Kc instead of 3 Kc. Summing up, Big John will get 9 Kc of energy if he decides to wait and 4 Kc if he decides to climb. Thus, he will decide to wait and Little John will climb (Gintis, 2009).

3.1.2 Fundamental Concepts

Following the previously introduced definition of strategy, it is imperative to present the concepts of pure and mixed strategies. *Pure strategies* are the actions a player may take for any situation faced, thus showing a complete description of how a player will play a game. The player's strategy set is the set of pure strategies available to that player (Gibbons, 1992). A *mixed strategy* is a probabilistic combination of pure actual strategies. It is an active randomization (*i.e.*, assigning probabilities to each pure stratey) that determines the player's decision. In an extreme case, a mixed strategy can be the choice of one pure strategy, meaning that the chosen strategy gets 100% probability and the others get 0%. Using mixed strategies can be very useful to interpret one player's uncertainty about what another player will do (Heap and Varoufakis, 1995).

Strategic dominance is an important concept related to strategies. It occurs when one strategy always gives a better payoff than another strategy for one player, no matter how the other players are playing. It weakly dominates the other strategy if it is always at least as good. This concept can be very helpful when solving simple games, however that is not what always happens. In more complex games, a strategy may be better or worse for one player depending on what the opponents do (Turocy and von Stengel, 2002).

Player preferences are represented in the form of payoff functions which associate a value to the outcome of each action in a way such that actions with higher numbers are more favorable to the player and are hence preferred. Mathematically, for any two strategies a and b in a set of strategies, with $u(a)$ and $u(b)$ said to represent the payoff functions and taking into account that these also depend on the other players' decisions, $u(a) > u(b)$ implies that the player prefers action a over b for the specific set of strategies chosen by the other players (Osborne, 2004).

Information plays a relevant role in GT. An information set for a player establishes all the possible moves that could have taken place in the game so far, given what that player has observed. A game has perfect information when, at any point in time, the player making the move knows exactly all the actions that have been made until then, *i.e.*, the player knows the point reached at that stage of the game and knows the moves previously made by all other players. Otherwise, if the game has imperfect information some players cannot be sure about what has taken place so far in the game and what their position is. Complete information is sometimes confused with perfect information, however they have different implications. Complete information means that every player in the game knows the strategies and the payoffs available to the other players, but not necessarily the actions taken previously (Gibbons, 1992).

To solve extensive-form games of perfect information, a commonly used technique is backward induction. It starts analyzing the last moves in the game, determines the best moves in each case and assumes them as given future actions. Then, it proceeds backwards in time again determining the best moves until the first move of the game is reached (Turocy and von Stengel, 2002).

One of the most important concepts of game theory is the *Nash equilibrium*. As previously mentioned, dominance reasoning is not the key to foresee what might happen in many games. In these circumstances, the solution is given by the Nash equilibrium concept, named after its creator John Nash. It is a list of strategies, one for each player, which has the property that no player can unilaterally change his strategy and get a better payoff. This is based on the idea that players would not want to change their strategies given what the others had chosen to do. It is important to notice that in a two-player game, an equilibrium point (and thus a Nash equilibrium) is a pair of strategies that are best replies to each other, that is, maximize the player's payoff given the strategy chosen by the other player (Heap and Varoufakis, 1995; Colman, 2003).

Lastly, a *subgame perfect Nash equilibrium* is a refinement of the Nash equilibrium used in sequential (dynamic) games. Games represented by extensive forms can be conceptualized as bundle of several subgames (as many as the number of decisions the main game contains). A strategy combination is a subgame perfect equilibrium if it represents a Nash equilibrium of every subgame of the original game. Intuitively, this means that if the players played any shorter game that could be represented by one subgame of the larger game, their behavior would represent a Nash equilibrium of that smaller game. Every finite extensive game has a subgame perfect equilibrium, that can be detected after backward induction (Gibbons, 1992).

To conclude this section, it is essential to remark that some of the previous concepts are substantiated by some assumptions. The main assumption in game theory is that all players are rational, which implies they all will play with the only goal of maximizing their own payoff. Another relevant assumption is common knowledge. A fact is common knowledge if all players know it, and know they all know it, and so on (Turocy and von Stengel, 2002).

3.2 Literature Review

This section provides a literature review of game theory applications to health care problems and also to other domains where it is considered to be a very important tool.

To begin this review it is interesting to briefly take a look at how medical consultations can be seen through game theory perspective. Tarrant et al. (2004) states that the medical consultation is "best understood as a two-way social interaction involving interactive decision making". This means that the outcomes of a consultation will be dependent on the actions and choices of both participants. Consequently, game theory is a promising tool to study consultations. Nevertheless, not all consultations run the same way since they depend on several factors, starting with the patient's disease. This is why there is a variety of game structures to be applied to provide insights of the underlying dynamics of the interaction between the doctor and the patient.

Some examples of game structures can be the Prisoners' Dilemma, the Assurance game, and the Centipede game, to be detailed ahead. Note that there is little evidence that shared decisions are recurring practice in consultations, but it is obvious that even if it is not the case of shared decision, the outcome will still be influenced by both players' decisions - the doctor's decision about treatment or management and the patient's decision about whether to follow the advice or look for a second opinion, for example. We shall now proceed to drill-down the previous structures using some medical context as examples.

3.2.1 Prisoners' Dilemma

The Prisioners' Dilemma can simulate a consultation interaction if some simplifying assumptions are made. The doctor may cooperate and act in the patient's best interests (C) or for some reason (*e.g.* medical error, misjudgment, lack of skills, or personal interests) take a decision which will not bring the best benefits for the patient (D). The patient in turn may decide to follow the physician's advice (C) or not (D). A hypothetical situation for this game could be the case of a patient going to see a doctor on a busy day complaining from a sore throat. The patient has results from some exams revealing a red throat, a slight fever, and a slightly swollen cervical lymph nodes. The doctor checks the exams and may decide whether to prescribe antibiotics and spend no more than five minutes dealing with the patient, or to fully undertake a lifestyle assessment and give personalized advice about self-management, which would extend the consultation for ten minutes. The patient then can choose between following the doctor's advice and prescription or withhold treatment and seek for a second opinion. We end up with four possible outcomes:

– (C,C): the doctor fully engages and the patient follows the advice;

– (C,D): the doctor fully engages but the patient seeks for a second opinion;

– (D,C): the doctor gives a prescription and the patient follows the treatment;

– (D,D): the doctor gives a prescription but the patient seeks for a second opinion.

Following intuition, one would suggest (C,C) to be the best all round - the doctor would do the best for the patient and the patient follows the advice saving up valuable time of other doctors. However, this pair of strategies is not a Nash equilibrium as explained previously. The unique Nash equilibrium in this game is the joint non-cooperation (D,D), since both players avoid the worst outcome possible (cooperating while the other does not). For all that, this would not have good consequences for the quality of care and this outcome is undesirable (Tarrant et al., 2004). That is why the Prisoner's Dilemma is sometimes considered paradoxical and problematic, especially when considering a single game as the previous one where cooperation is shown not to be a rational strategy by game theoretic principles. However, the situation changes when considering interactions that are assumed to last infinitely in the future, called the infinitely repeated games. When analyzing infinitely repeated Prisoner's Dilemma game it is possible to find some cooperative strategies that constitute Nash equilibrium, advocating cooperation as a rational strategy in these cases (Gibbons, 1992). There are some factors intrinsic to these infinitely repeated games that promote cooperation, including anticipation of future interactions (with the possibility of foreseeing cooperation payoffs), ability to recognize each other and recall past interactions, and lastly the threat of reprehension from the other player in future interactions. In the medical consultation context, both the doctor and the patient are encouraged to cooperate by these factors. The doctor is likely to dedicate time to ensure the patient gets the best assistance and feels pleased being followed by him. It will be rewarding to carry all the process until completion. In turn, the patient is more willing to follow treatment if there is an expectation that the doctor will monitor the future progress (Tarrant et al., 2004).

It is important to mention that payoffs in the medical context are not equitable. The stakes are higher for the patient who will be worse off if the doctor does not cooperate than the opposite. This way, payoffs will have a higher impact on the patient than the impact they have on the doctor.

The previous explanation on the limitations of applying the Prisoner's Dilemma game to medical consultations leads us to presenting other structures of games that better incorporate cooperation.

3.2.2 The Assurance Game

The Assurance game was introduce by Sen (1969) and models interactions where cooperation leads to the best outcomes, but carries some inherent risks. It is also known as the "Stag hunt game" which was a situation described by Rousseau and Cranston (1984). Two individuals go out on a hunt and simultaneously choose to hunt a stag or a hare. In order to successfully hunt a stag, a player requires

Table 3.1: Payoff matrix for Stag hunt game. Adapted from Girtz et al. (2017).

Stag Hunt Game		**Hunter 2**	
		Stag	Hare
Hunter 1	Stag	5, 5	0, 4
	Hare	4, 0	2, 2

cooperation from the other player. An individual can get a hare by himself, but a hare is worth less than a stag. The payoff matrix is depicted in table 3.1

Looking at table 3.1 one concludes that if both hunters one and two choose to hunt a stag then they will both get a payoff of 5. But if hunter one takes a hare and hunter two a stag then the first player will get 4 and the second gets 0, and so on.

Contrasting with Prisoners' Dilemma, this game has two pure strategy Nash equilibria - one that is risk dominant another that is payoff dominant. The strategy pair (Stag, Stag) is payoff dominant since payoffs are higher for both players compared to the other pure Nash equilibrium, (Hare, Hare). On the other hand, the latter strategy risk dominates the former since there is uncertainty about the other player's action. The more uncertain players are about the actions of the others, the more likely they will choose the strategy corresponding to it. Both players prefer one equilibrium to the other - hunting a stag is Pareto optimal and Hicks optimal. An outcome is Pareto efficient if there is no other outcome that increases at least one player's payoff without decreasing anyone else's. Pareto efficiency is a weaker form of efficiency because it does not make comparisons between players. A Hicks optimal outcome is an outcome in which the total payoff for all of the players of a game is the most it could possibly be. Hicks efficiency implies Pareto efficiency (Srivastava et al., 2005).

In addition to the pure strategy Nash equilibria there is one mixed strategy Nash equilibrium. This equilibrium depends on the payoffs, but the risk dominance condition places a bound on the mixed strategy Nash equilibrium. No payoffs (that satisfy the above conditions including risk dominance) can generate a mixed strategy equilibrium where Stag is played with a probability higher than one half (Rousseau and Cranston, 1984; Sen, 1969).

The notion of trust is then compulsory to address. Although it may be complex to define trust within the doctor-patient relationship in a game theory perspective, it has been proved to be correlated with some determinants. Consecutive interactions reinforce the trust between doctor and patient. This can be achieved by adequate and regular contact between them, thus showing involvement from the doctor. Time is a valuable variable to enrich trust since it allows to build a solid and trustfully relationship. Time can be used by physicians as a tool to show patients they care (used as a signal). Therefore, it is possible to bridge this medical context to signaling games as well. Signaling games are incomplete information games where the more informed player has to decide whether to signal in some way her/his true type, and the less informed player has to decide how to respond to both the uncertainty about her/his opponent's type and the signal sent, recognizing that signals may be strategically chosen. Since medicine is a credence good, a physician-patient interaction will always be considered an incomplete information game, with the physician being the more informed party (Sobel, 2007). The physician can show the patient that she/he cares and will make an effort to deliver the best treatment possible so the patient should trust her/him. This can be done by signaling, and spending time with the patient can be considered a signal. Time spent with a patient can also be seen as representing "continuity" of a game, since a physician is not likely to spend time with a patient she/he does not intend to see in the future (Huttegger et al., 2014). Other contributing factor to build up trust are the expectations about treatment and care in a sense of the patient being able to rely on the doctor if something unexpected is needed. The patient expects the doctor to be available when needed. Lastly, the verbal and non verbal signals transmitted during the consultation are key elements to reinforce trust. The attitudes from both participants may contribute to a trustful relation. These signals may range from the way the physician adjusts the transmitted information to the way the patient expresses himself or if she/he listens carefully to what the doctor has to say (Riva et al., 2014).

Trust and assurance go hand in hand in this type of games. If players expect cooperative behavior from each other then cooperation will be a probable outcome. If there is history of past interactions between both players there is more information through which players may create their expectations. If there is prediction of future interactions both players can clearly commit to cooperate in order to achieve the best outcome. Here, communication plays a critical role in the assessment of trust and assurance (Tarrant et al., 2004).

Tarrant et al. (2004) give an example of the Assurance game in the health care context, which is a doctor initiating a patient into a smoking cessation programme. Both players (doctor and patient) will benefit the most from mutual cooperation, but the doctor might be subject to some worse payoffs if the patient is not interested in cooperating. The worst outcome the doctor can have is to worthlessly dedicate time and effort to a patient who is not willing to commit to the programme. If the relationship is not a trustful one, and there is no assurance that the patient will cooperate, then the doctor should consider defection instead of cooperation.

3.2.3 The Centipede Game

The Centipede game is captivating attention in health care studies since it models a repetition of interactions between a pair of players (Rosenthal, 1981). The Centipede game has many characteristics in common with the Prisoner's Dilemma because in both games players would be better off cooperating but they face a temptation in each decision. The specific feature of the Centipede game is the increasing benefit of cooperating throughout the successive encounters. The downside is the subsequent rise in the non-cooperative outcome for unilateral non-cooperation, so temptation to defect is always enhanced in each move. The Centipede game can reflect several factors of the doctor-patient relationship. The more time the doctor and the patient invest on this relationship the more personal knowledge the doctor will have on the patient, thus enabling better care (appropriate diagnosis and management plans). On the patient's side, more time spent with the doctor is likely to increase the patient's confidence on the treatment. However, none of this is possible if the two players are not willing to cooperate. A defection move by a patient may be the decision to see another doctor or not accepting the doctor's advice. A doctor may defect by recommending another doctor or act in a way such that trust will be compromised (Tarrant et al., 2004). This game structure was introduced by Rosenthal (1981) who showed that the only rational strategy is to defect, and the Nash equilibrium resides on the first player defecting at the initial move, thus ending the game. The aforementioned conclusion is not in line with intuition and experiments proving that people can be very cooperative.

An example of a representation of centipede game is show in figure 3.3. The payoffs are represented at the tip of each non-cooperative branch, except for the last payoff which represents the outcome of cooperation throughout the entire game.

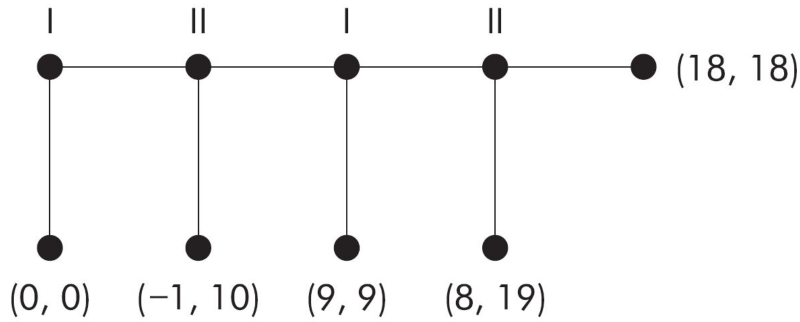

Figure 3.3: Centipede game. Extracted from Tarrant et al. (2004).

Player I has the first move (on the left) and defection will lead to a null outcome for both players. If the first player chooses cooperation, then player II gets to choose. If defection is chosen, player I gets a negative payoff while player II is rewarded with a payoff of 10. If neither defect, the game goes on until the last move. If one player defects at some point before the end, then the game stops and the players get their respective payoffs. It is possible to confirm cooperation is fostered as the game moves on, but defecting while the other player cooperates is also increasingly rewarding.

3.2.4 Game Theory and Health care

Two examples of games applied to health care are now presented. Note that both of them are structured as a Prisoner's Dilemma game, putting in evidence the lack of studies in this area.

Diamond et al. (1986) establish differences between game theory and decision analysis and uses the concept of expected utility, which can be explained as the product between utility (measure of one's degree of preference relative to a specific goal - payoff) of an outcome and its probability of occurrence. In decision analysis, the best outcome is the one carrying the highest expected utility and it relies upon one agent's decision. In contrast, game theory requires more than one agent to make a decision, and outcomes will be dependent on the actions taken by all agents. If there is uncertainty embodied in the game one might consider the payoffs as expected utilities, otherwise the players will just want to maximize their utility based on the others' actions.

The clinical problem is based on a decision a doctor may have to take when dealing with liver diseased patients: the patients may have one of two diseases, each one demanding different treatment paths. The hypothesis are: chronic progressive hepatitis, 20% of the cases, and cirrhosis, 80% of the cases. The doctor may decide whether to proceed with a treatment with steroids or ask for a biopsy. This latter decision will lead to a perfect diagnosis and appropriate treatment path but exposes the patient to a 0.1% risk of dying. The decision the doctor faces is whether to perform a biopsy or not.

It is necessary to know the utilities of each treatment to approach this problem. The possible paths to be taken are presented in figure 3.4, which was adapted from the article in question. The software Precision Tree from Palisade was used to draw this decision tree. Note that utility is quantified in two year survival, shown in percentage below the probability of occurrence of that branch. There is no interaction here, it only represents the doctor's decision and the utilities it may bring up based on the uncertain events that are more or less likely to happen (each uncertain event has a different probability of happening).

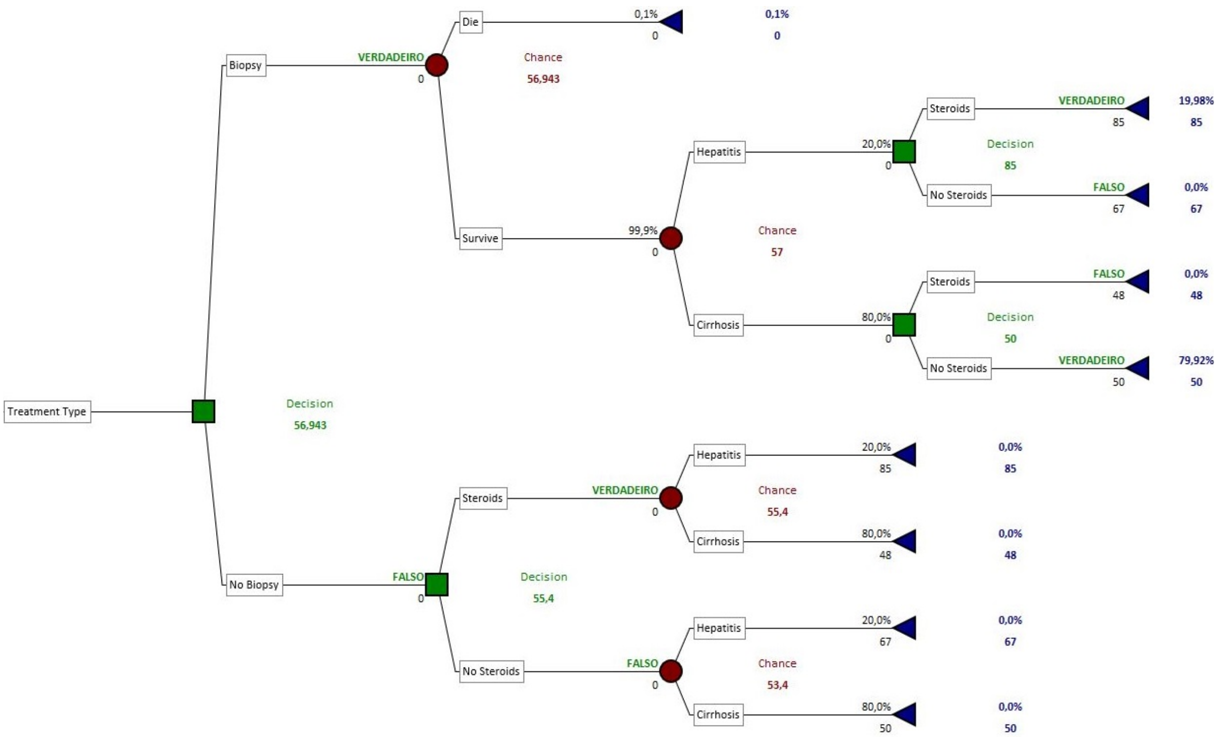

Figure 3.4: Decision tree for the treatment of chronic liver failure. Choice odes are denoted by a square and chance nodes by a circle. Survival is in percent. This decision tree was built using the software Precision Tree from Palisade. Adapted from Diamond et al. (1986).

Diamond et al. (1986) first solve the problem using decision analysis. The expected utility of each path is the product of its utility (here considered two be two-year survival) and the probability of its occurrence. As an example, the expected utility of having hepatitis and being treated with steroids is 85% times 20% which gives 17%. For the case of treating cirrhosis with steroids, the expected utility is given by multiplying 48% by 80% which gives 38.4%. The expected utility of each strategy is then given by the sum of all the expected utilities that strategy may comprise. The expected utility of choosing steroid treatment without previously performing a biopsy is then 17% plus 38.4%, giving a total of 55.4%.

The same reasoning is applied to the treatment without steroids, resulting in a expected utility of 53.4%. If the doctor decides to perform a biopsy, there is the chance of dying, with its expected utility being obviously null. When choosing the treatment after doing biopsy, for the case of biopsy survival, the expected utility for each treatment must be multiplied by the probability of surviving the biopsy, 99.9%. But the difference now is that the doctor knows exactly which treatment to choose because the disease was unveiled by the biopsy. Thus, the utility of treating hepatitis with steroids and not using steroids for cirrhosis after biopsy increases to 57%. However, the 0.1% death rate must be taken into account and this utility lowers to 56.9%. Given that this value is still higher than the expected utility of treating all patients with steroids without performing a biopsy, as a conclusion, the doctor should always perform a biopsy and only treat with steroids patients with chronic progressive hepatitis.

The main difference between game theory and decision theory is that in the former outcomes depend on the other players' decisions. Now it is assumed that the patient may reject the doctor's recommendation, whatever this may be.

There are now four possible strategies for the decision of whether to perform a biopsy or not, summarized in table 3.2 jointly with the payoffs.

Table 3.2: Payoff matrix for Biopsy game. Adapted from Diamond et al. (1986).

Biopsy Game		**Physician**	
		Biopsy	No Biopsy
Patient	Biopsy	55.6, 55.6	56.9, 56.9
	No Biospy	54.4, 54.4	55.4, 55.4

It is important to mention that payoffs are still measured in two year survival of the patient, explaining why payoffs are equal for both players. It is also assumed both players want to maximize the two year survival, but now the biopsy risk of death is 2.8% (corresponding to the threshold pointed by the authors).

One big difference is that now one has to consider four strategies instead of two. The author sets two probabilities, p is the probability that the physician recommends the biopsy and q the probability that the patient accepts that recommendation. Since the physician and the patient share the same utilities, it is feasible to assume the previous probabilities to be equal, and that the two-year survival is a function of p and the utility matrix. Reminding that players are assumed to be rational, the optimal strategy sets the value of p that will maximize the two year survival, computed through the following equation:

$$p = \frac{2d - c - b}{2(a - b - c + d)} \tag{3.1}$$

where a represents the utility of the strategy in which the biopsy is recommended by the doctor and accepted by the patient, b the utility of the strategy in which the physician does not recommend the biopsy but the patient does not accept that and struggles to find other doctor willing to perform it. Utility c represents the opposite from b, the patient rejects the biopsy recommended by the physician, and d

shows the utility of both players rejecting the biopsy.

The computation of this equation is shown in appendix A.

When replacing the utility letters with the values from the utility matrix, one finds the optimal value of p to be exactly $5/6$. This result contrasts with the deterministic strategy (*always recommend biopsy*) given by decision theory, where the patient's decision is disregarded. This value of p suggests that the doctor should recommend biopsy to five of every six patients, and only five out of every six patients should accept the recommendation (since q was assumed to be equal to p). The solution to this game is in fact a mixed-strategy.

It is important to reinforce the fact that the goal of this game was to maximize the number of patients surviving. It is also of greatest importance to mention that the doctor was considered to be altruist, instead of egoist. Both players had the goal to maximize the patient's utility, which can sometimes not be case in real life and change the game's results. Physicians also have their interests, and their payoffs must be represented and be considered separately, which can sometimes create conflict (Diamond et al., 1986).

Djulbegovic et al. (2015) describe a different problem recurring to game theory, adopting the Prisoners' Dilemma structure. Interestingly, the authors add emotions to this game, namely trust, regret, guilt, and frustration. All of the previous emotions will account for the payoffs and will be briefly explained now. Trust was previously described in detail, so the only important thing to remember is that it is helpful to avoid the Prisoners' Dilemma game, since trust promotes cooperation (Riva et al., 2014). A very important notion is that medicine is a credence good, meaning the situation between physician and patient is asymmetrical: patients are vulnerable because doctors have much more knowledge, which can sometimes lead to trust abuse circumstances. Additionally, physicians are incapable of assuring absolutely correct treatment or recommendations every time they treat a patient. This incapacity may sometimes lead to a feeling of regret, that can be felt by both players. An example of regret is when a doctor gives unnecessary treatment to a patient that can eventually be harmful. The feeling of guilt may affect physicians who realize that they did not act taking into account the patients' best interests. Last of all, frustration can be considered a feeling of disappointment when a player is unable to do something. As an example, a doctor may get frustrated if the patient refuses the recommended treatment. All these previous feelings will have impact on any of diverse clinical outcomes (Djulbegovic et al., 2015).

Now, after introducing the elements added to the game (namely to the payoffs matrix) the clinical situation is explained. A physician who is seeing a patient, does not know if the patient has a disease or not, and has to decide whether to recommend a specific treatment assuming it is not feasible to obtain more information, *i.e.*, it is not possible to do more diagnostic testing. In turn, the patient may choose between accepting or rejecting the doctor's recommendation.

The payoffs matrix is represented in table 3.3:

Note that Rx and $NoRx$ stand for recommend and not recommed treatment, respectively. Regarding the payoffs, P_{11} represents the utility of the patient's outcome when she/he trusts the doctor and accepts the recommended treatment. The letter D was chosen for the doctor's outcomes. As an example, D_{21} is the utility of the doctor's outcome when the treatment was recommended but the patient did not trust

Table 3.3: Payoff matrix for Treatment game. Adapted from Djulbegovic et al. (2015)

Treatment Game		**Doctor**	
		Rx	$NoRx$
Patient	$Trust$	(P_{11}, D_{11})	(P_{12}, D_{12})
	$NoTrust$	(P_{21}, D_{21})	(P_{22}, D_{22})

the doctor, thus rejecting the treatment.

The first step taken is to access if there is strategic dominance. If $P_{11} > P_{21}$ and $P_{12} > P_{22}$ then the patient has a dominant strategy, which is to always trust the doctor's decision. For the doctor, using the same reasoning one gets that if $D_{11} > D_{12}$ and $D_{21} > D_{22}$ then the doctor should always choose treatment (Rx) over no treatment ($NoRx$).

As previously mentioned, these payoffs are related to utilities that refer to a wide range of clinical outcomes (*e.g.* life expectancy, mortality and morbidity rates, amongst others). The utilities are denoted by V for the physician and U for the patient, and are expected to differ even for similar outcomes, since patient and doctor have different expectations and interests. The clarification of the different outcomes is depicted in table 3.4:

Table 3.4: Outcome utilities for patient and doctor, arising from the medical situation. Based on information in Djulbegovic et al. (2015).

Outcome Utilities for Patient (U) and Doctor (V)	
V_1, U_1	Treatment administered when disease is present
V_2, U_2	Treatment administered when disease is absent
V_3, U_3	Treatment not administered when disease is present
V_4, U_4	Treatment not administered when disease is absent

After defining the utilities, it is important to state some assumptions that will allow to appraise each one in relation to the others. The first assumption is that the doctor values more outcomes associated with treatment administered when the disease is present than treatment not administered when the disease is absent ($V_1 > V_4$). It is considered to be a plausible assumption since the willingness to act is higher than to do nothing. It is considered to be worse failing to administer treatment when necessary than administering treatment when it would not have been necessary. Secondly, outcomes associated with no treatment when the disease is absent will obviously be more valued than outcomes associated with treatment administration without the presence of the disease ($V_4 > V_2$). This assumption is in line with the “first do no harm” principle of the practice of medicine. Last assumption for the doctor’s utilities ordering is that outcomes associated with treatment administration when the disease is absent are more valued than the failure to administer treatment when necessary, *i.e.*, when the disease was present ($V_2 > V_3$). The utilities for the patient’s outcomes are assumed to follow the same pattern as the doctor’s. A patient usually sees a doctor when she/he does not feel right, and creates expectations about a diagnostic and a recommended treatment. It is also good to know there is no disease and no necessary treatment, but here it is assumed that having the disease and being treated matches the patient’s expectations better than not having the disease and not being treated ($U_1 > U_4$). When the disease is absent, it is obvious that any patient would rather not receive treatment than being overtreated ($U_4 > U_2$). However, when comparing the cases of being unnecessarily overtreated and undetreated when the disease was present, any patient would rather be overtreated than not receiving the treatment needed and not having her/his treatment expectations met ($U_2 > U_3$).

Although the authors restricted their assumptions to the previous ones in order to allow better interpretation of the results, other assumptions and utilities comparisons could have been made. However, these comparisons would require solid and logic rationale. If, for some reason, the assumptions made would change the sorted utilities then all the results of the game could also change. This could be used to perform sensitivity and robustness analysis to the model.

It is also credible to assume that the patient’s utility of being treated when sick is higher than the doctor’s utility when failing to administer treatment to a patient in need ($U_1 > V_3$). Analogously, the patient will benefit more of not being treated when unnecessary than the doctor when unnecessarily treating the patient ($U_4 > V_2$).

In order to make it all clearer, the assumptions considered are the following:

– $V_3 < V_2 < V_4 < V_1$

– $U_3 < U_2 < U_4 < U_1$

– $V_2 < U_4$ and $V_3 < U_1$

The last step to finish engineering the model is to define how the previously explained emotions correlate with the utilities in order to obtain the final payoffs. In this study, trust was taken into account on the patient’s decision, so it does not directly contribute to the payoff. Instead, without trust the patient chooses the opposite of the doctor’s recommendation. Regret (R) is considered to be related with the difference in utility between the outcome obtained from the decision taken and the best outcome possible. Guilt (G) was considered to be possibly felt only by physicians when there is an abuse of patient’s trust whether it is deliberate or by honest mistake. G represents the decrease in the doctor’s utility by a fraction of the difference between her/his and the patient’s utilities, when unnecessary treatment is prescribed or the necessary treatment is withheld. Ultimately, frustration (F) represents the difference in utilities between the patient and the doctor, or vice versa, when one of the players is forced by the other to do what is less optimal and will obtain smaller utilities. Similarly to the previous game example, this study only considered one interaction, but addressed two different situations: one where the patient demands treatment, but does not get it if the doctor does not recommend it, and the other in which the patient gets the demanded treatment (*e.g.*, if the patient sees another physician who is willing to administer treatment). As these two situations will use slightly different utilities but the same reasoning, only the first situation is considered to avoid unnecessary repetitions.

All the possible decisions, uncertainties, and the utilities associated to each outcome are organized in a decision tree, shown in figure 3.5. Note that p represents the probability of the disease being present, and therefore $(1-p)$ represents the probability of its absence.

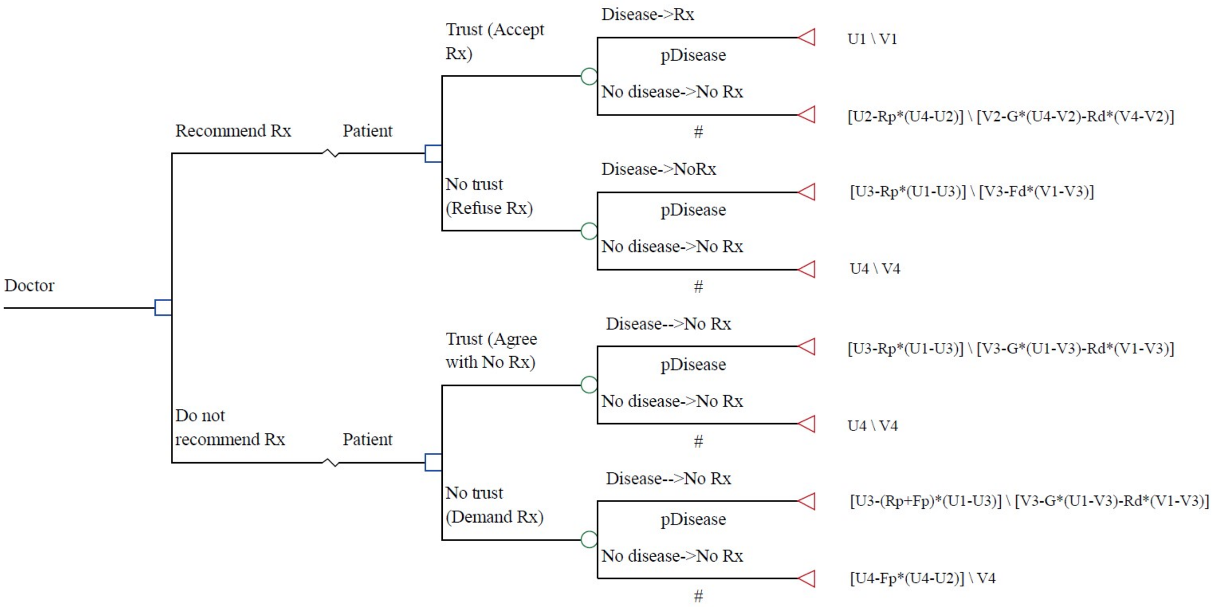

Figure 3.5: A game theory model related to decision whether a physician should give treatment when no further diagnostic testing is available, and whether a patient should accept the recommendation (The patient demands treatment but does not get it). Extracted from Djulbegovic et al. (2015).

Now it is possible to compute the payoffs. The method to compute them was the same as used in decision theory, computing the expected value E. Hence, the expected payoffs are obtained by multiplying the probability of an outcome with the utility associated to that outcome. It is important to mention that each strategy will have two possible outcomes: one for the presence of the disease and the other for its absence. For example, for case of the doctor choosing to recommend treatment and the patient not trusting this recommendation, one will have two outcomes depending on whether the disease is present, with different utilities each. The computation of all the payoffs is shown below:

$$P_{11} = E[Trust, Rx] = p \cdot U_1 + (1-p) \cdot (U_2 - R_p \cdot (U_4 - U_2)) \tag{3.2}$$

$$P_{21} = E[NoTrust, Rx] = p \cdot (U_3 - R_p \cdot (U_1 - U_3)) + (1-p) \cdot U_4 \tag{3.3}$$

$$P_{12} = E[Trust, NoRx] = p \cdot (U_3 - R_p \cdot (U_1 - U_3)) + (1-p) \cdot U_4 \tag{3.4}$$

$$P_{22} = E[NoTrust, NoRx] = p \cdot (U_3 - (R_p + F_p)(U_1 - U_3)) + (1 - p) \cdot (U_4 - F_p \cdot (U_4 - U_2)) \tag{3.5}$$

$$D_{11} = E[Rx, Trust] = p \cdot V_1 + (1 - p) \cdot (V_2 - G \cdot (U_4 - V_2) - R_d \cdot (V_4 - V_2)) \tag{3.6}$$

$$D_{21} = E[Rx, NoTrust] = p \cdot (V_3 - F_d \cdot (V_1 - V_3)) + (1 - p) \cdot V_4 \tag{3.7}$$

$$D_{12} = E[NoRx, Trust] = p \cdot (V_3 - G \cdot (U_1 - V_3) - R_d \cdot (V_1 - V_3)) + (1 - p) \cdot V_4 \tag{3.8}$$

$$D_{22} = E[NoRx, NoTrust] = p \cdot (V_3 - G \cdot (U_1 - V_3) - R_d \cdot (V_1 - V_3)) + (1 - p) \cdot V_4 \tag{3.9}$$

To make it clear for the reader, payoffs P_{11} and D_{11} will be explained and the same reasoning is followed for all the other payoffs.

P_{11} is the patient's payoff when the doctor decided to recommend treatment and this recommendation was trusted by the patient (thus explaining why $P_{11} = E[Trust, Rx]$). The element $p \cdot U_1$ refers to the utility the patient gets in the case of the disease being present (represented by the probability p). The second element, $(1 - p) \cdot (U_2 - R_p \cdot (U_4 - U_2))$, refers to the case when the disease is absent (given by the probability $(1 - p)$), knowing the treatment was administered. U_2 is the utility the patient gets when unnecessary treatment is administered and the patient's regret factor, R_p, is due to having trusted the doctor when she/he should not have done so. It is important to remember that regret was set to denote the difference in utilities for the player (patient in this case). For the case of disease absence, the best utility for the patient would have been not being treated (thus, U_4), and that is why it appears $R_p \cdot (U_4 - U_2)$.

Now looking at the doctors payoff, D_{11}, the first element, $p \cdot V_1$, is analogous to the patient's case: V_1 is the utility the doctor gets if the disease is present (with probability p), and the treatment was administered. The differences from the patient's payoff appear when calculating the utility for the case when the disease is absent (again with probability $(1 - p)$). V_2 is the previously mentioned utility for the doctor when unnecessary treatment is administered, but this time it is not only doctor's regret (R_d) that is added to the equation, one can see a term referring to guilt felt by the doctor, G. Once again, regret was set to denote the difference in utilities for the player (now it is the doctor), and for the case of the disease absence the doctor would have preferred to not treat the patient (V_4). Therefore the regret term appears as $R_d \cdot (V_4 - V_2)$. The doctor feels guilty for prescribing unnecessary treatment, and this is represented by the decrease in her/his utility by a fraction of the difference between the patient's best utility for the disease context (present or absent) and the actual doctor's utility, thus ending with $G \cdot (U_4 - V_2)$.

The last step of the model analysis is to study all the possible strategies. In the first place, the pure strategies are addressed and only after that the authors focus on mixed strategies.

Some pure strategies were previously mentioned in the strategic dominance assessment, but they are repeated now.

1. If $P_{11} > P_{21}$ and $P_{12} > P_{22}$ then the patient has a dominant strategy, which is to "$Trust$" the doctor's decision. Now the doctor knows the patient will trust (1^{st} row of the payoff matrix), so her/his strategy only depends on the payoffs D_{11} and D_{12}, choosing Rx if the former is larger and $NoRx$ in the other case.

   On the other hand, if $P_{11} < P_{21}$ and $P_{12} < P_{22}$ then the patient will rationally choose "$NoTrust$". The doctor will now choose Rx if $D_{21} > D_{22}$ and $NoRx$ if $D_{22} > D_{21}$.

2. Analogously for the doctor, if $D_{11} > D_{12}$ and $D_{21} > D_{22}$ then the doctor has a dominant strategy, which is to recommend treatment Rx. Knowing this, the rational patient will choose to $Trust$ the doctor if $P_{11} > P_{21}$ and $NoTrust$ the other way round.

   The doctor will recommend no treatment $NoRx$ whenever $D_{11} < D_{12}$ and $D_{21} < D_{22}$. In this case, the patient chooses $Trust$ if $P_{12} > P_{22}$ and $NoTrust$ if $P_{12} < P_{22}$.

After determining the necessary conditions for the existence of equilibrium in dominant strategies, we now look for Nash equilibria in pure strategies. Although the author did not perform this step, it was considered valuable to carry it out in this work. To do so, one must look for conditions for a player's strategy to be a best response to the other player's strategy and vice-versa:

- If $P_{21} > P_{11}$ and $D_{21} > D_{22}$ the action profile (Rx, $NoTrust$) is a Nash equilibrium, the strategies are best replies to each other;

- If $P_{12} > P_{22}$ and $D_{12} > D_{11}$ the action profile ($NoRx$, $Trust$) is a Nash equilibrium, the strategies are best replies to each other;

- If $P_{11} > P_{21}$ and $D_{11} > D_{12}$ the action profile (Rx, $Trust$) is a Nash equilibrium, the strategies are best replies to each other;

- If $P_{22} > P_{12}$ and $D_{22} > D_{21}$ the action profile ($NoRx$, $NoTrust$) is a Nash equilibrium, the strategies are best replies to each other.

Moving towards mixed strategies, the authors seek for the circumstances in which the patient will be neutral disregarding the doctor's choice and afterwards do the same for the doctor's case.

Starting with the patient, and assuming the doctor chooses Rx x per cent of the time and $NoRx$ $(1-x)$ per cent of the time, then the patient will be neutral if $E[Trust] = E[NoTrust]$. For the doctor's case, and assuming the patient chooses to $Trust$ y percent of the time and $NoTrust$ $(1-y)$ per cent of the time, the doctor will be neutral if $E[Rx] = E[NoRx]$. By the previous order, one gets:

for the patient

$$E[Trust] = x \cdot P_{11} + (1-x) \cdot P_{12} = x \cdot (P_{11} - P_{12}) + P_{12} \tag{3.10}$$

$$E[NoTrust] = x \cdot P_{21} + (1-x) \cdot P_{22} = x \cdot (P_{21} - P_{22}) + P_{22} \tag{3.11}$$

then one gets

$$x \cdot (P_{11} - P_{12}) + P_{12} = x \cdot (P_{21} - P_{22}) + P_{22} \tag{3.12}$$

which gives

$$x = \frac{P_{22} - P_{12}}{(P_{11} - P_{21}) + (P_{22} - P_{12})} = \frac{1}{1 + \frac{P_{11} - P_{21}}{P_{22} - P_{12}}} \tag{3.13}$$

whilst for the doctor

$$E[Rx] = y \cdot D_{11} + (1 - y) \cdot D_{21} \tag{3.14}$$

$$E[NoRx] = y \cdot D_{12} + (1 - y) \cdot D_{22} \tag{3.15}$$

Following the previous reasoning the authors come to

$$y = \frac{D_{22} - D_{21}}{D_{11} - D_{12} - D_{21} + D_{22}} = \frac{1}{1 + \frac{D_{11} - D_{12}}{D_{22} - D_{21}}} \tag{3.16}$$

So if the doctor plays Rx during x percent of the time and $NoRx$ the rest of the time, the patient will have the same payoff regardless the chosen strategy. The same happens for the doctor since the patient should choose $Trust$ during y per cent of time. As a result, there is a Nash equilibrium in mixed strategies for (Patient,Doctor)=(y,x). As an important note, the fractions $\frac{P_{11}-P_{21}}{P_{22}-P_{12}}$ and $\frac{D_{11}-D_{12}}{D_{22}-D_{21}}$ are both considered to be positive so that $0 < x < 1$ and $0 < y < 1$ and makes it possible to consider mixed strategies.

For interpretation purposes, in Djulbegovic et al. (2015) utilities are used to denote benefits and harms of the treatment, both for the patient who receives it and for the doctor who administrates it. On the one hand, for the patient one has $B_P = U_1 - U_3$ and $H_P = U_4 - U_2$. On the other hand, for the doctor it is set that $B_D = V_1 - V_3$ and $H_D = V_4 - V_2$. To determine the benefit of the treatment it makes sense to narrow the population to the cases of patients who suffer from the disease, leaving us with utilities U_1/V_1 (disease present and treatment administered) and U_3/V_3 (disease present but treatment not administered). This way, it is obvious that the benefit of the treatment can be computed by subtracting the utilities obtained when disease is present but the treatment is not administered, to the ones earned when the disease is also present but the necessary treatment was administered. Regarding the harms, it is only calculated for the case when the disease is absent, U_4/V_4 and U_2/V_2. Following the previous reasoning, the harm of the treatment can be computed by subtracting the utilities that administration of unnecessary treatment generates to the utilities of correctly withholding the unnecessary treatment (Pauker and Kassirer, 1975, 1980). Using these notations, the author computes P and D conditioned by the utilities, and benefits and harms both players get. The resulting expressions for x and y are shown in appendix A.

In conclusion, the authors summarize the best possible strategies to follow (now taking into consideration the benefits and harms):

1. If $P_{11} - P_{21} > 0$, or equivalently $\frac{p}{1-p} \cdot \frac{B_D}{H_D} > 1$ then the patient has a dominant strategy, which is $Trust$. The doctor should then assess whether D_{11} is larger than D_{12} and choose Rx in that case or $NoRx$ in the other case. Note that having $\frac{p}{1-p} \cdot \frac{B_D}{H_D} > 1$ is equivalent to $p > \frac{1}{1+\frac{B_D}{H_D}}$, which

shows that the most rational strategy for the patient is to $Trust$ whenever the doctor's treatment assessment predicts a higher expected net benefit (B_D) than the expected net harms (H_D).

2. If $D_{11} - D_{12} > 0$ and $D_{22} - D_{21} < 0$ the doctor has a dominant strategy: Rx. Thus, it is rational for the patient to select $Trust$ when $-[1 - \frac{p}{1-p}\frac{B_P}{H_P}]$ and $NoTrust$ otherwise.

3. If $D_{11} - D_{12} < 0$ and $D_{22} - D_{21} > 0$ contrasting with the previous strategy, the doctor now has the opposite dominant strategy: $NoRx$. It was shown that $P_{22} - P_{12} < 0$, so the patient in this case should always choose to $Trust$ the doctor. In other words, it is rational for the player to trust when the doctor recommends no treatment.

4. Lastly, when $\frac{P_{11}-P_{21}}{P_{22}-P_{12}} > 0$ and $\frac{D_{11}-D_{12}}{D_{22}-D_{21}} > 0$ the best solution are mixed strategies. The patient should choose $Trust$ y per cent of times and $NoTrust$ $(1 - y)$ percent of the times. The same reasoning is applied to the doctor, concluding that it is rational to choose Rx x per cent of times and $NoRx$ $(1 - x)$ per cent of times. This shows a Nash equilibrium: (Patient,Doctor)$=$ (y, x).

It is not impossible to avoid a Prisoner's Dilemma game in a clinical interaction. For that it is imperative to reinforce trust between patients and physicians, for example by bringing closer their interests. It is important to reverse the discredit of health care and make it more transparent. This can be achieved by encouraging doctors to give clearer explanations and help patients managing the huge amounts of information they can access nowadays (Djulbegovic et al., 2015).

3.3 Other Applications of Game Theory

Game theory is also used in several domains, such as economics, political science and psychology, logic and computer science, and even biology. Actually, it is applied to a wide range of behavioral relations is considered a valuable tool for the science of logical decision making in humans, animals, and computers.

Some application examples in different study fields are presented in this section, namely Sociology in subsection 3.3.1, Biology in subsection 3.3.2, and Economics in subsection 3.3.3.

3.3.1 Game Theory in Sociology

It is pretty common to find game theory applied to sociological and anthropological studies, for instance exploring how human beings interact and live as a society. Santos et al. (2018) address cooperation in a society, and how it can influence each person's way of acting with others. The main focus was on how reputation can regulate cooperation between individuals. For this, it was considered a binary system for reputation - it is either good or bad. The individual's reputation is dependent on the settled social norms that define what is considered to be a good or a bad action, and consequently improve or deteriorate each individual's reputation. Santos et al. (2018) use the Donation Game as a major basis for the study. This type of game assumes there is a donor and a receptor, with the former one offering some sort of benefit to the latter one at the expense of loosing something (in an altruistic act). The typical matrix for these games is presented in table 3.5.

Table 3.5: General payoff matrix for the Donation game.

Donation Game		**Player 2**	
		$Cooperate$	$Defect$
Player 1	$Cooperate$	$b-c, b-c$	$-c, b$
	$Defect$	$b, -c$	$0, 0$

If both players donate, then both will get the cost of their donation subtracted to the benefit from the others' donations. If both players defect, then nothing happens and the payoffs are null. Lastly, if one player cooperates by donating, and the other defects, then the donor only gets the negative cost - as a loss - of the donation, while the receptor gets the benefit of the donation received.

The concept of indirect reciprocity is anchored to this study, and it is considered to be "the most elaborate and cognitively demanding of all known cooperation mechanisms, and is the most specifically human, because it involves reputation and status" (Santos et al., 2018, page 1). It suggests that when an individual helps other there is no prospect of any kind of retribution from the individual who was helped, but from a third party which observes this donation.

One of the conclusions reached is quite simple and straightforward: "help good people and refuse help otherwise, and we shall be nice to you; otherwise, you will be punished". There is a critical underlying concept behind this conclusion, which is stern judging. "Under stern-judging, helping a good individual or refusing help to a bad individual leads to a good reputation, whereas refusing help to a good individual or helping a bad one leads to a bad reputation" (Pacheco et al., 2006). It is as simple as help the good ones (and refuse to help the bad ones) to become good and to maintain that reputation. Acting contrarily will make individuals lose their good labels.

Santos et al. (2018) state that this results "show that cooperation under indirect reciprocity can emerge even when the cognitive capacity of individuals is limited", making reference to toddlers behaviors. Additionally, the association of high cooperation and low complexity may indicate that even the most uncomplicated social norms can trigger cooperation in complex environments.

Other studies in sociology study how participants in a society gather in groups, and uses game theory to explain organization within these groups - many times it is hierarchical - and also how elements of the groups interact between themselves. An example of groups formation can be the gangs created in prisons (Burns et al., 2017). Kaminski (2004) dwells on the life at a polish prison invoking game theory to do so. Despite seeming to be inserted in an irrational and unpredictably violent environment, prisoners must be very keen on strategic decision-making in order to increase their chances of surviving through the imprisonment period. A clever move can shorten a sentence; a bad decision can lead to rape, beating, or social isolation. Kaminski (2004) reinforces this idea by saying that inmates act rationally and are scheming most times, contrasting with the general idea that they would act driven by pathological emotion.

Besides sociology, research in other fields, like political science, have applied game theory. Levy and Razin (2004) used a game-theoretic approach to point out that the democratic peace can be due to the public and open debates in democracies, that send clear and reliable information regarding the intentions to other states. On the contrary, nondemocratic leaders are not so clear when sending signals. Therefore, it is not so easy to decode what their intentions are, and if they will keep their promises. Thus there will be mistrust and unwillingness to make concessions if at least one of the parties in a dispute is nondemocratic.

3.3.2 Game Theory in Biology

Other fields of application of game theory are for example biology and animal behavior studies. Related to this, one must introduce Evolutionary Game Theory, which studies dynamic populations in biology. It defines a framework of contests, strategies, and analytics into which Darwinian competition can be modelled. This specific application of game theory emerged from the need of explanation of some aspects of animal behavior. Following classical game theory, each player will act selfishly. But that is not what happens when observing animal behavior. Although it goes against Darwin's thoughts of natural selection occurring at an individual level, gentlemanly actions made by animals suggest they may act for benefit of the species instead of for own profit.

This theory was first introduced in Smith and Price (1973), where game theory was applied to demonstrate that "limited war" strategy benefits not only individual animals but also the species they belong to. Limited wars is the term used to characterize intraspecific animal fights where there is no serious injury infliction to none of the participants. Usually, two male animals fight for territory, food, dominance, a female reproduction partner, amongst other advantages. Intuitively, and following the natural selection reasoning, the winner will transmit its genes to the next generation at higher frequencies than the loser, so it would not be surprising if "natural selection would develop deathly weapons or fighting styles for a 'total war' strategy of battles between males to the death". However, this is not what happens and the most accepted justification is that it would possibly lead to the species extinction.

Smith and Price (1973) focus both conflicts where serious injury is possible and also where it is impossible, with the winner individual being the one who lasted longer. Before performing the simulations, the authors introduce the concept of evolutionary stable strategy (ESS): "roughly, an ESS is a strategy such that, if most of the members of a population adopt it, there is no "mutant" strategy that would give higher reproductive fitness". A set of five strategies with different levels of hostility and precaution were defined for simplification purposes, along with the assumptions necessary to determine the payoffs associated with the hostile encounters of individuals with the distinct strategies. One of the five strategies is a "total war" strategy while the others are "limited war" strategies. The simulation results emphasized the advantage of "limited wars" strategies when compared with the "total war" strategy, named Hawk strategy. It is important to mention that this result is subject to changes when the assumptions made to determine the payoffs change. Following this simplification, the authors try to infer how the model would behave if applied to real animals, concluding that dangerous attacks are rarely used in intraspecific encounters. However, the best response to an "escalated" attack is to escalate in return. To finish, study the conflict in contests where injury is impossible. As previously mentioned, in these situations the winner is the individual who lasts longer, *i.e.*, the one who endures until the opponent retreats. The authors show that no pure strategy represents an ESS, but it is possible to find a mixed strategy which is an ESS. As a consequence, species that have genetically polymorphic populations will be favoured, meaning that individuals whose behavior changes from conflict to conflict are more likely to succeed. Therefore, there can not exist populations with a stable uniform behavior.

3.3.3 Game Theory in Economics

To illustrate how game theory can be applied to Economics, a study intended to identify the conditions under which the stakeholders in the health care market could interact in a cooperative way - benefiting the collective - was chosen. Agee and Gates (2013) compare the traditionally used in the USA fee-for-service pricing system with an alternative framework. This alternative framework suggests altered pricing and cooperation incentive strategies for doctors, hospitals, and insurers. It assumes that providers agree to manage all outpatient bills while the insurer agrees to deal with all inpatient bills, and that insurance premiums are tied to patients' healthy behaviors.

If doctors and hospitals act only for own profit, making pricing decisions autonomously approximate to the Nash equilibrium, and not taking into account the consequences these decisions might have on the other parties (insurers included), then providers will collectively increase health care prices, undermining the value of insurance, and at the same time reduce profits (Wright, 2006).

Game Theory sets two crucial aspects that promote cooperation between providers and insurers: the first one is the communication between entities so that each stakeholder knows what are the other stakeholders' interests; the second aspect relies in formalizing price contracts between interested parties so no entity is hampered by the others' choices of final price.

As a conclusion, Agee and Gates (2013) state that data gathered and treated during the study prove that the alternative pricing framework - encompassing the assumptions mentioned above - would allow providers and insurers to achieve lower administrative costs and higher profits. Additionally, patients could obtain lower health insurance costs.

Another remarkable example of game theory application is the creation of the revolutionary system for matching kidney donors with patients in need of a kidney transplantation (Roth, 2015). The author is the 2012 Nobel prize for economics joint winner, for working with operational investigation models (especially matching models). With this revolutionary system, it is easier to match biologically compatible donors and receptors. As an example, imagine someone who is very sick and needing a kidney transplantation to survive. A family member is willing to donate a kidney but they are not a biological match. Certainly, there are more diseased people in need for a kidney transplant, and certainly more closely related people willing to donate one. The kidney exchange system locates and matches the couples: donate a kidney which is compatible with the stranger, while the stranger has a closely related person willing to donate her/his biological matching kidney in exchange; the operations are performed simultaneously to make sure no one backs out. This system illustrates an economist's understanding of incentives: if you can't get someone to give an organ out of altruism, and you can't pay him either, find two parties who are desperate to align their incentives.

3.4 Summary

Game Theory is widely used to study interactions between two or more entities. It is used in areas as disparate as political science and biology, economics and philosophy, and so on and so forth. In the health care context, there have been some efforts to develop studies applying game theory to interactions between private and public stakeholders, and also modelling the physician-patient interaction. However, there is still a lot of work to be done since it looks like we are still in an embryonic stage.

A game comprises the players, their strategies, and the payoffs each player gets from a specific combination of strategies. The Prisoners' Dilemma is a simple approach to an interaction in the consultation context. However, when used to study a single interaction, it commonly results in conflict and non-cooperative payoffs, which may contrast with real life situations. Repeated Prisoners' Dilemma or other game structures, such as the Centipede game, avoid the non-cooperative results and can resemble medical interactions with patients. In general, physicians and patients cooperate in order to achieve a common goal which will benefit them both.

There is still a long way toward developing game theory applications to health care situations but efforts are being put into it and upgraded models will arise. Game theory provides a new picture of the whole medical consultation scenario, specially when considering the interaction between doctors and patients. Both agents have estimable goals and aspirations, which are interdependent. It is necessary to find game structures where cooperation constitutes an equilibrium and investigate deeper what are the dynamics that may lead to outcomes enhancing patients' health and the quality of care provided. Additionally, and due to the enormous variety of health issues, each different situation may comprise very complex features and games should be robustly built taking that diversity into consideration.

Chapter 4

Model

4.1 Model Description

This section describes the model, including the players, the strategies, the payoffs, and the game equilibria. Subsection 4.1.1 introduces the players and explains of the environment in which they interact. The strategies for each player are presented in subsection 4.1.2, while the payoffs each player gets from the different combinations of strategies are fully detailed in subsection 4.1.3.

4.1.1 Players

Every game theory model is built by firstly defining the players. It should be clear by now that the players in this game are the doctor - commonly a hepatologist - and the patient. Nevertheless, the doctor in this case can be seen as an entity representing the entire multidisciplinary team described in section 2.2, usually incorporating surgeons, the hepatologist and the psychologist that follow the patient more closely, and two nurses. Since no one is making decisions on their own, assuming the doctor to represent the multidisciplinary team is considered to be reasonable. Contrasting with the joint decision taking team, the patient decides on her/his own. It is clear that the patient's decision can always be influenced by family or friends, but this will not be considered separately here. Therefore, the patient is a single decision maker.

4.1.2 Strategies

The patient has two pure strategies: $Cooperation$, and $No\ Cooperation$. By cooperating, the patient follows the doctor's advice and complies with the requirements (most importantly staying abstinent). The patient may choose to not cooperate by drinking alcohol or stop following the doctor's recommendations in any other way (taking any forbidden risk behavior, missing appointments, among others).

The doctor has a larger set, containing three pure strategies: $Cooperation$, $No\ Cooperation\ T$ (temporary), and $No\ Cooperation\ D$ (definitive). The doctor's cooperation can be seen as a reward to the patient, keeping the patient as a candidate in the liver transplantation waiting list (or introduce the patient's name if it was not there yet). By not cooperating, the doctor excludes the patient from that list. However, this can be done temporarily, changing the treatment focus for keeping the patient abstinent.

If by any reason the doctor permanently excludes the patient from the list, the game ends and the most likely outcome for the patient is death.

4.1.3 Payoffs

Defining the payoffs is a crucial task when designing a game since they are the drivers for the actions taken. So, payoffs must be set meticulously and should be as approximate as possible to reality, which can sometimes represent a big challenge. As explained in subsection 3.1.2, payoffs are the numerical representation of the players' preferences regarding the outcomes. Usually, they are represented in utility measures. The payoffs were defined as a generic value represented by a letter - depending on the player they are associated to - and a number - depending on the actions taken that led to that payoff. It is then possible to create a payoff order.

Logically, the payoffs will be assigned with the letter P for the patient, and with the letter D when they refer to the doctor. Furthermore, the numbers assigned regarding the actions taken will be set according to the following reasoning:

– for the doctor, 1 means cooperation, 2 means temporary non-cooperation, and 3 means definitive non-cooperation

– for the patient, 1 means cooperation and 2 means non-cooperation

The resulting payoffs are represented in table 4.1.

Table 4.1: Payoffs for the Doctor (D) and for the Patient (P), dependent on the actions taken by both players.

Payoffs for Doctor (D) and Patient (P)	
D_{11}, P_{11}	Mutual cooperation (C, C)
D_{21}, P_{21}	Doctor's temporary non-cooperation and Patient's cooperation (NC_t, C)
D_{31}, P_{31}	Doctor's definitive non-cooperation and Patient's cooperation (NC_d, C)
D_{12}, P_{12}	Doctor's cooperation and Patient's non-cooperation (C, NC)
D_{22}, P_{22}	Doctor's temporary non-cooperation and Patient's non-cooperation (NC_t, NC)
D_{32}, P_{32}	Doctor's definitive non-cooperation and Patient's non-cooperation (NC_d, NC)

The next step is to sort the payoffs by order in a substantiated way. With the help of an expert judgment, a final payoff order was accomplished as well as the establishment of the positive and negative payoffs. This expert is an experienced doctor, working in a reference hospital in Lisbon in the area of liver transplantation. Starting with the doctor's payoffs, one gets $D_{11} > D_{22} > D_{32} > D_{21} > D_{31} > D_{12}$.

From these, the only payoff considered positive with certainty is D_{11}. The negative payoffs are D_{21}, D_{31}, and D_{12}. Lastly, D_{22} and D_{32} are considered to be possibly negative or positive.

The explanation is as follows:

– $D_{11} > D_{22}$: every doctor would rather have the patient cooperating, and cooperate according to that, than having to exclude (even if temporarily) a non cooperative patient;

– $D_{22} > D_{32}$: assuming one of the doctor's interests is the patient's well-being, then it is preferred to exclude a patient temporarily, targeting the treatment to the patient's abstinence, than to do it definitely – almost certainly leading to the patient's death;

– $D_{32} > D_{21}$: any doctor would prefer to exclude a patient who did not cooperate than a patient who did. This patient gets excluded temporarily by any external reason, for example the suspicion of a manifestation of an absolute contraindication. Furthermore, excluding definitely a non-cooperative patient can also be considered relieving for the doctor because it will save her/him time, that can be dedicated to cooperative patients;

– $D_{21} > D_{31}$: it represents the same situation as in the previous case, but here it is stated that the doctor would rather exclude temporarily a cooperating patient than to do it definitely, which is considered to be obvious if the objective is the patient's well-being;

– $D_{31} > D_{12}$: lastly, the worst payoff for the doctor (and for the society) comes from cooperating with a non cooperative patient. If the non cooperative patient gets away and ends up receiving the transplant, she/he might keep drinking alcohol and waste a liver graft that could have been given to a cooperative patient who had a better prognostic.

For the patient, the payoffs order is $P_{11} > P_{12} > P_{21} > P_{22} > P_{31} > P_{32}$. The payoffs P_{11}, P_{12}, and P_{21} are considered to be positive, while P_{22}, P_{31}, and P_{32} are regarded as negative payoffs.

The reasoning for this ordering is the following:

– $P_{11} > P_{12}$: the patient is better off cooperating than not cooperating, even if continuity on the list is not at stake (which is the case here). Cooperation by not drinking is beneficial for patient's health, while drinking or not cooperating may be harmful for the patient's health;

– $P_{12} > P_{21}$: the patient is always better off staying on the list than being excluded (even if only temporarily);

– $P_{21} > P_{22}$: in case of being excluded from the list, it is better for the patient to cooperate and stay alcohol abstinent. This will benefit her/his health and might reduce the time the patient is excluded from the list;

– $P_{22} > P_{31}$: for the patient long-term well-being, being excluded definitely is always worse than temporarily, since it will lead to death sooner or later;

– $P_{31} > P_{32}$: although both situations are bad, the patient will always benefit from being alcohol abstinent, instead of damaging her/his health even more by drinking.

Similarly to what is done in Djulbegovic et al. (2015), the payoffs can be expressed in terms of utilities. For that, three utilities per player are defined. The letter U concerns the patient's utilities and the letter V refers to the doctor's utilities. U_1 and V_1 denote the utility the patient and the doctor respectively earn if the patient is kept on the list, eventually ending up with the transplantation being performed. U_2 and V_2 are utilities referring to the case when the patient is temporarily removed from the list. Finally, U_3 and V_3 pertain to the case when the patient is definitely excluded from the list. Naturally, $U_1 > U_2 > U_3$ and $V_1 > V_2 > V_3$.

As in Djulbegovic et al. (2015), it was assumed that the both the patient and the doctor may feel regret (R) or frustration (F) in some final results. Additionally, the doctor may also feel guilt (G). These emotions are further explained below. The emotions obtained in temporary decisions are distinguishable from the ones arising after definitive decisions using the superscript t or d, respectively. Emotions appear as fractions of the difference between the obtained utility and the highest utility (U_1 or V_1). The last elements of the payoff expressions are the benefit (B) a patient gets for staying alcohol abstinent and the harm (H) resulting from drinking alcohol.

Lastly, there are two additional parameters, β and γ. β represents how much the doctor values saving a liver graft to a patient with a better prognostic, instead of giving it to a patient who might have a worse prognostic. Thus, this value will be added to the doctor's payoffs correspondent to the definitive non-cooperation. The value of γ represents the instant pleasure the patient gets by drinking (or not cooperating). Therefore, γ is added to the patient's payoffs representing her/his non-cooperation. Different doctors may be associated with different values of β and different patients may be assigned with different values of γ. The payoffs are depicted in table 4.2.

Since mutual cooperation results in the best payoffs, it is associated with the best utilities both for the doctor and the patient. The patient receives the additional health benefit for cooperating. If the patient does not cooperate but the doctor does, the patient gets the utility of staying in the list (U_1) but with a subtracting element that represents the harm that drinking carries. However, this negative element may be offset by the instantaneous pleasure of drinking the patient gets.

As previously mentioned, a player may feel regret or frustration. The former is associated with the cases where the player did not choose the strategy that would be a best response to the other player's strategy, for that situation. This can be seen as feeling regret for causing the other player to loose utility. As an example, in D_{21} the doctor regrets choosing $No\ Cooperation\ T$ because the patient chose $Cooperation$, and it would have been better for both players if the doctor had chosen $Cooperation$. Note that the doctor's non-cooperation does not necessarily mean the doctor made a mistake, there might have been suspicions of a contraindication or any other cause. The same reasoning applies for the explanation of the regret felt by the patient. In turn, frustration is defined to represent a player's disappointment for not being able to persuade the other player to cooperate and act as it would be best for herself/himself. To illustrate this, payoff D_{32} is analyzed. Since the doctor definitely excludes the patient from the list, the utility she/he gets is V_3. Although the doctor probably took this decision to punish the patient's non-cooperation, there will still be a feeling of frustration for not being able to persuade the patient to cooperate. This is represented by F_D^d (the d refers to a definitive decision),

Table 4.2: Representation of the payoffs in terms of utilities, for the doctor and for the patient.

	Payoffs represented in terms of utilities
Doctor	$D_{11} = E[C, C] = V_1$
	$D_{12} = E[C, NC] = V_1 - G \cdot (V_1 - V_2)$
	$D_{21} = E[NC_t, C] = V_2 - R_D^t \cdot (V_1 - V_2)$
	$D_{22} = E[NC_t, NC] = V_2 - F_D^t \cdot (V_1 - V_2)$
	$D_{31} = E[NC_d, C] = V_3 - R_D^d \cdot (V_1 - V_3) + \beta$
	$D_{32} = E[NC_d, NC] = V_3 - F_D^d \cdot (V_1 - V_3) + \beta$
Patient	$P_{11} = E[C, C] = U_1 + B$
	$P_{12} = E[C, NC] = U_1 - H + \gamma$
	$P_{21} = E[NC_t, C] = U_2 - F_P^t \cdot (U_1 - U_2) + B$
	$P_{22} = E[NC_t, NC] = U_2 - R_P^t \cdot (U_1 - U_2) - H + \gamma$
	$P_{31} = E[NC_d, C] = U_3 - F_P^d \cdot (U_1 - U_3) + B$
	$P_{32} = E[NC_d, NC] = U_3 - R_P^d \cdot (U_1 - U_3) - H + \gamma$

multiplied by the loss in utility relative to the highest one, $V_1 - V_3$. Apart from the frustration, the doctor receives an additional utility for preserving a liver graft and saving it to a patient with a better prognostic, represented by β.

To finish, it is important to mention a payoff that is an exception, specifically the worst payoff for the doctor: D_{12}. This payoff refers to the case when the doctor cooperates with the non-cooperative patient, which can end up with this patient receiving liver transplant. If a non-cooperative patient is rewarded with a liver transplantation there is a loss for society because the liver graft could have been allocated to a patient with a better prognostic, and the doctor feels guilty (G) about it because she/he contributed for that loss. G is assumed to have the highest value of all the emotions and is only attributable to the doctor. Regarding the other two emotions, and analogously for both players, regret (where $R^d > R^t$) is assumed to be greater than frustration (where $F^d > F^t$). This is considered to be a reasonable assumption since regret refers to an action taken by the player that affected negatively the other player. It is acceptable that one feels worse when the other is impaired by an action taken be her/him, than when she/he is not able to persuade the other to act according to her/his own interests. Naturally, definitive emotions are stronger than the temporary ones. Additionally, it seems reasonable to admit that $R^d - F^d > R^t - F^t$, that is, when the decision is definite the difference between regret and frustration is larger than when the decision is temporary. This requisite was validated by an expert. Concerning the benefits or harms that alcohol abstinence or consumption may bring to patient's health, with the judgment of an expert it was set that the harm of drinking alcohol is greater in absolute value than the benefit that alcohol abstinence represents. In sum, $|H| > |B|$. This presumption is based on the fact that if the patient keeps drinking alcohol she/he will likely die, while staying alcohol abstinent is unlikely to lead to absolute healing or make the transplantation unnecessary. It may happen but only in a limited number of cases, while alcohol consumption at this stage is almost always fatal. In addition, H and B are lower than the values of the utilities U and V.

4.2 Model Equilibria

This section computes the different equilibria this game might encompass. Subsection 4.2.1 solves the game in strategic form and finds the Nash equilibria in pure strategies. In subsection 4.2.2 the equilibrium in mixed strategies is computed. Lastly, subsection 4.2.3 shows the game in extensive form, and finds the equilibria for different layouts of the game.

4.2.1 Strategic Form equilibria in Pure strategies

The liver transplantation process involves repeated interactions through time. Actually, it even is sometimes an infinite process. A single interaction of the repeated game is considered here. In the normal form game both players act simultaneously or, in other words, players act without observing the other's choice. This means that the doctor chooses to maintain or exclude the patient from the list without knowing if she/he cooperated or not. Likewise, the patient decides to cooperate or not without knowing if she/he is maintained or excluded from the list. Sequential games (see ahead) allow for more information when deciding.

The simultaneous game can be represented by the matrix depicted in table 4.3.

Table 4.3: Representation of the simultaneous game in normal form.

Normal Form		**Doctor**		
		$Cooperation$	$No\ Cooperation\ T$	$No\ Cooperation\ D$
Patient	$Cooperation$	P_{11}, D_{11}	P_{21}, D_{21}	P_{31}, D_{31}
	$No\ Cooperation$	P_{12}, D_{12}	P_{22}, D_{22}	P_{32}, D_{32}

Initially, the values of γ and β are considered to be low enough so that the order of preferred strategies remains unchanged (and equal to the previously mentioned one) for both players. The resulting boundary conditions for the parameters are shown in appendix B. Later on, γ and β will be allowed to fluctuate and the changes to the results are analyzed.

Remember that the ordered payoffs are:

– doctor: $D_{11} > D_{22} > D_{32} > D_{21} > D_{31} > D_{12}$;

– patient: $P_{11} > P_{12} > P_{21} > P_{22} > P_{31} > P_{32}$.

Scanning the doctor's payoffs order one notices that $D_{11} > D_{21} > D_{31}$ and that $D_{22} > D_{31} > D_{12}$, showing that there are always preferred actions to the definitive non-cooperation for the different situations - patient's cooperation and non-cooperation. Therefore, the doctor's strategy of permanently excluding the patient from the list is dominated by others, and this strategy will never be rationally chosen.

Now going through the patient's payoffs order one has $P_{11} > P_{12}$, $P_{21} > P_{22}$, and $P_{31} > P_{32}$, so the patient always prefers to act cooperatively no matter what the doctor chooses to do. Hence, $Cooperation$ is a dominant strategy. This may be considered a reasonable result if the health benefits of staying abstinent are substantial. Given that the patient always cooperates, the doctor will also cooperate, and (C, C) is the equilibrium.

The subsequent step is to determine if there are any Nash equilibria in pure strategies. In order to do so, it is necessary to find two strategies that are best replies to each other. In other words, if one player chooses one strategy, the other player will choose the strategy that carries the best payoff taking into account the first player's choice. To check if this strategy pair represents a Nash equilibrium, one has to reverse the order and check if the strategy chosen by the first player is also a best response to the strategy chosen by the second one.

The patient's best reply to a cooperative action from the doctor is to also cooperate (because $P_{11} > P_{12}$). Reversing the situation, the doctor's best reply to a cooperative action from the patient is also to cooperate (because $D_{11} > D_{21}$). Therefore, the strategy pair (C, C) constitutes a Nash equilibrium. Checking now for a non-cooperative move from the doctor, the patient will rationally choose to act cooperatively (because $P_{21} > P_{22}$ and $P_{31} > P_{32}$). Inversely, the doctor's best reply to a cooperative move from the patient is also a cooperative move (because $D_{11} > D_{21} > D_{31}$). If the patient chooses the strategy $No\ Cooperation$, the doctor will reply with a non-cooperative decision (because $D_{22} > D_{32} > D_{12}$). Thus, there are no more Nash equilibrium strategy pairs.

As a conclusion, (C, C) is the only Nash equilibrium. Although it would be great for society if everyone always cooperated, this does not correspond to what happens in real life. The inclusion of sufficiently high β and γ may change this result. Let us define the boundary conditions for β and γ.

Starting with the patient's parameter γ, one has to check the necessary conditions to make the patient not cooperate for the different choices the doctor may do. In other words, and since the patient was considered to be always cooperative - $P_{11} > P_{12}$, $P_{21} > P_{22}$, and $P_{31} > P_{32}$ - it is essential to find the intervals of γ that change this payoff order. The resulting conditions are shown in table 4.4. It is easy to see that $\gamma_{NC_d} > \gamma_{NC_t} > \gamma_C$. The extended demonstration of these results is addressed in appendix B.

The same reasoning can be applied for the doctor's payoffs, yet with some disparities. The parameter β is added to payoffs D_{31} and D_{32}, since they are the ones referring to a definitive exclusion of the patient from the waiting list. This value represents the benefit the doctor gets from saving a liver graft for a patient with a better prognostic, which is not guaranteed to happen if the doctor excludes the patient only temporarily. Once again, it is necessary to find the value intervals of β that will change the doctor's payoff order. In response to a cooperative action from the patient, the doctor prefers to cooperate, then to exclude temporarily, and lastly to exclude definitively ($D_{11} > D_{21} > D_{31}$). If the doctor is dealing with

Table 4.4: Boundary conditions for γ

Conditions for the payoff order variation as a function of γ
$P_{12} > P_{11} \Leftrightarrow \gamma > B + H = \gamma_C$
$P_{22} > P_{21} \Leftrightarrow \gamma > B + H + (U_1 - U_2) \cdot (R_P^t - F_P^t) = \gamma_{NC_t}$
$P_{32} > P_{31} \Leftrightarrow \gamma > B + H + (U_1 - U_3) \cdot (R_P^d - F_P^d) = \gamma_{NC_d}$

a non-cooperative patient, the most preferred strategy will be to not cooperate temporarily, followed by a definitive non-cooperation, and lastly a cooperative action ($D_{22} > D_{32} > D_{12}$). All in all, one has to find the values of β that will lead to $D_{31} > D_{21}$, $D_{31} > D_{11}$, and $D_{32} > D_{22}$. Note that D_{12} is not taken into account here because it is the least preferred payoff, and since it does not have β added it will stay as the worst. Additionally, it is important to remark that even if $D_{31} > D_{21}$ the doctor will still rationally choose to cooperate, which will have implications in the results. The resulting conditions for β are shown in table 4.5.

Table 4.5: Boundary conditions for β

Conditions for the payoff order variation as a function of β
$D_{32} > D_{22} \Leftrightarrow \beta > (V_2 - V_3) + F_D^d \cdot (V_1 - V_3) - F_D^t \cdot (V_1 - V_2) = \beta_{NC}$
$D_{31} > D_{21} \Leftrightarrow \beta > (V_2 - V_3) + R_D^d \cdot (V_1 - V_3) - R_D^t \cdot (V_1 - V_2) = \beta_C$
$D_{31} > D_{11} \Leftrightarrow \beta > (V_1 - V_3) + R_D^d \cdot (V_1 - V_3) = \beta_{CC}$

The extended demonstration of these results and the proof that $\beta_{CC} > \beta_C > \beta_{NC}$ are shown in detail in appendix B.

After having set the intervals in which β and γ may vary, all the possible combinations were checked to see if there were any Nash equilibria for them. The results are presented in table 4.6.

A more detailed explanation of the findings is as follows:

– $\gamma < \gamma_C < \gamma_{NC_t} < \gamma_{NC_d}$: as previously mentioned, with this values of γ the patient would have no incentives to not cooperate. This is noticeable since the player chooses to cooperate in all Nash equilibria obtained with this values of γ. Evaluating the doctor's choices, one observes that there are also no incentives to not cooperate, except for when $\beta > \beta_{CC}$. So, (C, C) is a Nash equilibrium. This is a reasonable deduction since the doctor prefers to cooperate with a cooperative patient. The exception arises when $D_{31} > D_{11}$ due to the high value of β. This is still an acceptable result because sometimes doctors are obliged to exclude patients, not due to their non-cooperation but due to the appearance of absolute contraindication, for example the development of hepatocellular

Table 4.6: Nash Equilibria obtained from variation of parameters γ and β.

Nash Equilibria as a function of β and γ			
$\gamma < \gamma_C < \gamma_{NC_t} < \gamma_{NC_d}$	$\beta < \beta_{NC} < \beta_C < \beta_{CC}$	$P \Rightarrow C$; $D \Rightarrow C$	$D \Rightarrow C$; $P \Rightarrow C$
	$\beta_{NC} < \beta < \beta_C < \beta_{CC}$	$P \Rightarrow C$; $D \Rightarrow C$	$D \Rightarrow C$; $P \Rightarrow C$
	$\beta_{NC} < \beta_C < \beta < \beta_{CC}$	$P \Rightarrow C$; $D \Rightarrow C$	$D \Rightarrow C$; $P \Rightarrow C$
	$\beta_{NC} < \beta_C < \beta_{CC} < \beta$	$P \Rightarrow C$; $D \Rightarrow NC_d$	$D \Rightarrow NC_d$; $P \Rightarrow C$
$\gamma_C < \gamma < \gamma_{NC_t} < \gamma_{NC_d}$	$\beta < \beta_{NC} < \beta_C < \beta_{CC}$	$No\ Equilibrium$	$No\ Equilibrium$
	$\beta_{NC} < \beta < \beta_C < \beta_{CC}$	$No\ Equilibrium$	$No\ Equilibrium$
	$\beta_{NC} < \beta_C < \beta < \beta_{CC}$	$No\ Equilibrium$	$No\ Equilibrium$
	$\beta_{NC} < \beta_C < \beta_{CC} < \beta$	$P \Rightarrow C$; $D \Rightarrow NC_d$	$D \Rightarrow NC_d$; $P \Rightarrow C$
$\gamma_C < \gamma_{NC_t} < \gamma < \gamma_{NC_d}$	$\beta < \beta_{NC} < \beta_C < \beta_{CC}$	$P \Rightarrow NC$; $D \Rightarrow NC_t$	$D \Rightarrow NC_t$; $P \Rightarrow NC$
	$\beta_{NC} < \beta < \beta_C < \beta_{CC}$	$No\ Equilibrium$	$No\ Equilibrium$
	$\beta_{NC} < \beta_C < \beta < \beta_{CC}$	$No\ Equilibrium$	$No\ Equilibrium$
	$\beta_{NC} < \beta_C < \beta_{CC} < \beta$	$P \Rightarrow C$; $D \Rightarrow NC_d$	$D \Rightarrow NC_d$; $P \Rightarrow C$
$\gamma_C < \gamma_{NC_t} < \gamma_{NC_d} < \gamma$	$\beta < \beta_{NC} < \beta_C < \beta_{CC}$	$P \Rightarrow NC$; $D \Rightarrow NC_t$	$D \Rightarrow NC_t$; $P \Rightarrow NC$
	$\beta_{NC} < \beta < \beta_C < \beta_{CC}$	$P \Rightarrow NC$; $D \Rightarrow NC_d$	$D \Rightarrow NC_d$; $P \Rightarrow NC$
	$\beta_{NC} < \beta_C < \beta < \beta_{CC}$	$P \Rightarrow NC$; $D \Rightarrow NC_d$	$D \Rightarrow NC_d$; $P \Rightarrow NC$
	$\beta_{NC} < \beta_C < \beta_{CC} < \beta$	$P \Rightarrow NC$; $D \Rightarrow NC_d$	$D \Rightarrow NC_d$; $P \Rightarrow NC$

carcinoma. If this is the case, then it is expectable that the value the doctor gets by not letting the patient receive the transplant will rise steeply. The doctor will not cooperate with the patient no matter what, but the patient will cooperate even if the doctor does not cooperate because the pleasure experienced by drinking alcohol is not sufficiently high. Additionally, the patient knows that staying alcohol abstinent is beneficial for her/his health. Therefore, (NC_d, C) constitutes a Nash equilibrium.

– $\gamma_C < \gamma < \gamma_{NC_t} < \gamma_{NC_d}$: as expected, now the patient has some incentives to choose $No\ Cooperation$. If a patient experiences more pleasure by drinking, he is tempted to not cooperate if it is known that the doctor will not exclude her/him from the list. This is why the previous Nash equilibrium (C, C) does not exist anymore. The only existent equilibrium for this conditions is the same as the previous one for the case where $\beta > \beta_{CC}$, (NC_d, C).

– $\gamma_C < \gamma_{NC_t} < \gamma < \gamma_{NC_d}$: now γ is higher than γ_{NC_t}, meaning that the patient will prefer to not cooperate if it is known that the doctor will exclude her/him temporarily, whereas the doctor will choose temporary non-cooperation if the patient does not cooperate if $\beta < \beta_{NC}$ (because $D_{22} > D_{32}$). This way, and if these parameter conditions are verified, (NC_t, NC) is a Nash equilibrium. Furthermore, analogously to the previous cases, if $\beta > \beta_{CC}$, the Nash equilibrium (NC_d, C) is once again evidenced, for the same reasons as before.

– $\gamma_C < \gamma_{NC_t} < \gamma_{NC_d} < \gamma$: patients associated with such a high γ will never cooperate because alcohol provides them a high level of pleasure. For low values of β, the doctor will still prefer to exclude the patient temporarily. So, (NC_t, NC) is a Nash equilibrium, equivalently to the previous case. For doctors associated with higher β (high enough so that $D_{32} > D_{22}$), the preferred action to respond to the patient's non-cooperation is $No\ Cooperation\ D$. As the patient never cooperates, $No\ Cooperation$ will also be the best response to the doctor's definitve non-cooperation. That is why (NC_d, C) constitutes a Nash equilibrium for the last three ranges of values for β.

A simpler game would be obtained by restraining β to its lower values and vary γ to verify how the equilibria change along the different values of pleasure patients feel when drinking. This would allow to make the problem computationally more tractable. Additionally, it seems reasonable to exclude very high β other than very high γ, since it is very common to have patients struggling to keep the abstention. If this is done, then the doctor has a dominated strategy: $No\ Cooperation\ D$. Therefore, the game matrix can be reduced into a 2×2 matrix. The new matrix representing the game is presented in table 4.7.

Table 4.7: Representation of the abbreviated simultaneous game in normal form. The dominated strategy for $\beta < \beta_{NC}$ was eliminated.

Normal Form Short		**Doctor**	
		$Cooperation$	$No\ Cooperation\ T$
Patient	$Cooperation$	P_{11}, D_{11}	P_{21}, D_{21}
	$No\ Cooperation$	P_{12}, D_{12}	P_{22}, D_{22}

The results found meet the expectations, since lower values of γ imply that the patient is more prone to cooperate, and so is the doctor. For higher values of γ, it is exactly the opposite. However, there is an interval for γ where there is no Nash equilibrium: $\gamma_C < \gamma < \gamma_{NC_t} < \gamma_{NC_d}$. This happens because, for such γ, $P_{12} > P_{11}$ and $P_{21} > P_{22}$, meaning that the patient feels enough pleasure when drinking to not cooperate if the doctor does so, trying to deceive her/him. Yet, the patient cooperates if the doctor decides to temporarily exclude her/him. The patient knows that her/his health is at steak, and it is important to be reintegrated into the waiting list, thus it is better to cooperate in case of being excluded. This temporary exclusion can then be seen as a warning, to which the patient responds cooperatively (for $\gamma < \gamma_{NC_t}$), and non-cooperatively (for $\gamma_{NC_t} < \gamma$). The Nash equilibria for each γ is presented in table 4.8.

Table 4.8: Nash Equilibria (or absence of it) obtained from variation of parameter γ having $\beta < \beta_{NC}$.

Nash Equilibria (or absence of it) as a function of γ			
$\gamma < \gamma_C < \gamma_{NC_t} < \gamma_{NC_d}$	$P \Rightarrow C$; $D \Rightarrow C$	$D \Rightarrow C$; $P \Rightarrow C$	(C, C)
$\gamma_C < \gamma < \gamma_{NC_t} < \gamma_{NC_d}$	$P \Rightarrow C$; $D \Rightarrow C$	$D \Rightarrow C$; $P \Rightarrow NC$	——
$\gamma_C < \gamma_{NC_t} < \gamma < \gamma_{NC_d}$	$P \Rightarrow NC$; $D \Rightarrow NC_t$	$D \Rightarrow NC_t$; $P \Rightarrow NC$	(NC_t, NC)
$\gamma_C < \gamma_{NC_t} < \gamma_{NC_d} < \gamma$	$P \Rightarrow NC$; $D \Rightarrow NC_t$	$D \Rightarrow NC_t$; $P \Rightarrow NC$	(NC_t, NC)

4.2.2 Strategic Form equilibria in Mixed strategies

Let us now determine the conditions for the patient to be indifferent regardless what the doctor does, and the conditions for the doctor to be indifferent regardless what the patient does. Being indifferent means that one player will receive the same expected payoff under any chosen action, no matter how the other player acts.

Starting with the doctor, considering low values for β ($\beta < \beta_{NC}$) and respecting the payoff order obtained with the judgment of our expert, we are left with a 2×2 matrix as in table 4.7. The expected payoff for the doctor when choosing $No\ Cooperation\ D$ is inferior to the expected payoffs for the other two strategies under the probability p that leads to $E_D[C] = E_D[NC_t]$. Thus the strategy $No\ Cooperation$ D is dominated. This strategy will never be chosen and can be eliminated. The complete validation is computed in appendix C.

Assuming the patient cooperates p percent of the time and acts non-cooperatively $(1-p)$ per cent of the time, one has to check the conditions for the doctor's indifference between $Cooperation$ and No $Cooperation\ T$. This requirement is expressed by equation 4.1, where E_D refers to the expected value for the doctor for a chosen strategy.

$$E_D[Cooperation] = E_D[NoCooperationT] \tag{4.1}$$

Explicitly writing this equation it is possible to obtain the value of p that leads to the doctor's indifference.

$$\begin{gathered} p \cdot D_{11} + (1-p) \cdot D_{12} = p \cdot D_{21} + (1-p) \cdot D_{22} \\ pV_1 + (1-p)[V_1 - G \cdot (V_1 - V_2)] = p[V_2 - R_D^t \cdot (V_1 - V_2)] + (1-p)[V_2 - F_D^t \cdot (V_1 - V_2)] \\ p = \frac{G - F_D^t - 1}{G + R_D^t - F_D^t} \end{gathered} \tag{4.2}$$

In conclusion, the doctor will be indifferent between playing $Cooperation$ or $No\ Cooperation\ T$ if the patient plays $Cooperation$ in p percent of the time, and $No\ Cooperation$ the rest of the time (*i.e.*, $1-p$ percent of the time), with $p = \frac{G-F_D^t-1}{G+R_D^t-F_D^t}$.

Doing the same for the patient, if the doctor chooses $Cooperation\ q$ per cent of the time and No

$Cooperation\ T$ $(1-q)$ per cent of the time, the patient's indifference is obtained when the expected values for the patient (E_P) of both strategies are equal, *i.e.*, if equation 4.3 is verified.

$$E_P[Cooperation] = E_P[NoCooperation] \tag{4.3}$$

The value of q is then obtained by solving this equation.

$$q \cdot P_{11} + (1-q) \cdot P_{21} = q \cdot P_{12} + (1-q) \cdot P_{22}$$
$$q \cdot (U_1 + B) + (1-q) \cdot [U_2 - F_P^t \cdot (U_1 - U_2) + B] =$$
$$= q \cdot (U_1 - H + \gamma) + (1-q) \cdot [U_2 - R_P^t \cdot (U_1 - U_2) - H + \gamma]$$
$$q = 1 - \frac{\gamma - (B+H)}{(R_P^t - F_P^t) \cdot (U_1 - U_2)} \tag{4.4}$$

Obviously, the probabilities p and q must respect $0 < p < 1$ and $0 < q < 1$. The extended proof of both cases is presented in appendix C, where some conditions derived from the payoffs' order will have influence on these two values. Nevertheless, a brief display of proof and remarks on the resulting conclusions is made now.

For $p = \frac{G - F_D^t - 1}{G + R_D^t - F_D^t}$ to be positive, both the numerator and denominator signs must match. From $D_{31} > D_{12}$ one gets that $G > \frac{(V_1 - V_3) \cdot (1 + R_D^d) - \beta}{V_1 - V_2}$, and when the value of β is replaced by its upper bound (for the case when β is such that the payoffs order given by the expert is respected), then one obtains $G > 1 + F_D^t$. This means that the numerator is positive, so it is mandatory that the denominator has the same sign. Since $G > R_D^t > F_D^t$, it is a trivial task to conclude that $G + R_D^t - F_D^t > 0$. Thereby, it is now proved that $p > 0$. Additionally, it is now also pretty obvious that $G - F_D^t - 1 < G + R_D^t - F_D^t$, which allows one to conclude that $p < 1$. Therefore, it has been proven that $0 < p < 1$.

Equivalently, it is necessary that $q = 1 - \frac{\gamma - (B+H)}{(R_P^t - F_P^t) \cdot (U_1 - U_2)}$ is positive and less than 1 for the existence of an equilibrium in mixed strategies. For $q > 0$, one must check that $B + H - \gamma > -(R_P^t - F_P^t) \cdot (U_1 - U_2)$. This implication is easily confirmed with the fact that $B + H > \gamma$ arising from $P_{11} > P_{12}$. Consequently, $B + H - \gamma$ is a positive value, and naturally higher than the negative value of $-(R_P^t - F_P^t) \cdot (U_1 - U_2)$. The problem appears when checking the conditions for $q < 1$. Solving this inequality for γ leads to $\gamma > B + H$, which is exactly the opposing condition for the previous requirement. Therefore, for values of γ such that $\gamma < B + H$ there is no equilibrium in mixed strategies. However, γ is not fixed and it varies from patient to patient. If γ is considered to be higher than $B + H$, then $q < 1$ is verified and one needs to check for $q > 0$. Solving this inequality for γ one obtains $\gamma < B + H + (R_P^t - F_P^t) \cdot (U_1 - U_2)$, which is higher than just $B + H$ (actually it corresponds to γ_{NC_t}). In conclusion, the necessary condition for $0 < q < 1$ to be verified, and consequently for the existence of equilibrium in mixed strategies, is that $\gamma_C < \gamma < \gamma_{NC_t} < \gamma_{NC_d}$. That is, for very low values of γ the patient always prefers to cooperate, so does not randomize between cooperating and not cooperating. For high values, she/he always does not cooperate. Only for intermediate γ does the patient randomize between the two strategies.

It is interesting to see which of the two players is more cooperative under the mixed strategies equilibrium. This is done by comparing the values of p and q. The extensive computation of this comparison is done in appendix C, and here only the results are presented. It was verified that the relation between these two probabilities is dependent on the value of γ, which, as already seen, is bounded by $\gamma_C = B + H < \gamma < B + H + (R_P^t - F_P^t) \cdot (U_1 - U_2) = \gamma_{NC_t}$. The results are:

– If $\gamma_C < \gamma < \frac{(R_D^t+1)\cdot(R_P^t-F_P^t)\cdot(U_1-U_2)}{G-F_D^t+R_D^t} + B + H$, then $q > p$ and the doctor will cooperate with more probability than the patient;

– If $\frac{(R_D^t+1)\cdot(R_P^t-F_P^t)\cdot(U_1-U_2)}{G-F_D^t+R_D^t} + B + H < \gamma < \gamma_{NC_t}$, then $p > q$ and the patient will cooperate with more probability than the doctor.

In short, for lower values of γ the doctor cooperates with higher probability, and for higher values of γ the patient cooperates with higher probability. Though this result may seem counterintuitive as a patient associated with higher γ is more prone to not cooperate, one is dealing with an equilibrium in mixed strategies, so the players must be indifferent between choosing one of their strategies. For higher values of γ, the patient has to cooperate with higher probabilities to maintain the doctor's indifference of choosing between $Cooperation$ and $No\, Cooperation\, T$. Otherwise, the doctor will not be indifferent anymore and will tend to adopt the non-cooperative strategy with higher probability (because q decreases with the increase of γ). It is also possible to assess how the other parameters contribute to determine which player cooperates with higher probabilities. This is done in the comparative statics analysis below.

Let us now analyze how the probabilities q and p are influenced by the other variables - a comparative statics analysis. The resulting partial derivatives for both q and p are presented in table 4.9.

Table 4.9: Comparative statics for q and p.

Partial Derivatives of q and p	
$q = 1 - \frac{\gamma-(B+H)}{(R_P^t-F_P^t)\cdot(U_1-U_2)}$	$\frac{\partial q}{\partial B} = \frac{1}{(R_P^t-F_P^t)\cdot(U_1-U_2)} > 0$
	$\frac{\partial q}{\partial H} = \frac{1}{(R_P^t-F_P^t)\cdot(U_1-U_2)} > 0$
	$\frac{\partial q}{\partial \gamma} = -\frac{1}{(R_P^t-F_P^t)\cdot(U_1-U_2)} < 0$
	$\frac{\partial q}{\partial (R_P^t-F_P^t)} = \frac{\gamma-(B+H)}{(R_P^t-F_P^t)^2\cdot(U1-U2)} > 0$
	$\frac{\partial q}{\partial (U_1-U_2)} = \frac{\gamma-(B+H)}{(R_P^t-F_P^t)\cdot(U1-U2)^2} > 0$
$p = \frac{G-F_D^t-1}{G-F_D^t+R_D^t}$	$\frac{\partial p}{\partial G} = \frac{F_D^t-G+1}{(G-F_D^t+R_D^t)^2} + \frac{1}{G-F_D^t+R_D^t} > 0$
	$\frac{\partial p}{\partial R_D^t} = \frac{F_D^t-G+1}{(G-F_D^t+R_D^t)^2} < 0$
	$\frac{\partial p}{\partial F_D^t} = \frac{G-F_D^t-1}{(G-F_D^t+R_D^t)^2} - \frac{1}{G-F_D^t+R_D^t} < 0$

Starting this analysis for the doctor, one has to figure out how q will fluctuate when a variable changes. Additionally, checking how some partial derivatives depend on the other variables may possibly lead to interesting results. The conclusions reached are the following:

- $\frac{\partial q}{\partial B} > 0$: it may seem counterintuitive but, as $\gamma_C < \gamma < \gamma_{NC_t}$, it results that $P_{12} > P_{11}$, and the patient tends to not cooperate when the doctor does so. Since this analysis refers to the equilibrium in mixed strategies, both players must be indifferent between their strategies. So, if B increases the patient will have more incentives to cooperate because the benefit for her/his health will be larger. This will lead to a patient's preference for cooperating, unbalancing the indifference. To counteract this unbalance, the doctor has to cooperate with higher probability (higher q), so the patient feels safer when choosing $No\ Cooperation$, and keeping the patient's indifference. It is noticeable that the higher the difference between R_P^t and F_P^t or between U_1 and U_2, the weaker is the impact of the benefit for patient's health of staying alcohol abstinent on the doctor's probability of cooperating. The patient will attach higher importance to emotions or to the utility of staying in the list, and health benefit will not be seen as such a powerful incentive to cooperation.

- $\frac{\partial q}{\partial H} > 0$: in this case it is possible to address the same reasoning as the one used for the previous situation. H represents the harm for the patient's health that not cooperating carries. If H is higher, the patient will again have more incentives to cooperate, and the doctor cooperates with higher probability, increasing the patient's temptation to not cooperate. This way, the patient's indifference is maintained. Regarding the dependence of this partial derivative on other variables, it follows again the exact same reasoning. An increase in $(R_P^t - F_P^t)$ or in $(U_1 - U_2)$ will reduce the significance that the patient gives to the harm associated with drinking alcohol, thus having less effect on her/his choices, and consequently on the doctor's.

- $\frac{\partial q}{\partial \gamma} < 0$: patients associated with higher values of γ feel more pleasure when not cooperating. Thus, an increase in γ works as an incentive for the patient to not cooperate. It is important to mention again that γ ranges between γ_C and γ_{NC_t} (otherwise there is no equilibrium in mixed strategies). For this range, the patient has $P_{12} > P_{11}$ but $P_{21} > P_{22}$, *i.e.*, the patient prefers non-cooperation if the doctor cooperates but will act cooperatively if the doctor chooses to not cooperate. Employing the same rationale as before, the doctor will have to counteract the new patient's unbalance. If the patient has now incentives to choose $No\ Cooperation$, the doctor will have to choose $No\ Cooperation\ T$ with higher probability, encouraging the patient to cooperate. This result is in perfect accordance with reality, since a doctor will not want to cooperate with patients that are more prone to not cooperate. Similarly to the previous cases, this partial derivative depends on $(R_P^t - F_P^t)$ and on $(U_1 - U_2)$. The lower values these differences assume, the more the patient will be moved by her/his pleasure on drinking, thus deterring the doctor from cooperating if this pleasure is high.

- $\frac{\partial q}{\partial (R_P^t - F_P^t)} > 0$: R_P^t is assigned to the patient's payoffs associated with non-cooperative strategies, with a negative value. In turn, F_P^t is assigned to the patient's payoffs associated with cooperative behaviours along with the doctor's non-cooperation, also with a negative sign. The increase in this difference - whether for increase in R_P^t, decrease in F_P^t, or uneven variations on both values - will give the patient incentives to cooperate, due to the increasing regret of not cooperating or the decreasing frustration of cooperating with a non-cooperative doctor. Analogously to the previous cases, the doctor will cooperate with higher probability (q increases) to reverse the incentives for cooperation the patient gets. Since $P_{12} > P_{11}$, the patient is tempted to not cooperate if the doctor does so. This effect is more important for patients with higher propensity to drink (increases with γ).

– $\frac{\partial q}{\partial (U_1 - U_2)} > 0$: once again, the increase in this difference promotes the patient's cooperation, since U_1 will be higher in relation to U_2. The patient will be valuing more staying in the list as this difference increases. The doctor will foster the patient's non-cooperation by cooperating with higher probability. Once again, this effect is more important for patients with higher propensity to drink (increases with γ).

The same comparative statics may be performed for the probability of the patient's cooperation, p. The conclusions reached are the following:

– $\frac{\partial p}{\partial G} > 0$: G refers to the guilt a doctor feels when she/he cooperates with a patient that deceives her/him. An increase in this emotion of guilt will act as an incentive for the doctor's non-cooperation, since it is an emotion that the doctor wants to avoid. To offset this pattern, the patient will cooperate with higher probability p. If the patient feels the doctor will easily exclude her/him from the list, this is an incentive to cooperation.

– $\frac{\partial p}{\partial R^t_D} < 0$: the doctor feels R^t_D when choosing to exclude a cooperating patient. Higher levels of regret will promote the doctor's cooperation, hence unbalancing the equilibrium in mixed strategies. A decrease in p means that the patient will cooperate with lower probability. This patient's non-cooperative trend will boost the doctor to not cooperate, maintaining her/his indifference between the strategies.

– $\frac{\partial p}{\partial F^t_D} < 0$: analogously to the previous case, an increase in the doctor's frustration leads to a higher propensity to the doctor's cooperation. To compensate for that and maintain the doctor's indifference, the patient will tend to not cooperate and force the doctor to follow her/his footsteps and not cooperate as well. That is why p decreases with an increase of F^t_D.

After analyzing the properties of the probabilities of each player acting cooperatively, it is also interesting to assess the correspondent expected values for the players' payoffs, as well as how these will be influenced by the variables. Since the values of p and q are the ones creating the equilibrium in mixed strategies, *i.e.*, making the players indifferent between their two strategies (leading to equations 4.1 and 4.3), it is only necessary to analyze one of the expected values for the payoffs of one of the possible strategies. The validation of equations 4.1 and 4.3 is computed in appendix C. The expected payoffs for the equilibrium p and q, are given in equations 4.5 and 4.6.

$$E_D[C]_p = E_D[NC_t]_p = E_D = V1 - \frac{G \cdot (V_1 - V_2) \cdot (1 + R^t_D)}{G - F^t_D + R^t_D} \tag{4.5}$$

$$E_P[C]_q = E_P[NC]_q = E_P = U_1 + B - \frac{\gamma - (B + H)}{R^t_P - F^t_P} \cdot (1 + R^t_P) \tag{4.6}$$

The comparative statics analysis is presented in table 4.10

Table 4.10: Comparative statics for E_D and E_P.

Partial Derivatives of E_D and E_P	
$E_D = V_1 - \frac{G\cdot(V_1-V_2)}{G+R_D^t-F_D^t}\cdot(1+R_D^t)$	$\frac{\partial E_D}{\partial V_1} = 1 - \frac{G\cdot(1+R_D^t)}{G+R_D^t-F_D^t} < 0$ $\frac{\partial E_D}{\partial V_2} = \frac{G\cdot(1+R_D^t)}{G+R_D^t-F_D^t} > 0$ $\frac{\partial E_D}{\partial G} = -\frac{(1+R_D^t)\cdot(R_D^t-F_D^t)\cdot(V_1-V_2)}{(G+R_D^t-F_D^t)^2} < 0$ $\frac{\partial E_D}{\partial F_D^t} = -\frac{G\cdot(1+R_D^t)\cdot(V_1-V_2)}{(G+R_D^t-F_D^t)^2} < 0$ $\frac{\partial E_D}{\partial R_D^t} = -\frac{G\cdot(G-F_D^t-1)\cdot(V_1-V_2)}{(G+R_D^t-F_D^t)^2} < 0$
$E_P = U_1 + B - \frac{\gamma-(B+H)}{R_P^t-F_P^t}\cdot(1+F_P^t)$	$\frac{\partial E_P}{\partial U_1} = 1 > 0$ $\frac{\partial E_P}{\partial B} = \frac{F_P^t+1}{R_P^t-F_P^t} + 1 > 0$ $\frac{\partial E_P}{\partial H} = \frac{F_P^t+1}{R_P^t-F_P^t} > 0$ $\frac{\partial E_P}{\partial \gamma} = -\frac{F_P^t+1}{R_P^t-F_P^t} < 0$ $\frac{\partial E_P}{\partial F_P^t} = \frac{(B+H-\gamma)\cdot(R_P^t+1)}{(R_P^t-F_P^t)^2} < 0$ $\frac{\partial E_P}{\partial R_P^t} = \frac{(\gamma-(B+H))\cdot(F_P^t+1)}{(R_P^t-F_P^t)^2} > 0$

Beginning with the doctor's expected value, one has to figure out whether the partial derivatives are positive or negative to assess how E_D fluctuates when a variable changes. Additionally, checking how some partial derivatives depend on the other variables may lead to interesting inferences. The findings are presented now:

- $\frac{\partial E_D}{\partial V_1} < 0$: it may seem unreasonable that the doctor's expected value decreases with the increase of her/his best possible utility, V_1. However, and remembering again that this analysis is valid only for $\gamma_C < \gamma < \gamma_{NC_t}$, the patient's preferred response to the doctor's cooperation is a non-cooperative action ($P_{12} > P_{11}$). In this case, the doctor's payoff (D_{12}) is the worst possible and that is why the expected value for the doctor decreases with an increase in V_1.
- $\frac{\partial E_D}{\partial V_2} > 0$: V_2 contributes positively for the payoffs D_{21} and D_{22}, thus an increase in V_2 is expected to increase the doctor's expected payoff. Although V_2 contributes negatively in the previous payoffs, and additionally in D_{12}, this is not enough to offset the positive contribution because the negative values are multiplied by a fraction of an emotion (F_D^t or R_D^t), and are hence smaller than V_2.
- $\frac{\partial E_D}{\partial G} < 0$: an increase in the value of the doctor's guilt decreases the expected value of the doctor's payoff. This is a straightforward conclusion if one invokes the argument used in the first

example. The preferred patient's action to respond to a cooperative move from the doctor, is to be uncooperative. This leads the doctor to feel guilty, and as this guilt grows the expected value for the payoff declines.

- $\frac{\partial E_D}{\partial F_D^t} < 0$: the reasoning used previously can be employed here again. Frustration is felt by doctors when they are unable to persuade the patient to cooperate. Therefore, F_D^t contributes negatively for the payoff a doctor might get. If F_D^t increases, then the expected payoff naturally decreases.

- $\frac{\partial E_D}{\partial R_D^t} < 0$: it is once again possible to extend the rationale applied in the previous case to this one. Regret is felt by doctors when they do not cooperate with a cooperative patient, and consequently is added to the payoff with a negative sign. If R_D^t increases, it is then obvious that the expected payoff decreases.

A similar analysis is performed for the patient's expected value. The conclusions are the following:

- $\frac{\partial E_P}{\partial U_1} > 0$: the expected payoff for the patient varies in direct proportion with U_1. This is a natural result since U_1 represents the utility of staying in the list. The higher U_1 is, the better the expected payoff for the patient. Although U_1 also contributes negatively to the patient's payoff when she/he is excluded, it is reduced by U_2 and multiplied by a fraction of regret. Therefore, U_1 contribution is more significant in P_{11} and P_{12}, and, since it is positive, an increase in U_1 leads to an increase in the expected value of the patient's payoff.

- $\frac{\partial E_P}{\partial B} > 0$: the higher the benefit carried by alcohol abstinence, the more the patient cooperates. This leads to a substantial increase in the patient's expected payoff, enhanced as well by the positive contribution of B. This increase can even be reinforced if the emotion R_P^t decreases (since it is associated with the non-cooperative strategies), or if F_P^t increases (since it is associated with the cooperative strategies).

- $\frac{\partial E_P}{\partial H} > 0$: once again, the higher the harm provoked by alcohol consumption the more incentives for cooperation the patient has. And if the patient is more cooperative, the payoff rises. Additionally, this impact is more significant for higher values of F_D^t and for lower values of R_D^t, just like before.

- $\frac{\partial E_P}{\partial \gamma} < 0$: although γ contributes positively in two payoffs (P_{12} and P_{22}), it works as an incentive for non-cooperation. This leads to a decrease in the expected payoff for the patient. Now, the impact of γ is influenced by the emotions R_P^t and F_P^t in the reverse order as previously. A decrease in R_P^t strengthens the influence that γ has in the expected payoff, while an increase in F_P^t weakens it.

- $\frac{\partial E_P}{\partial F_P^t} < 0$: an increase in F_P^t leads to a decrease in the value of the payoffs correspondent to cooperative strategies chosen by the patient. If such value decreases, the patient is less willing to cooperate. Consequently, the patient's expected payoff decreases.

- $\frac{\partial E_P}{\partial R_D^t} > 0$: in contrast to the previous situation, an increase in this emotion will lead to an increase in the patient's expected payoff. This is due to the negative contribution of R_D^t for the payoffs arising from the patient's non-cooperative strategies. This will work has an incentive to the patient's cooperation, and eventually leads to an increase in the patient's expected payoff.

4.2.3 Extensive Form equilibria

Using a game in its extensive form is a more realistic approach to this problem. In such game, decisions are not taken simultaneously and this is closer to what happens in a medical consultation. It is reasonable to assume that the doctor is the first player to take a decision, since the doctor has the first choice of assigning the patients to the liver transplantation waiting list, depending on their diagnostic results. The patient then may decide to cooperate with the doctor or not. However, it is also interesting to investigate how the game changes if the patient is the first to take a decision. This can be seen as the patient going to the first consultation with the game already being played. If the patient has been consuming alcohol during the time period before the consultation, then she/he has not cooperated. Otherwise, it can be considered that the patient has cooperated.

The first step to analyze an extensive-form game is to perform backward induction, as described earlier in subsection 3.1.2. To do so, it is necessary to assess the second player's actions as a response to each possible choice taken by the first player. Then, the first-mover will choose the strategy that will benefit her/him the most.

To begin with the simpler case, both the previously mentioned games will be considered to be played for low enough β such that the doctor's strategy $No\ Cooperation\ D$ is dominated, and thus being excluded from the game.

As mentioned in subsection 3.1.1, an extensive form game can be represented by a tree. Let us start by analyzing the game where the doctor chooses first, and then the same analysis is performed for the game in which the patient decides before the doctor. The results are then compared. The tree referring to the first play of the game starting with the doctor's decision is illustrated in figure 4.1.

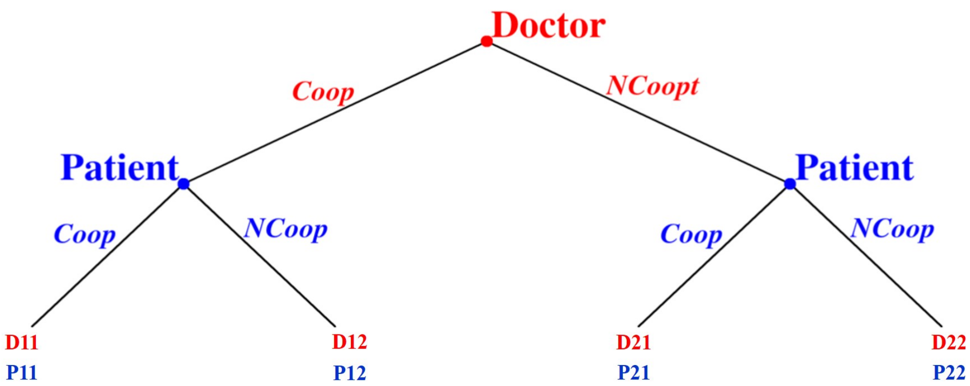

Figure 4.1: Tree representation of the extensive form game, illustrating the interaction between the doctor and the patient in a consultation. Each player has two strategies, and the doctor decides first.

For $\gamma < \gamma_C$, the patient will have no incentive to not cooperate, that is, $P_{11} > P_{12}$ and $P_{21} > P_{22}$. So, the patient always cooperates. Given that $D_{11} > D_{21}$, the doctor replies with cooperation. Therefore, for $\gamma < \gamma_C$ the equilibrium is the strategy pair (C, C).

When γ exceeds γ_C but not γ_{NC_t}, the patient has incentives to try to deceive the doctor if she/he cooperates, which means $P_{12} > P_{11}$. When the doctor does not cooperate, the patient still prefers to cooperate (because $P_{21} > P_{22}$). Hence, the doctor chooses not to cooperate, since $D_{21} > D_{12}$.

For values of γ greater than γ_{NC_t}, the patient never cooperates ($P_{12} > P_{11}$ and $P_{22} > P_{21}$). Here the doctor chooses again to not cooperate with the patient ($D_{22} > D_{12}$). The results are summed up in table 4.11, where the backward induction equilibrium is presented for each interval of γ.

Table 4.11: Backward induction equilibrium for each interval of γ, when the doctor decides first, and resulting payoffs.

Backward Induction equilibria as a function of γ; D chooses first		
γ	Equilibrium Strategies (D, P)	Equilibrium Payoffs
$\gamma < \gamma_C < \gamma_{NC_t} < \gamma_{NC_d}$	(C, C)	(D_{11}, P_{11})
$\gamma_C < \gamma < \gamma_{NC_t} < \gamma_{NC_d}$	(NC_t, C)	(D_{21}, P_{21})
$\gamma_C < \gamma_{NC_t} < \gamma < \gamma_{NC_d}$	(NC_t, NC)	(D_{22}, P_{22})
$\gamma_C < \gamma_{NC_t} < \gamma_{NC_d} < \gamma$	(NC_t, NC)	(D_{22}, P_{22})

Let us now consider the tree referring to the first play of the game starting with the patient's decision (figure 4.2).

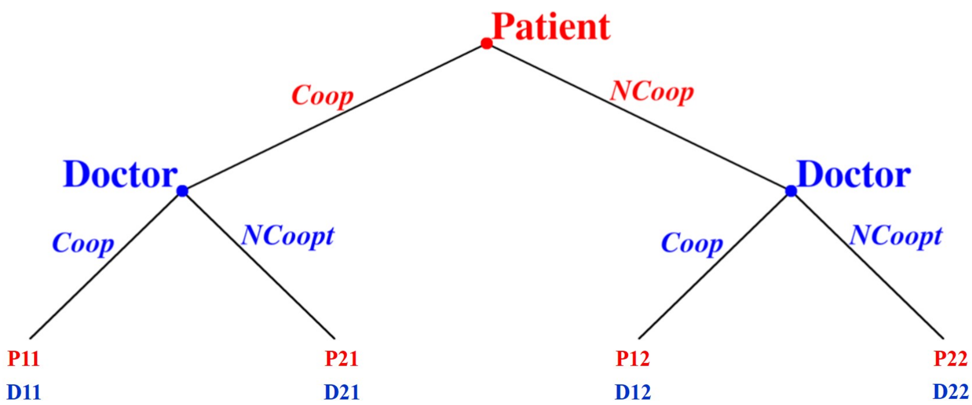

Figure 4.2: Tree representation of the extensive form game, illustrating the interaction between the doctor and the patient in a consultation. Each player has two strategies, and the patient decides first.

For $\gamma < \gamma_C$, the patient has no incentives to deceive the doctor, thus choosing always $Cooperation$. The doctor also prefers to cooperate, since $D_{11} > D_{21}$. As a result, the equilibrium is the strategy pair (C, C).

Increasing γ the patient starts feeling tempted to choose $No\ Cooperation$. For $\gamma_C < \gamma < \gamma_{NC_t}$, the patient deceives the doctor if she/he cooperates, but acts cooperatively if the doctor decides to not cooperate. The doctor cooperates as a reply to a cooperative move ($D_{11} > D_{21}$) but does not cooperate if the patient chooses $No\ Cooperation$ ($D_{22} > D_{12}$). Since $P_{11} > P_{22}$, the patient chooses to cooperate. Once again, the strategy pair (C, C) is the equilibrium.

It is possible to extend the previous reasoning to the remaining intervals of γ ($\gamma_{NC_t} < \gamma < \gamma_{NC_d}$ and $\gamma_{NC_d} < \gamma$). The patient always has to choose between cooperating and getting the payoff P_{11} or not cooperating and getting the payoff P_{22}. As the former payoff is always higher than the latter, the patient prefers to cooperate no matter what.

Summing up, the only subgame perfect Nash equilibrium for this game is the strategy pair (C, C), regardless of the value of γ. The results are presented in table 4.12.

Table 4.12: Backward induction equilibrium for each interval of γ, when the patient decides first, and resulting payoffs.

Backward Induction equilibria as a function of γ; P chooses first		
γ	Equilibrium Strategies (P, D)	Equilibrium Payoffs
$\gamma < \gamma_C < \gamma_{NC_t} < \gamma_{NC_d}$	(C, C)	(P_{11}, D_{11})
$\gamma_C < \gamma < \gamma_{NC_t} < \gamma_{NC_d}$	(C, C)	(P_{11}, D_{11})
$\gamma_C < \gamma_{NC_t} < \gamma < \gamma_{NC_d}$	(C, C)	(P_{11}, D_{11})
$\gamma_C < \gamma_{NC_t} < \gamma_{NC_d} < \gamma$	(C, C)	(P_{11}, D_{11})

Changing the order of play may completely change the results, as observed in this analysis. Remarkably, the first observable result is that if the patient decides first the game ends up being "much more cooperative", in a sense that the strategy pair (C, C) is the equilibrium more often. This can be explained by the fact that if the patient decides first, she/he gets the chance to anticipate the doctor's choices as a reply to her/his own. The patient finds out that the doctor will act non-cooperatively as a response to a non-cooperative move, and that the doctor will cooperate if the patient acts likewise. In sum, the doctor will follow the patient's decision. This allows the patient to choose the strategy that will provide him the best payoff, and in this case it is $Cooperation$, resulting in P_{11} - which is higher than P_{22} that would have been obtained if the strategy pair was (NC, NC_t).

The game with the doctor being the first to decide helps her/him to choose what is best for her/him taking into account the patient's type (defined by the value of γ). For patients associated with low values of γ, the doctor tends to be more cooperative. But as this value rises the doctor will be cautions and tend to not cooperate. Note that for intermediate values of γ ($\gamma_C < \gamma < \gamma_{NC_t}$) the best strategy for the doctor to lead the patient to cooperate is to choose $No\,Cooperation\;T$. If the doctor cooperates, the patient will prefer to not cooperate (because $P_{12} > P_{11}$). Thus, the doctor has the first-mover advantage and can induce the player to cooperate, except if $\gamma > \gamma_{NC_t}$. For high values of γ, the doctor will always choose $No\,Cooperation\;T$. To sum up, the doctor may induce the patient to cooperate by cooperating if γ is low, and by not cooperating temporarily if γ is intermediate. Otherwise, the patient will never cooperate and the doctor will act in the same way. Being able to predict the patient's decision allows the doctor to act in order to avoid being deceased and, if possible, induce the patient to cooperate.

Let us now allow β to take higher values so that the doctor has more incentives to not cooperate and the strategy $No\,Cooperation\;D$ may no longer be dominated.

Both the games with the doctor and the patient choosing first will be analyzed again, but this time the doctor is allowed to exclude the patient definitely from the list. It is important to stress that the game may not even take place when the doctor decides first. Since one is dealing with a game with one play, namely representing the first consultation after the doctor had access to the patient diagnostic tests (process explained in section 2.2), if the doctor decides to not even include the patient in the waiting list for liver transplantation, then there will be no game. In this case the doctor gets $D_{30} = \beta$ as payoff and the patient gets $P_{30} = 0$. This situation may represent cases where patients' health status is already too serious for considering the possibility of being transplanted. Therefore, the doctor will save a liver graft to a patient with better prognostics, thus being rewarded with β. The game in which the doctor decides first is shown in figure 4.3.

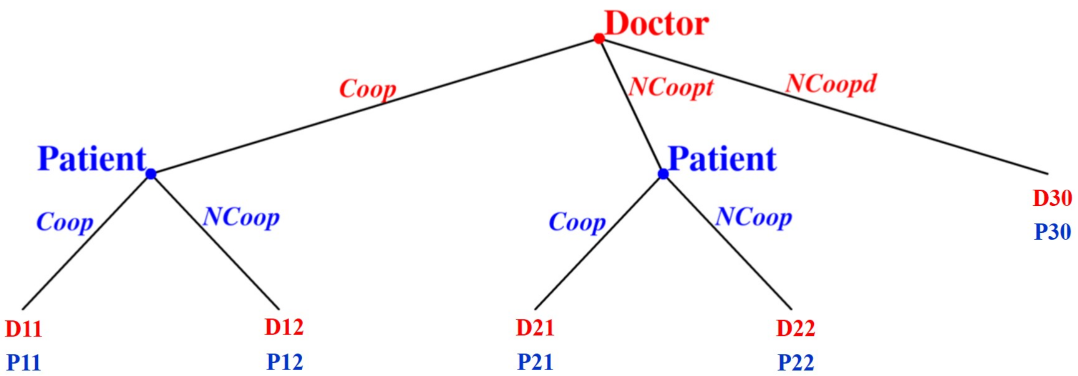

Figure 4.3: Tree representation of the extensive form game, illustrating the interaction between the doctor and the patient in a consultation, with the doctor acting first.

Once again, it is necessary to apply backward induction through all the possible combinations of γ and β. To comprehend the reasoning applied to this analysis, it is important to remember the boundary conditions for these parameters, presented in tables 4.4 and 4.5.

If γ and β take low values ($\gamma < \gamma_C$ and $\beta < \beta_C$ - note that β_{NC} is not even taken into consideration because the results only change when $\beta > \beta_C$), then neither player will have an incentive to not cooperate and (C, C) is the only equilibrium. Players receive D_{11} and P_{11}, which are the best payoffs for them.

Maintaining $\gamma < \gamma_C$ keeps the patient without incentive to not cooperate, so she/he cooperates no matter what the doctor does. Therefore, the doctor decides by comparing the payoffs D_{11}, D_{21}, and D_{30}. As $D_{11} > D_{21}$ for any β, the doctor does not choose the strategy $No\ Cooperation\ T$ when dealing with such low values of γ. Increasing β, (C, C) may no longer be the equilibrium. For the interval $\beta_C < \beta < \beta_{CC}$, the equilibrium solution will depend on the relation between V_1 and β. If $V_1 > \beta$, the doctor's payoff for mutual cooperation will be higher than the payoff for $No\ Cooperation\ D$ ($D_{11} > D_{30}$), and the strategy pair (C, C) is again a subgame perfect Nash equilibrium. Conversely, if $V_1 < \beta$ ($D_{11} < D_{30}$) the doctor will choose to exclude definitely the possibility of inserting the patient in the list and there is no game. Taking β to its highest values ($\beta > \beta_{CC}$) will make the doctor choose immediately $No\ Cooperation\ D$, and again the games ends before it has even started. For the last two cases, the players get D_{30} and P_{30} as payoffs.

Increasing γ to $\gamma_C < \gamma < \gamma_{NC_t}$, the patient feels tempted to not cooperate more often. If the doctor cooperates, the patient chooses $No\ Cooperation$ and this leads to the payoff combination (D_{12}, P_{12}). As this represents the worst payoff for the doctor, it is possible to conclude that she/he will never cooperate with patients associated with such values of γ. This leaves us with the choice between $No\ Cooperation\ T$ and $No\ Cooperation\ D$. If the doctor chooses $No\ Cooperation\ T$, the patient chooses $Cooperation$ (because $P_{21} > P_{22}$). If the doctor chooses $No\ Cooperation\ D$, the game does not even begin and the doctor gets $D_{30} = \beta$ as payoff. So, the doctor must take the decision by comparing D_{21} and D_{30}. If $D_{21} > \beta$, the doctor will choose $No\ Cooperation\ T$ and induce the patient's cooperation. But if β takes values such that $D_{21} < \beta$, the doctor will choose the definitive non-cooperation strategy and there will be no game.

Patients associated with higher γ values, high enough so that $\gamma_{NC_t} < \gamma < \gamma_{NC_d}$, tend to be much less cooperative. So it is expected that the doctor assumes a much more cautious attitude towards the patient. Analogously to the previous situation, the branch corresponding to the doctor's cooperation can be disregarded for such range of γ, since the doctor never chooses to cooperate. This leads us once again to the situation where the doctor must choose between the temporary and the definitive exclusion of the patient from the list. This time, the patient prefers to not cooperate if the doctor chose $No\ Cooperation\ T$ ($P_{22} > P_{21}$). So, if the doctor chooses $No\ Cooperation\ T$, she/he gets D_{22} as payoff. Yet again, deciding to definitely not insert the patient in the list gets the doctor the payoff D_{30}. In a simple way, the doctor's decision will now be dependent on how the values D_{22} and $D_{30} = \beta$ are related. The doctor will choose $No\ Cooperation\ T$ whenever $D_{22} > \beta$, and will choose $No\ Cooperation\ D$ otherwise.

Lastly, for even higher values of γ ($\gamma_{NC_d} < \gamma$), the results will be exactly the same as for the previous case. This is due to the fact that the doctor will never choose cooperation again, and that the patient will still choose $No\ Cooperation$ as a reply to being temporarily excluded from the list. This reduces the game to the doctor's decision between the temporary and the definitive exclusion of the patient, respectively carrying D_{22} and D_{30} as payoffs. The doctor's choice is thus the temporary exclusion if $D_{22} > D_{30}$, or the definitive exclusion otherwise.

The results of the previous analysis are summed up in table 4.13.

To wrap it up, doctors associated with high values of β are much more prone to not cooperate with patients and the probability that they will not give patients a chance to enter the list is high. As the doctor has the first-mover advantage, she/he will be able to predict the patient's decision based on their type (value of γ). Doctors will be much more cautious with patients associated with high values of γ, thus resulting in much less cooperation. If the doctor and the patient at stake are associated with low values of β and γ, respectively, the game has much more chances of becoming collaborative. For $\gamma_C < \gamma < \gamma_{NC_t}$ and $\beta < D_{21}$, the doctor may induce the patient to cooperate by choosing the temporary non-cooperative strategy.

When the games with two strategies for each player were compared, the conclusion reached was that if the patient has the first-mover advantage, then the game ends up comprising much more cooperation. To see if this results expand to the more complex game, the previous analysis is now performed with the patient as first mover. The tree representing this game is shown in figure 4.4.

Table 4.13: Backward induction equilibrium for each interval of γ and β, when the doctor decides first, and resulting payoffs.

Equilibria as a function of β and γ; D chooses first			
γ	β	Equilibrium Strategies (D, P)	Equilibrium Payoffs
$\gamma < \gamma_C < \gamma_{NC_t} < \gamma_{NC_d}$	$\beta < \beta_C$	(C, C)	(D_{11}, P_{11})
	$\beta_C < \beta < V_1 < \beta_{CC}$	(C, C)	(D_{11}, P_{11})
	$\beta_C < V_1 < \beta < \beta_{CC}$	$(NC_d, -)$	$(\beta, 0)$
	$\beta_{CC} < \beta$	$(NC_d, -)$	$(\beta, 0)$
$\gamma_C < \gamma < \gamma_{NC_t} < \gamma_{NC_d}$	$\beta < D_{21}$	(NC_t, C)	(D_{21}, P_{21})
	$D_{21} < \beta$	$(NC_d, -)$	$(\beta, 0)$
$\gamma_C < \gamma_{NC_t} < \gamma$	$\beta < D_{22}$	(NC_t, NC)	(D_{22}, P_{22})
	$D_{22} < \beta$	$(NC_d, -)$	$(\beta, 0)$

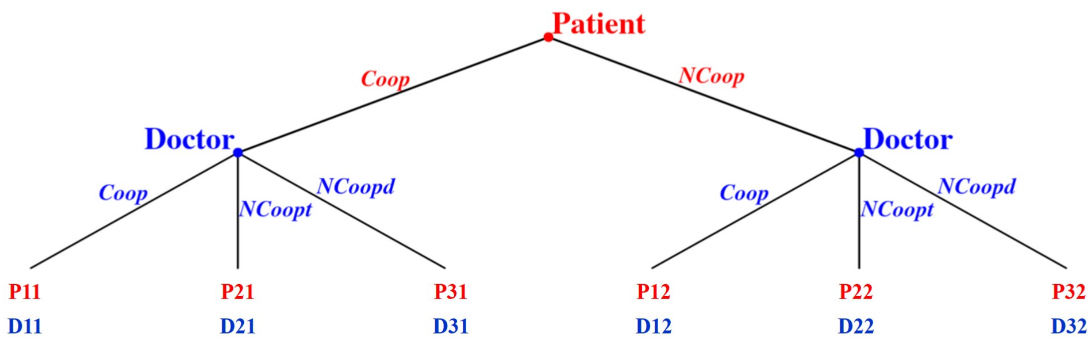

Figure 4.4: Tree representation of the extensive form game, illustrating the interaction between the doctor and the patient in a consultation, with the patient acting first.

Starting for $\gamma < \gamma_C$, the subgame perfect Nash equilibrium is once again (C, C), except for $\beta_{CC} < \beta$. For such values of γ, the patient has no incentives to not cooperate and always prefers to choose $Cooperation$, no matter what the doctor may possibly decide to do. Therefore, the doctor's decision is based on the comparison between D_{11}, D_{21}, and D_{31}. For $\beta < \beta_{CC}$, D_{11} is always the best payoff the doctor can get. This way, the strategy pair (C, C) is the equilibrium solution. The exception arises for $\beta_{CC} < \beta$, where $D_{31} > D_{11}$ and the doctor chooses $No\ Cooperation\ D$ even if the patient cooperated. For such case, the subgame perfect Nash equilibrium is the strategy pair (C, NC_d).

Patients associated with higher values of γ are expected to cooperate less. However, this is not what happens when the patient has the first-mover advantage. The patient can predict how the doctor will act in reply to a patient's non-cooperative strategy. For $\gamma_C < \gamma < \gamma_{NC_t}$, $P_{12} > P_{11}$ and the patient prefers to not cooperate in reply to a cooperative move from the doctor. However, if the patient chooses to not cooperate, the doctor replies by excluding the patient from the list (temporarily or definitely, depending on β). But if the patient cooperates, the doctor replies with cooperation for any $\beta < \beta_{CC}$. As $P_{11} > P_{22}$ and $P_{11} > P_{32}$, *i.e.*, the payoff for staying in the list is higher than the payoff for any kind of exclusion, the patient chooses to cooperate and the doctor cooperates back. So, (C, C) is an equilibrium solution for $\gamma_C < \gamma < \gamma_{NC_t}$ and $\beta < \beta_{CC}$. For $\beta_{CC} < \beta$, one obtains the same subgame perfect equilibrium as before - (C, NC_d).

Using the previous reasoning in the entire range of γ, one reaches the conclusion that, as long as $\beta < \beta_{CC}$, the strategy pair (C, C) constitutes the only subgame perfect Nash equilibrium.

The difference arises when $\beta_{CC} < \beta$, values for which the doctor always chooses the strategy No $Cooperation\ D$, regardless of what the patient might have chosen before. This way, the patient's decision is only based on the comparison between P_{31} and P_{32}. If $\gamma < \gamma_{NC_d}$, $P_{31} > P_{32}$ and the patient prefers to cooperate, resulting in the equilibrium (C, NC_d). For $\gamma_{NC_d} < \gamma$, $P_{31} < P_{32}$ and the patient chooses No $Cooperation$. Hence, the equilibrium when both parameters take the highest values is (NC, NC_d).

Summarizing the results, for $\beta < \beta_{CC}$ the subgame perfect Nash equilibrium is (C, C), no matter the value of γ. For $\beta > \beta_{CC}$, the equilibrium will depend γ. The patient anticipates the doctor's non-cooperation, but prefers to cooperate if she/he is associated with a γ such that $\gamma < \gamma_{NC_d}$. This leads to the equilibrium strategy pair (C, NC_d). Otherwise, if $\gamma_{NC_d} < \gamma$, then the patient prefers to not cooperate after predicting the doctor's definitive non-cooperation, thus resulting in the subgame perfect Nash equilibrium (NC, NC_d).

The results are presented in table 4.14.

Table 4.14: Backward induction equilibrium for each interval of γ and β, when the patient decides first, and resulting payoffs.

Equilibria as a function of β and γ; P chooses first			
γ	β	Equilibrium Strategies (P, D)	Equilibrium Payoffs
$\forall\ \gamma$	$\beta < \beta_{CC}$	(C, C)	(P_{11}, D_{11})
$\gamma < \gamma_{NC_d}$	$\beta_{CC} < \beta$	(C, NC_d)	(P_{31}, D_{31})
$\gamma_{NC_d} < \gamma$	$\beta_{CC} < \beta$	(NC, NC_d)	(P_{32}, D_{32})

Not surprisingly, the game in which the patient takes the first-mover advantage can be considered "more cooperative" than the game in which it is the doctor taking the first decision, in the sense that the equilibrium (C, C) is reached for a larger parameter range. This result goes in accordance with what was observed in the simplified game, and the explanation follows the same reasoning. Having the first-mover advantage allows the patient to predict how the doctor will react and reply to her/his choices. This gives the patient leverage over the doctor, and allows her/him to choose the strategy that will lead to the best payoff possible for her/him. So, the patient reaches the conclusion that it is almost always better to cooperate, in order to avoid the risk of being excluded. Additionally, the doctor will never get the chance to exclude the patient temporarily, a misfortune the patient could possibly bear. But being definitely excluded means the patient will eventually die, sooner than later. And that is why the patient prefers to cooperate most of the times and avoid such exclusion that would jeopardize her/his survival chances. Furthermore, the doctor will also act much more cooperatively, except when $\beta_{CC} < \beta$.

4.3 Summary

In this game, the players are the doctor and the patient. Both of them can cooperate or not. Players have own incentives for non-cooperation, which may vary and are represented by the parameters γ (corresponding to the patient's pleasure on drinking alcohol) and β (corresponding to the importance the doctor gives to saving the liver graft to another patient).

The Nash equilibria for the strategic form game depends on the values of γ and β. For lower values of these two parameters, the equilibrium is (C, C). But as these parameters increase, the players start having more incentives to not cooperate. If patients and doctors are associated with higher values of γ and β, the equilibria starts being constituted by non-cooperative strategies. The equilibrium in mixed strategies is obtained when players are indifferent regarding their options and randomize them. There only exists an equilibrium in mixed strategies if $\gamma_C < \gamma < \gamma_{NC_t}$. The results show that it depends on γ who is more cooperative. For lower γ, the doctor cooperates with higher probability. Otherwise, it is the patient cooperating with higher probability. The probabilities with which the players cooperate are also dependent on parameters such as B, H, G, and both players' emotions of regret and frustration.

From the analysis of the extensive form game it was shown that a change in the first mover makes the results contrast sharply. The game where the doctor decides first has the strategy pair (C, C) as an equilibrium for low values of γ. As this values increases, the doctor anticipates the patient's non-cooperation, and becomes more cautious to avoid being deceived. But when the patient is set to be the first mover, the only equilibrium is (C, C), for any value of γ (except for $\beta_{CC} < \beta$). This happens because the patient anticipates the doctor's non-cooperation as a reply to her/his own non-cooperation.

Chapter 5

Discussion

This chapter discusses the results obtained in this dissertation. The findings are presented and discussed, and some possible implications are debated. Two different game theory dynamics were applied and different results were obtained.

The strategic form game is shown in table 4.3. The payoffs were ordered with the judgment of an expert. Sticking to this payoff order, both the patient and the doctor would never have incentives to not cooperate. Therefore, the game would have mutual cooperation as unique equilibrium. However, things do not work this way in reality and usually players do have incentives to not cooperate with each other. So, parameters γ and β represent those incentives.

Observing table 4.6, it is possible to draw some conclusions on how cooperative both players will be depending on the type of incentive to not cooperate they are associated to. For low values of γ ($\gamma < \gamma_C$) the patient is very cooperative, and prefers to cooperate no matter what the doctor does. For $\beta < \beta_{CC}$ the doctor repays with cooperation, since the incentive for non-cooperation is not high enough. Nevertheless, for $\beta_{CC} < \beta$ this incentive is high enough to make the doctor exclude the patient definitely from the list. So, for the lowest possible values of γ there are two possible Nash equilibria: (C, C) and (NC_d, C).

Oddly, for $\gamma_C < \gamma < \gamma_{NC_t}$ the only equilibrium is the same as for the lowest values of γ and highest values of β - (NC_d, C). For values of β below β_{CC}, there are no equilibria. This type of patient has now some incentives to try to deceive the doctor, and will prefer to not cooperate when the doctor cooperates. As expected, the doctor's reservations about the patient will now make her/him more cautious, and less cooperative. However, this type of patient is concerned with her/his health and aware of the harms alcohol consumption carries, and would prefer to cooperate if excluded from the list (whether temporarily or definitely). This move can be seen as a warning given by the doctor.

For $\gamma_{NC_t} < \gamma$ in combination with low values of β ($\beta < \beta_{NC}$), the Nash equilibria is the strategy pair (NC_t, NC). The patient has significant incentives for non-cooperation, but this type of doctor is more benevolent and chooses to exclude the patient only temporary, giving her/him time to become abstinent. Higher values of β mean harsher doctors and they will tend to choose the strategy $No\, Cooperation\ D$. If $\gamma < \gamma_{NCd}$ the patient is aware that drinking alcohol will accelerate her/his death, and despite feeling high levels of pleasure when drinking she/he is able to suppress the desire for alcohol and cooperate with the doctor's advice, even if the latter excluded her/him definitely from the list. This situation is described by the strategy pair (NC_d, C), which constitutes a Nash equilibrium for the conditions at issue. Lastly, if $\gamma_{NC_d} < \gamma$ the only possible Nash equilibrium is (NC_d, NC) for values of β such that $\beta_{NC} < \beta$.

To sum up, patients and doctors associated with lower values of γ and β, respectively, tend to be more cooperative. But as these parameters are raised, cooperation tends to disappear and be replaced by non-cooperation. Some benevolent doctors may not immediately exclude patients with higher levels of pleasure obtained by drinking, but under specific conditions.

It would be interesting then to find ways to shape the parameters in order to increase cooperation rates. However, it would not be recommendable to decrease β a lot, so that any patient would be maintained in the list, even if she/he had not cooperated. This could possibly lead to major losses for society. The ideal setting would be having patients associated with the lowest γ possible, but doctors associated with intermediate β to maintain selectivity. By trying to decrease γ, one is making an attempt to deviate patients from the temptation of drinking alcohol and alert them to the importance of following the rules of the process strictly. Patients are very often referred to communities such as Alcoholics Anonymous, but some may quit for several reasons. One could suggest a more incisive approach, such as private lectures with the patients showing them shocking images of what alcohol does to health. Additionally, it could possibly be very helpful if patients going through this extremely difficult process could talk and listen to someone who successfully made through it. Having a cured patient, who got the liver transplant, giving speeches could inspire others.

The analysis of the equilibrium in mixed strategies yields the probability that both players cooperate when they are indifferent between cooperating or not. There exists equilibrium in mixed strategies only for $\gamma_C < \gamma < \gamma_{NC_t}$. This condition is actually pretty feasible. If γ assumes lower values, the patient will always prefer to cooperate, and this strategy is dominant, thus being chosen with probability $p = 1$. For higher values of γ the dominant strategy would be $No\ Cooperation$, and the strategy $Cooperation$ would be chosen with probability $p = 0$. Therefore, the necessary conditions for the existence of mixed strategies is that the patient has slight incentives for non-cooperation.

For low values of γ the doctor cooperates with higher probability, and for higher values of γ it is the patient who cooperates with higher probability.The rationale is the following: the equilibrium in mixed strategies implies that both players are indifferent between choosing one of their possible strategies, if the other player chooses $Cooperation$ with a certain probability. For higher values of γ, the patient has to cooperate with higher probabilities to maintain the doctor's indifference of choosing between $Cooperation$ and $No\ Cooperation\ T$. Otherwise, the doctor will not be indifferent anymore and will not randomize her/his choices. Actually, she/he tends to adopt the non-cooperative strategy with higher probability.

A comparative statics analysis is performed to assess how the probability of cooperation (for both players) varies as a function of the parameters. The probability, q, with which the doctor cooperates depends on the γ associated with the patient, as well as with the values of benefit and harm, and the differences between emotions felt or utility values. Both B and H are positively correlated with q, so an increase in each of the parameters (or both) works as an incentive to the patient's cooperation. Bridging the model with reality, this goes in accordance with what would happen. A doctor will cooperate with a higher probability if the patient is also more cooperative. But the parameter explanation is that the doctor will cooperate in order to foster the patient's non-cooperation and maintain the indifference (remember that $\gamma_C < \gamma < \gamma_{NC_t}$ so $P_{12} > P_{11}$). In contrast, γ is negatively correlated with the probability of the doctor's cooperation. Once again, it is perfectly plausible that a doctor will cooperate less with a patient who feels higher levels of pleasure from drinking alcohol, and is naturally more prone to not cooperate.

The previously mentioned policy recommendations may also be applied to increase the patients' perception on how the parameters B and H are influenced by alcohol abstinence or not. By increasing B or H, patients would be more aware of the benefits from stop drinking and of the harms provoked by alcohol consumption, and would naturally cooperate with higher probabilities. Consequently, doctors would also end up being more cooperative.

The value of p indicates the probability with which the patient cooperates. It is dependent basically only on the emotions felt by the doctor. As G increases, the doctor will be much more cautious to avoid that a patient deceives him. Therefore, it will be much easier for this doctor to exclude the patient. To assure her/his maintenance in the list, the patient has to cooperate with higher probability. The other two emotions (R_D^t and F_D^t) have the opposite effect. An increase in each of them works as an incentive for the doctor to cooperate and, as this patient has a slight incentive for not cooperate, it may reduce the probability with which the patient will cooperate.

The way a doctor interacts with a patient may be a key aspect for the patient's adherence to the recommendations. If the doctor shows real commitment, and makes the patient feel respected and esteemed, the patient will definitely feel worse when ending up disappointing the doctor. To avoid this, the patient cooperates with higher probability. Talking to the patient, asking about her/his personal and professional life is definitely a great trick to make the patient open up with the doctor. But more important than asking these information on the first consultation, is to remember that information the next time the patient comes. This way, the patient feels the doctor cares about her/him, and is more likely to cooperate.

The extensive form game may simulate the reality in a closer way, since decisions are not taken without knowing what the other player is doing. For example, the patient usually knows if she/he has been excluded from the list. If, for some reason, she/he does not known, it is unlikely that the doctor hides this information if asked by the patient. On the other hand, the only way the doctor does not know if the patient cooperated or not is if she/he successfully deceived the doctor, which is unlikely given that medical exams give information about what the patient has been drinking. Two different games were constructed: one with the doctor being the first-mover and one with the patient deciding first. This was helpful because it is not straightforward to establish a beginning moment for this game. It may be with the doctors' decision of integrating or not the patient in the list after analyzing the diagnostic exams; yet, it may also be considered to begin much before that. The patient going to the consultation has a long track record of risky behaviours, and it may be considered that the patient was given the chance to cooperate or not before the first consultation. The results obtained differ in a very interesting way.

Starting with the game in which the doctor decides first, it is possible to conclude that the results go in accordance with the ones obtained in the strategic form analysis. For low values of γ one obtains an equilibrium arising from mutual cooperation. As γ increases, the equilibrium changes to non-cooperative strategies. The main difference comparing to the strategic form is that the doctor is much less cooperative, even when β is in its lowest possible interval. This may be explained by the fact that the doctor can anticipate what the patient will do as a reply to her/his own decision. If the doctor can anticipate the slightest patient's temptation to deceive her/him, the doctor will choose to not cooperate. Note that the doctor's non-cooperation can also be used to induce the patient's cooperation if the latter is associated with values of γ in the range $\gamma_C < \gamma < \gamma_{NC_t}$. This can be seen as a warning sent by the doctor to the patient.

The interesting part comes when the game changes and the patient now is the first to play. The results show that the only equilibrium will be mutual cooperation (C, C), no matter the value of γ. In other words, no matter how much pleasure the patient gets from drinking alcohol, she/he will always prefer to cooperate. Once again, this can be explained by the fact that the patient can now anticipate the doctor's move, knowing that the doctor knows how the patient acted. And the patient knows the doctor will not cooperate knowing the patient did not. So, the patient knows she/he will not get away with a non-cooperative decision and will end up cooperating no matter what.

These conclusions extend to the game in which the doctor may exclude the patient definitely. However, a major difference now is that the game may not even begin if the doctor gets the chance to decide first and excludes definitely the patient. In this situation, the doctor gets β and the patient gets a null payoff. This β is now free to fluctuate and it will have an influence in the results. As previously, low values of γ combined with low values of β give rise to cooperative games, and as these increase the cooperation is replaced by non-cooperative strategies. High values of β will make the doctor choose to not even integrate the patient in the list - if $\beta > V_1 = D_{11}$. In two circumstances, the doctor might choose temporary non-cooperation to try to induce the patient to cooperate. For $\gamma_C < \gamma < \gamma_{NC_t}$ this warning results, but for higher values it will not produce the desired effect.

Lastly, and in accordance with the abbreviated game, having the patient as the first mover will also enhance cooperation. There are only two non-cooperative equilibria. In one of these the patient cedes to the pleasure of drinking and will not cooperate (if $\gamma_{NC_d} < \gamma$), whilst in the other the patient prefers to cooperate even if the doctor excludes her/him. Note that these two situations only happen for excessively high values of β.

The conclusion drawn from the extensive form game suggest that an empowered patient tends to be more cooperative. It could be useful to make the patient understand that her/his behaviors are fundamental for her/his survival. Showing the patient that it is the doctor who is deciding based on what she/he did, and not the other way round, could lead to more cooperative patients. Instead of giving them orders or requisites they must comply with, doctors could give them options to make.

Chapter 6

Conclusions and Future Work

Game theory arises as a looming tool to solve an immense amount of socio-economic issues. It is widely used to study multiperson decision problems, in the most diverse areas. However, its full potential has not been properly explored and applied to health care problems, such as the one addressed in this dissertation.

The liver transplantation process is very complex, especially because it involves a specific type of patients going through an extremely delicate period in their lives. There are not two equal patients and all of them must receive the needed care. The main problem in all this process is the unbalance between liver grafts available for transplantation and the number of patients that are in need of one. Unfortunately, the latter exceeds the former since liver grafts available for transplantation are a very scarce resource. This unbalance demands that the patients considered eligible for liver transplantation are selected after passing a very strict series of requirements. Knowing this, patients will try as hard as possible to be considered eligible, even if that implies sometimes trying to deceive doctors. And it is in this negotiation process between the doctor and the patient where game theory can be very useful.

The first obstacle in building such a game arose when defining the payoffs for both players. This is a difficult task because one is trying to quantify abstract concepts. Additionally, and as emotions play a crucial role in processes like this, they had to be taken into account. Moreover, the values for the benefit or harm to the health of the patient are also difficult to measure precisely. This leaves us with parameters γ and β, inserted to replicate possible incentives to non-cooperative behaviours.

It was proven that doctor's cooperation would be more likely to happen for lower values of β. However, low β may lead the patient to think that the doctor has no incentives to exclude her/him from the list no matter what she/he does, and it would possibly raise the temptation for the patient's non-cooperation. However, a doctor associated with high values of β might also be too intransigent and eventually turn the patient off. It would thus be advisable for the doctor to take an intermediate attitude, not too rigid but also not too benevolent. Nevertheless, it is important that the doctor is able to adapt her/his behaviour to the different patients she/he faces.

The parameter that can be best managed is γ. It may be possible to help the patient experience lower values of pleasure when drinking, thus reducing the value of γ she/he is associated with. This dissertation demonstrated that patient's cooperation (and consequently the cooperative equilibrium) occurs for low values of γ. If it is possible to lower this parameter, then the health policy implications arising from this dissertation advise to do so.

There are entities creating targeted programmes for alcoholic patients, such as the Alcoholics Anonymous. However, these programmes may not be enough to scare off patients, for numerous reasons. As an example, talks with fellow patients who successfully went through the transplantation process could work as an eye-opening experience, as well as having private sections with doctors where they were shown chocking illustrations of how harmful alcohol can be. Additionally, it is not uncommon to see alcoholic patients totally abandoned by their families, patients who lost their jobs, and so on. A crucial pillar for this type of patients is life stability, and having their time occupied. The more occupied a patient is, the less time she/he will spend drinking or thinking how enjoyable a drink would feel. Although it can be hard to bring the family back together, it would not be so hard to provide patients an occupation. It was stated that a major share of patients suffering from alcoholic liver disease that are admitted at hospitals are aged between 20 and 60 years, being the most socioeconomically active age group (da Rocha et al., 2017). These patients can be encouraged to stay abstinent if they are shown they are able to work, or if they are taught a task in order to find a job. In Germany there is a law stating that "All employers (public and private) with a workforce of 20 employees or more are required to fill 5% of their jobs with severely disabled employees" (Kock, 2004). In fact, employers are not obliged to create jobs for disabled people nor lay off non-disabled people in order to replace them by disabled people. But if they do not comply with this law, they must pay a monthly fee. Following this example, one could think of a sort of courses or workshops where alcoholic patients could learn some tasks and then be hired by some specific companies. This way, patients would feel they are worth something and that they have a reason to stay away from relapse drinking, and at the same time rebuild their lives, whether it is socially or professionally.

Another key aspect to bear in mind is that the patient tends to be much more cooperative if she/he is given the first-mover advantage, as proven in the extensive form game analysis. This fact could serve as basis for further health policy implications. Perhaps, the way doctors give patients the news, whether good or bad, could change in order to make them feel more empowered. Instead of making the patient feel she/he is deciding as a reply to the doctor's decision, let the patient take the lead. As an example, the patient should be aware that she/he ought to cooperate even before the first consultation, and that this cooperation would be taken into account. In fact, it would be much more pleasant for both the doctor and the patient if the first consultation runs in a friendly environment and if the patient could be congratulated and stimulated to keep up with the good effort. Everyone prefers to be cheered up than to take a reprimand in the first contact.

Although the study is completed, there are some suggestions that may lead to very important extensions in future works. These games were analyzed assuming perfect and complete information scenarios. By this, one means that players watch the other player's moves and both players' type and payoffs are common knowledge. Yet, this does not correspond to what happens in real life very often. Medicine is a credence good, and the doctor is more (and better) informed than the patient, and she/he might decide how much of that information to give to the patient. Additionally, the patient may possess information that would be useful for the doctor but decide to hide it for any reason. Thus, there may be asymmetry of information. As an example, the doctor may not tell the patient she/he was excluded from the list, although this is not very common. Another example could be the patient hiding that she/he drank alcohol, which much more common. In this game, γ and β were common-knowledge for both players, but it might not be what happens in reality.

Game theory comprises a big amount of game dynamics. Signaling games are a tool to deal with some asymmetric information scenarios. The doctor may send a signal to the patient about his severity type (β). Or, in other hand, the patient may send a signal to the doctor to show her/his type (γ, namely). An example of a signal sent by the patient could be showing awareness for the problem she/he possesses and autonomously joining communities like the Alcoholics Anonymous. In principle, the doctor would possess a higher amount of information than the patient, and could play with it. Additionally, and as it happens commonly in real life negotiations, threats and promises could enrich the game and lead to more fruitful conclusions. Using repeated games can also enrich conclusions. In fact, infinitely repeated games can be applied in situations like the liver transplantation process once it is impossible to stipulate the end of the game. Actually, if one considers the post-transplantation surveillance patients are submitted to, the process can be considered to be infinite since it only ends when the patient dies.

This study could be used as a ground for deeper and more complex studies applying game theory to health care issues. It is important to strike problems in processes such as the liver transplantation for alcoholic liver diseased patients, since problems like these are harmful for the society and end up creating large expenses for the governments. But ALD is not the only disease representing a burden for governments and society, that involves negotiation between the doctor and the patient. Obesity, for instance, is also a disease for which the treatment process could also be modeled using game theory.

It is important that tools like game theory start being employed more frequently to try to solve such issues. And even if it is not possible to reach a one-fits-all solution, at least it should be possible to deliver more efficient and complete decision aid tools to doctors and other responsible entities. Doctors and practitioners must take decisions dealing with human lives, so they should have access to as many efficient tools as possible in order to be aided and reduce the number of erroneous decisions.

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Appendix A

Background Articles

A.1 Mixed strategy in Diamond et al. (1986)

The clinical game is characterized by two pure strategies for each of the two players, with the possible outcomes being praised equally by both players. So it is possible to define a 2×2 matrix M, comprising the four different outcomes:

$$M = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

The probability of the doctor recommending biopsy was set to be p and q refers to the probability of the patient accepting the recommendation. It was also assumed that $p = q$, so it is plausible to look at p as the probability of each player choosing one strategy and $(1 - p)$ the probability of choosing the other strategy. With this, it is possible to define an utility function U given by:

$$\begin{aligned} U &= a \cdot (p) \cdot (p) + b \cdot (p) \cdot (1 - p) + c \cdot (1 - p) \cdot (p) + d \cdot (1 - p) \cdot (1 - p) \\ &= (a - b - c + d) \cdot p^2 + (b + c - 2d) \cdot p + d \end{aligned}$$

As a quadratic function, it will have a single extremum which shows the point where the derivative of the function is zero. Therefore, to find p one has to differentiate the function with respect to p and set it equal to zero.

$$\begin{aligned} \frac{dU}{dp} &= 2(a - b - c + d) \cdot p + (b + c - 2d) = 0 \\ p &= \frac{2d - c - b}{2(a - b - c + d)} \end{aligned}$$

Since one is looking for a maximum utility, it is required that the second derivative of the utility function is negative (in this case it is the denominator of the second equation). Given that p is a probability, it can neither be negative or higher than one. Therefore, the numerator must be also negative and lower in absolute value so that p lies in the interval (0,1). This occurs when $2d < (b + c)$, since $(2d - c - b) < 0$, and when $2a < (b + c)$, since $(2d - c - b) < 2(a - b - c + d)$ (Diamond et al., 1986).

A.2 Expressions for x and y in Djulbegovic et al. (2015)

For the patient one has

$$
\begin{aligned}
P_{11} - P_{21} &= p \cdot U_1 + (1-p) \cdot (U_2 - R_p \cdot (U_4 - U_2)) - p \cdot (U_3 - R_p \cdot (U_1 - U_3)) - (1-p) \cdot U_4 \\
&= -(1+R_p)(1-p)(U_4 - U_2)\left[1 - \frac{p}{1-p}\frac{U_1 - U_3}{U_4 - U_2}\right]
\end{aligned}
\tag{A.1}
$$

and

$$
\begin{aligned}
P_{22} - P_{12} &= p \cdot (U_3 - (R_p + F_p)(U_1 - U_3)) + (1-p) \cdot (U_4 - F_p \cdot (U_4 - U_2)) \\
&- p \cdot (U_3 - R_p \cdot (U_1 - U_3)) - (1-p) \cdot U_4 \\
&= -F_p \cdot (1-p)(U_4 - U_2)\left[1 + \frac{p}{1-p}\frac{U_1 - U_3}{U_4 - U_2}\right]
\end{aligned}
\tag{A.2}
$$

thus ending up with

$$
x = \frac{1}{1 + \frac{-(1+R_p)(1-p)(U_4-U_2)\left[1-\frac{p}{1-p}\frac{U_1-U_3}{U_4-U_2}\right]}{-F_p \cdot (1-p)(U_4-U_2)\left[1+\frac{p}{1-p}\frac{U_1-U_3}{U_4-U_2}\right]}} = \frac{1}{1 + \frac{(1+R_p)\left[1-\frac{p}{1-p}\frac{B_p}{H_p}\right]}{F_p \cdot \left[1+\frac{p}{1-p}\frac{B_p}{H_p}\right]}}
\tag{A.3}
$$

Considering that $1-p$ is assumed to be greater than or equal to zero and $U_4 > U_2$, the sign of $P_{11} - P_{21}$ is the same as the sign of $-\left[1 - \frac{p}{1-p}\frac{B_p}{H_p}\right]$. Equivalently, $P_{22} - P_{12}$ is always less than zero since it was assumed that F_p, $(1-p)$, and $U_4 - U_2$ are all non-negative.

Analogously, for the doctor one has

$$
\begin{aligned}
D_{11} - D_{12} &= p \cdot V_1 + (1-p) \cdot (V_2 - G \cdot (U_4 - V_2) - R_d \cdot (V_4 - V_2)) \\
&- p \cdot (V_3 - G \cdot (U_1 - V_3) - R_d \cdot (V_1 - V_3)) - (1-p) \cdot V_4 \\
&= p \cdot (1 + R_d) \cdot (V_1 - V_3) - (1-p) \cdot (1 + R_d) \cdot (V_4 - V_2) \\
&- G \cdot (1-p) \cdot (U_4 - V_2) + G \cdot p \cdot (U_1 - V_3) \\
&= p \cdot (1 + R_d) \cdot B_D - (1-p) \cdot (1 + R_d) \cdot H_D \\
&+ G \cdot p \cdot (U_1 - V_3) - G \cdot (1-p) \cdot (U_4 - V_2) \\
&= -(1-p) \cdot H_D \left[(1+R_d) \cdot (1 - \frac{p}{1-p} \cdot \frac{B_D}{H_D}) + G \cdot (\frac{U_4 - V_2}{H_D} - \frac{p}{1-p} \cdot \frac{U_1 - V_3}{H_D})\right]
\end{aligned}
\tag{A.4}
$$

and

$$
\begin{aligned}
D_{22} - D_{21} &= p \cdot (V_3 - G \cdot (U_1 - V_3) - R_d \cdot (V_1 - V_3)) + (1+p) \cdot V_4 \\
&- p \cdot (V_3 - F_d \cdot (V_1 - V_3)) - (1-p) \cdot V_4 \\
&= -G \cdot p \cdot (U_1 - V_3) - R_d \cdot p \cdot (V_1 - V_3) + F_d \cdot p \cdot (V_1 - V_3) \\
&= -(R_d - F_d) \cdot p \cdot (V_1 - V_3) - G \cdot p \cdot (U_1 - V_3) \\
&= -(1-p) \cdot H_D \cdot \frac{p}{1-p}\left[(R_d - F_d) \cdot \frac{B_D}{H_D} + G \cdot \frac{U_1 - V_3}{H_D}\right]
\end{aligned}
\tag{A.5}
$$

hence giving

$$
y = \frac{1}{1 + \frac{D_{11} - D_{12}}{D_{22} - D_{21}}} = \frac{1}{1 + \frac{(1+R_d)(1 - \frac{p}{1-p} \cdot \frac{B_D}{H_D}) + G \cdot (\frac{U_4 - V_2}{H_D} - \frac{p}{1-p} \cdot \frac{U_1 - V_3}{H_D})}{\frac{p}{1-p} \cdot \left[(R_d - F_d) \cdot \frac{B_D}{H_D} + G \cdot \frac{U_1 - V_3}{H_D}\right]}}
\tag{A.6}
$$

As a remark for the doctors case, and since $(1-p) \geq 0$ and $H_D \geq 0$, the sign of $D_{11} - D_{12}$ is the same a the sign of $-\left[(1+R_d)(1-\frac{p}{1-p} \cdot \frac{B_D}{H_D}) + G \cdot (\frac{U_4 - V_2}{H_D} - \frac{p}{1-p} \cdot \frac{U_1 - V_3}{H_D})\right]$. Likewise, the sign of $D_{22} - D_{21}$ is the same as the sign of $-\left[(R_d - F_d) \cdot \frac{B_D}{H_D} + G \cdot \frac{U_1 - V_3}{H_D}\right]$ (Djulbegovic et al., 2015).

Appendix B

Boundary Conditions for parameters

Deriving from the payoffs order, established with the judgment of an expert, it is possible to draw a set of boundary conditions for all the parameters that constitute the payoffs. Additionally, depending on whether the payoffs are positive or negative, it is also possible to determine more conditions. Recalling this order:

– doctor: $D_{11} > D_{22} > D_{32} > D_{21} > D_{31} > D_{12}$

– patient: $P_{11} > P_{12} > P_{21} > P_{22} > P_{31} > P_{32}$

For the doctor, the only payoff considered positive with certainty is D_{11}. The negative payoffs are D_{21}, D_{31}, and D_{12}. Lastly, D_{22} and D_{32} are considered to be possibly negative or positive. For the patient's case, P_{11}, P_{12}, and P_{21} are considered to be positive, while P_{22}, P_{31}, and P_{32} are regarded as negative payoffs.

Remember that it was assumed that $F_D^t < F_D^t < R_D^t < R_D^d < G$ for the doctor, and that $F_P^t < F_P^t < R_P^t < R_P^d$ and $B < H$ for the patient.

The conditions obtained are summarized in table B.1.

Looking at the boundary conditions, it is possible to conclude that some are redundant. It is then necessary to find the highest lower bound and the lowest upper bound for both γ and β, so they can be conditioned to an interval.

For β, the boundary conditions arise from $D_{22} > D_{32}$ and from $D_{32} > D_{21}$. As a result, one gets $(V_2 - V_3) + F_D^d \cdot (V_1 - V_3) - R_D^t \cdot (V_1 - V_2) < \beta < (V_2 - V_3) + F_D^d \cdot (V_1 - V_3) - F_D^t \cdot (V_1 - V_2)$. It is important to verify that the lower bound is in fact inferior to the upper bound. Equation B.1 validates it, since $R_D^t > F_D^t$.

$$\begin{aligned}(V_1 - V_3) - R_D^t \cdot (V_1 - V_2) &< (V_2 - V_3) + F_D^d \cdot (V_1 - V_3) - F_D^t \cdot (V_1 - V_2)\\ -R_D^t \cdot (V_1 - V_2) &< -F_D^t \cdot (V_1 - V_2)\\ R_D^t &> F_D^t \end{aligned} \tag{B.1}$$

Doing the same operation for γ, it resulted that the boundary conditions derived from conditions $P_{11} > P_{12}$ and $P_{12} > P_{21}$. In other words, the upper bound derives from $P_{11} > P_{12}$ and the lower bound

Table B.1: Boundary Conditions for the parameters that constitute the payoffs, obtained from the payoffs order.

	Boundary Conditions for parameters
Doctor	$D_{11} > D_{22} \Longrightarrow F_D^t > -1$
	$D_{22} > D_{32} \Longrightarrow \beta < (V_2 - V_3) + F_D^d \cdot (V_1 - V_3) - F_D^t \cdot (V_1 - V_2)$
	$D_{32} > D_{21} \Longrightarrow \beta > (V_2 - V_3) + F_D^d \cdot (V_1 - V_3) - R_D^t \cdot (V_1 - V_2)$
	$D_{21} > D_{31} \Longrightarrow \beta < (V_2 - V_3) + R_D^d \cdot (V_1 - V_3) - R_D^t \cdot (V_1 - V_2)$
	$D_{31} > D_{12} \Longrightarrow \beta > (V_1 - V_3) \cdot (1 + R_D^d) - G \cdot (V_1 - V_2)$
Patient	$P_{11} > P_{12} \Longrightarrow \gamma < B + H$
	$P_{12} > P_{21} \Longrightarrow \gamma > B + H - (U_1 - U_2) \cdot (1 + F_P^t)$
	$P_{21} > P_{22} \Longrightarrow \gamma < B + H + (U_1 - U_2) \cdot (R_P^t - F_P^t)$
	$P_{22} > D_{31} \Longrightarrow \gamma > B + H - (U_2 - U_3) + R_P^t \cdot (U_1 - U_2) - F_P^d (U_1 - U_3)$
	$D_{31} > P_{32} \Longrightarrow \gamma < B + H + (U_1 - U_3) \cdot (R_P^d - F_P^d)$

from $P_{12} > P_{21}$. As a result, one gets $B + H - (U_1 - U_2) \cdot (1 + F_P^t) < \gamma < B + H$. The other bounds may be considered redundant for being lower than the lower bound or higher than the upper bound. Once again, it is important to verify that the upper bound is superior to the lower bound. But in this case it is easier because the lower bound is obtained by subtracting $(U_1 - U_2) \cdot (1 + F_P^t)$ to the upper bound, thus resulting in a lower value.

The conditions obtained from the payoffs negativity analysis are shown in table B.2.

Note that the payoffs D_{22} and D_{32} are missing from the table. This is due to the fact that it is not straightforward to consider them positive or negative. However, from the first one it is obtained that F_D^t can be larger or lower than $\frac{V_2}{V_1 - V_2}$. Since from $D_{21} < 0$ one obtains that $R_D^t > \frac{V_2}{V_1 - V_2}$, and since $F_D^t < R_D^t$, it is possible to conclude that F_D^t can be greater or less than this value as long as it is less than R_D^t. The same happens for the latter payoff, where it results that F_D^d can be greater or less than $\frac{\beta + V_3}{V_1 - V_3}$. Since $F_D^d < R_D^d$, and $R_D^d > \frac{\beta + V_3}{V_1 - V_3}$, so F_D^d can be larger or lower than $\frac{\beta + V_3}{V_1 - V_3}$ as long as it is less than R_D^d.

For the patient's emotions R_P and F_P it is possible to solve the inequations, using the upper bound for γ, and cross the results with the magnitude order hypothesis. From this, it is possible to obtain:

- $F_P^t < \frac{U_2 + B}{U_1 - U_2}$; $F_P^d > \frac{U_3 + B}{U_1 - U_3}$
- $R_P^t > \frac{U_2 + B}{U_1 - U_2}$; $R_P^d > \frac{U_3 + B}{U_1 - U_3}$

Since $\frac{U_2 + B}{U_1 - U_2} > \frac{U_3 + B}{U_1 - U_3}$ and $F_P^t < F_P^t < R_P^t < R_P^d$, R_P^d must be also higher than $\frac{U_2 + B}{U_1 - U_2}$. F_P^d may also be higher than this bound, but always as long as it is lower than the two regrets.

Table B.2: Negativity Conditions for the parameters that constitute the payoffs.

	Negativity Conditions for parameters
Doctor	$D_{11} > 0 \Longrightarrow V_1 > 0$
	$D_{12} < 0 \Longrightarrow G > \frac{V_1}{V_1 - V_2} > 1$
	$D_{21} < 0 \Longrightarrow R^t_D > \frac{V_2}{V_1 - V_2}$
	$D_{31} < 0 \Longrightarrow R^d_D > \frac{\beta + V_3}{V_1 - V_3}$
Patient	$P_{11} > 0 \Longrightarrow U_1 + B > 0$
	$P_{12} > 0 \Longrightarrow U_1 > H - \gamma$
	$P_{21} > 0 \Longrightarrow U_2 + B > (U_1 - U_2) \cdot F^t_P$
	$P_{22} < 0 \Longrightarrow U_2 + \gamma < H + R^t_P \cdot (U_1 - U_2))$
	$D_{31} < 0 \Longrightarrow U_3 + B < (U_1 - U_3) \cdot F^d_P$
	$D_{32} < 0 \Longrightarrow U_3 + \gamma < H + (U_1 - U_3) \cdot R^d_P$

Lastly, the computation of the thresholds that γ and β may cross when their variation is allowed, is presented now. Starting with γ, the first barrier to surpass is the one that will make $P_{12} > P_{11}$. It is necessary that inequation B.2 is verified.

$$\begin{aligned} U_1 - H + \gamma &> U_1 + B \\ \gamma &> B + H = \gamma_C \end{aligned} \tag{B.2}$$

Increasing γ to a point where $P_{22} > P_{21}$, is the next barrier to be transcended. For that, γ must respect the condition shown in inequation B.3.

$$\begin{aligned} U_2 - R^t_P \cdot (U_1 - U_2) - H + \gamma &> U_2 - F^t_P \cdot (U_1 - U_2) + B \\ \gamma &> B + H + (U_1 - U_2) \cdot (R^t_P - F^t_P) = \gamma_{NC_t} \end{aligned} \tag{B.3}$$

Lastly, γ may be increased up to a point where a patient that is definitely excluded from the list will prefer to not cooperate than to cooperate, *i.e.*, $P_{32} > P_{31}$. For that, γ must respect the condition shown in inequation B.4.

$$\begin{aligned} U_3 - R^d_P \cdot (U_1 - U_3) - H + \gamma &> U_3 - F^d_P \cdot (U_1 - U_3) + B \\ \gamma &> B + H + (U_1 - U_3) \cdot (R^d_P - F^d_P) = \gamma_{NC_d} \end{aligned} \tag{B.4}$$

Note that it was validated by an expert that $(R^d_P - F^d_P) \cdot (U_1 - U_3) > (R^t_P - F^t_P) \cdot (U_1 - U_2)$, so it is possible to confirm that $\gamma_{NC_d} > \gamma_{NC_t} > \gamma_C$.

It is then necessary to apply the same reasoning for the boundary conditions of β. The first barrier that β overcomes when increasing is the one leading to $D_{32} > D_{22}$. For that, β must respect the condition shown in inequation B.5.

$$\begin{gathered} V_3 - F_D^d \cdot (V_1 - V_3) + \beta > V_2 - F_D^t \cdot (V_1 - V_2) \\ \beta > (V_2 - V_3) + F_D^d \cdot (V_1 - V_3) - F_D^t \cdot (V_1 - V_2) = \beta_{NC} \end{gathered} \tag{B.5}$$

The next change in the payoff order that the increase in β triggers is $D_{31} > D_{21}$. For that, β must be such that inequation B.6 is verified.

$$\begin{gathered} V_3 - R_D^d \cdot (V_1 - V_3) + \beta > V_2 - R_D^t \cdot (V_1 - V_2) \\ \beta > (V_2 - V_3) + R_D^d \cdot (V_1 - V_3) - R_D^t \cdot (V_1 - V_2) = \beta_C \end{gathered} \tag{B.6}$$

Lastly, the barrier demanding higher values of β is the one in which the doctor starts to prefer excluding definitely a cooperating patient than to cooperate, *i.e.*, $P_{31} > P_{11}$.

$$\begin{gathered} V_3 - R_D^d \cdot (V_1 - V_3) + \beta > V_1 \\ \beta > (V_1 - V_3) + R_D^d \cdot (V_1 - V_3) = \beta_{CC} \end{gathered} \tag{B.7}$$

It is then trivial to verify that $\beta_{NC} < \beta_C < \beta_{CC}$.

Appendix C

Equilibrium in mixed strategies

C.1 Proving the strategy NC_d is dominated

In order to assume that the strategy $No\ Cooperation\ D$ is dominated by others in mixed strategies it is necessary to show that $E_D[C] = E_D[NC_t] > E_D[NC_d]$. For that, it is necessary to verify this condition for the value of the probability that leads to the indifference between the strategies $Cooperation$ and $No\ Cooperation\ T$. To validate this, it is necessary that equation C.1 is respected.

$$E_D[NC_t] > E_D[NC_d] \tag{C.1}$$

For that, it is necessary to use the values of the expected payoffs: $E_D[NC_t] = V_2 - F_D^t \cdot (V_1 - V2) + p \cdot [(F_D^t - R_D^t) \cdot (V_1 - V_2)]$ and $E_D[NC_d] = V_3 + \beta - F_D^d \cdot (V_1 - V_3) + p \cdot [(F_D^d - R_D^d) \cdot (V_1 - V_3)]$, with the value of p being replaced by $p = \frac{G - F_D^t - 1}{G + R_D^t - F_D^t}$.

The inequation leads to a condition for β, shown in inequation C.2.

$$\begin{aligned} V_2 - F_D^t \cdot (V_1 - V2) + p \cdot [(F_D^t - R_D^t) \cdot (V_1 - V_2)] &> V_3 + \beta - F_D^d \cdot (V_1 - V_3) + p \cdot [(F_D^d - R_D^d) \cdot (V_1 - V_3)] \\ \beta &< (V_2 - V_3) + F_D^d \cdot (V_1 - V_3) - F_D^t \cdot (V_1 - V_2) \\ &+p \cdot [(R_D^d - F_D^d) \cdot (V_1 - V_3) - (R_D^t - F_D^t) \cdot (V_1 - V_2)] \end{aligned} \tag{C.2}$$

Therefore, it is necessary to compare this upper bound for β with the upper bound obtained from the payoff order - $\beta < (V_2 - V_3) + F_D^d \cdot (V_1 - V_3) - F_D^t \cdot (V_1 - V_2)$. If this upper bound is higher than the one obtained from the payoffs order, than β will always be below it and the necessary condition for $E_D[C] = E_D[NC_t] > E_D[NC_d]$ is always checked.

Given that $(R_D^d - F_D^d) > (R_D^t - F_D^t)$, it results that the parcel multiplied by p is positive. So, the new upper bound will be higher than the bound from the payoff order if $p > 0$, resulting that the new upper bound is equal to the older one summed up with a positive amount. Since p is a probability, it is positive. However, this demonstration is done next in this appendix. If $p > 0$ it is then demonstrated that the strategy $No\ Cooperation\ D$ is dominated and can be excluded from the analysis in mixed strategies.

C.2 Validating the probabilities p and q

In order for the equilibrium in mixed strategies to exist it is necessary that $0 < p < 1$ and $0 < q < 1$. These conditions will now be demonstrated here.

As previously mentioned, $p = \frac{G-F_D^t-1}{G-F_D^t+R_D^t}$ for the indifference between $E_D[C]$ and $E_D[NC_t]$. Let us analyze this probability and check if it respects the necessary conditions. In order for p to be higher than zero, both its numerator and denominator must be positive or negative. There is no doubt that the denominator is positive: $(G - F_D^t + R_D^t) > 0$, if $G > R_D^t > F_D^t$ as it was defined by hypothesis. The numerator will be positive if $G > 1 + F_D^t$. Consulting the boundary conditions for the parameters established in appendix B, it is possible to change the inequation obtained from $D_{31} > D_{12}$, and solve it in order to G. Then, as β shows up with a negative sign, it can be replaced by its upper bound ($\beta < (V_2 - V_3) + F_D^d \cdot (V_1 - V_3) - F_D^t \cdot (V_1 - V_2)$) and the result is the one in inequation C.3.

$$G > \frac{(V_1 - V_3) \cdot (F_D^d + 1) - \beta}{V_1 - V_2}$$
$$G > \frac{(V_1 - V_3) \cdot (F_D^d + 1) - (V_2 - V_3) - F_D^d \cdot (V_1 - V_3) + F_D^t \cdot (V_1 - V_2)}{V_1 - V_2}$$
$$G > 1 + F_D^t \tag{C.3}$$

So, it is validated that the numerator of p is also positive. Therefore, one can state that $p > 0$. Let us compare the numerator with the denominator to see if the latter is higher than the former and p will always be less than 1. The computation is shown in inequation C.4.

$$G - F_D^t - 1 < G - F_D^t + R_D^t$$
$$-1 < R_D^t \tag{C.4}$$

It is always true that $R_D^t > -1$. In conclusion, it is possible to safely state that $0 < p < 1$.

Proceeding to the probability $q = 1 - \frac{\gamma-(B+H)}{(R_P^t-F_P^t)\cdot(U_1-U_2)}$, one has to apply the exact same reasoning, but now taking into account that it depends on γ. Checking for $q > 0$, results in the inequation C.5.

$$1 - \frac{\gamma - (B + H)}{(R_P^t - F_P^t) \cdot (U_1 - U_2)} > 0$$
$$B + H - \gamma > -(R_P^t - F_P^t) \cdot (U_1 - U_2) \tag{C.5}$$

This result is possible for $\gamma < B + H = \gamma_C$. So, if $\gamma < \gamma_C$, it is possible to state that $q > 1$. Let us now check if $q < 1$. The demonstration is shown in inequation C.6.

$$1 - \frac{\gamma - (B + H)}{(R_P^t - F_P^t) \cdot (U_1 - U_2)} < 1$$
$$B + H - \gamma < 0$$
$$B + H < \gamma \tag{C.6}$$

This result goes against the requirement for $0 < q$. Thus, for $\gamma < \gamma_C$ there is no equilibrium in mixed strategies. However, it is possible to verify if for values of γ such that $\gamma_C < \gamma$, this equilibrium exists. For that, it is necessary to rearrange inequation C.7 and solve it in order to γ.

$$\begin{aligned} B + H - \gamma &> -(R_P^t - F_P^t) \cdot (U_1 - U_2) \\ B + H + (R_P^t - F_P^t) \cdot (U_1 - U_2) &> \gamma \end{aligned} \tag{C.7}$$

Interestingly, it results that $\gamma < \gamma_{NC_t}$ must be respected in order to have q positive. As this requirement does not go against the requirement for $q < 1$, it is possible to state that there will only be equilibrium in mixed strategies if $\gamma_C < \gamma < \gamma_{NC_t}$, and one will have $0 < q < 1$. This result makes sense, since if $\gamma < \gamma_C$ the patient would not have any incentive to not cooperate and would choose the strategy $Cooperation$ with probability 1. For $\gamma_{NC_t} < \gamma$, the patient would never cooperate, and consequently there would be no equilibrium in mixed strategies.

Lastly, to finish this analysis it is important to compare p and q in order to see which of the players will cooperate with higher probability. Let us assume, by hypothesis, that $q > p$, *i.e.*, the doctor cooperates with higher probability than the patient. The verification of this relation is shown below.

$$\begin{aligned} 1 - \frac{\gamma - (B + H)}{(R_P^t - F_P^t) \cdot (U_1 - U_2)} &> \frac{G - F_D^t - 1}{G - F_D^t + R_D^t} \\ \frac{B + H - \gamma}{(R_P^t - F_P^t) \cdot (U_1 - U_2)} &> \frac{G - F_D^t - 1 - (G - F_D^t + R_D^t)}{G - F_D^t + R_D^t} \\ \frac{B + H - \gamma}{(R_P^t - F_P^t) \cdot (U_1 - U_2)} &> \frac{-R_D^t - 1}{G - F_D^t + R_D^t} \\ -\gamma &> \frac{(-R_D^t - 1) \cdot (R_P^t - F_P^t) \cdot (U_1 - U_2)}{G - F_D^t + R_D^t} - B - H \\ \gamma &< \frac{(R_D^t + 1) \cdot (R_P^t - F_P^t) \cdot (U_1 - U_2)}{G - F_D^t + R_D^t} + B + H \end{aligned} \tag{C.8}$$

After this, it is necessary to compare the upper bound obtained in C.8 with the upper bound for γ obtained from the payoff order (for $\gamma_C < \gamma < \gamma_{NC_t}$). Recall that $\gamma_{NC_t} = B + H + (R_P^t - F_P^t) \cdot (U_1 - U_2)$. If the upper bound from C.8 is higher than γ_{NC_t}, then one has $q > p$ for any value of γ, and the doctor cooperates always with higher probability. This computation is shown below.

$$\begin{aligned} \frac{(R_D^t + 1) \cdot (R_P^t - F_P^t) \cdot (U_1 - U_2)}{G - F_D^t + R_D^t} + B + H &> B + H + (R_P^t - F_P^t) \cdot (U_1 - U_2) \\ \frac{(R_D^t + 1) \cdot (R_P^t - F_P^t) \cdot (U_1 - U_2)}{G - F_D^t + R_D^t} &> (R_P^t - F_P^t) \cdot (U_1 - U_2) \\ \frac{R_D^t + 1}{G + R_D^t - F_D^t} &> 1 \\ R_D^t + 1 &> G + R_D^t - F_D^t \\ F_D^t + 1 &> G \end{aligned} \tag{C.9}$$

Condition C.9 is impossible as previously shown. So, the upper bound obtained in C.8 is not superior to γ_{NC_t}. One can conclude then, that $q > p$, *i.e.*, the doctor will cooperate with higher probability for $\gamma_C < \gamma < \frac{(R_D^t+1) \cdot (R_P^t - F_P^t) \cdot (U_1 - U_2)}{G - F_D^t + R_D^t} + B + H$. Otherwise, if $\frac{(R_D^t+1) \cdot (R_P^t - F_P^t) \cdot (U_1 - U_2)}{G - F_D^t + R_D^t} + B + H < \gamma < \gamma_{NC_t}$, then $p > q$ and the patient cooperates with higher probability.

C.3 Validating $E_D[C] = E_D[NC]$ and $E_P[C] = E_P[NC]$

Starting with the doctor's payoffs, for the probability p one has $E_D[C] = V_1 - \frac{G\cdot(V_1-V_2)\cdot(1+R_D^t)}{G-F_D^t+R_D^t}$ and $E_D[NC_t] = V_2 + (V_1 - V2)\cdot[\frac{(F_D^t-R_D^t)\cdot(G-F_D^t-1)-F_D^t}{G-F_D^t+R_D^t}]$. From equation 4.1 one must have:

$$
\begin{gathered}
V_2 + (V_1 - V2)\cdot[\frac{(F_D^t - R_D^t)\cdot(G - F_D^t - 1) - F_D^t}{G - F_D^t + R_D^t}] = V_1 - \frac{G\cdot(V_1 - V_2)\cdot(1 + R_D^t)}{G - F_D^t + R_D^t} \qquad \text{(C.10)} \\
\frac{(F_D^t - R_D^t)\cdot(G - F_D^t - 1)}{G - F_D^t + R_D^t} - F_D^t = 1 + \frac{G\cdot(-1 - R_D^t)}{G - F_D^t + R_D^t} \\
(F_D^t - R_D^t)\cdot(G - F_D^t - 1) = (G - F_D^t + R_D^t)\cdot(1 + F_D^t) + G\cdot(-1 - R_D^t) \\
G\cdot F_D^t - (F_D^t)^2 - F_D^t - R_D^t\cdot G + R_D^t\cdot F_D^t + R_D^t = G - F_D^t + R_D^t + G\cdot F_D^t - (F_D^t)^2 + R_D^t\cdot F_D^t - G - R_D^t\cdot G \\
R_D^t = G + R_D^t - G
\end{gathered}
$$

It was then proven that $E_D[C] = E_D[NC_t]$ for the equilibrium value of p, thus validating equation 4.1. Doing the same for the patient one gets:

$$
\begin{gathered}
U_1 + B - \frac{\gamma - (B + H)}{R_P^t - F_P^t}\cdot(1 + F_P^t) = U_1 - H + \gamma - \frac{\gamma - (B + H)}{R_P^t - F_P^t}\cdot(1 + R_P^t) \qquad \text{(C.11)} \\
B + H - \gamma = \frac{B + H - \gamma}{R_P^t - F_P^t}\cdot(1 + R_P^t) - \frac{B + H - \gamma}{R_P^t - F_P^t}\cdot(1 + F_P^t) \\
B + H - \gamma = \frac{(B + H - \gamma)\cdot(1 + R_P^t - 1 - F_P^t)}{R_P^t - F_P^t} \\
B + H - \gamma = \frac{(B + H - \gamma)\cdot(R_P^t - F_P^t)}{R_P^t - F_P^t} \\
B + H - \gamma = B + H - \gamma
\end{gathered}
$$

It was now proven that $E_P[C] = E_P[NC]$ for the equilibrium value of q, thus validating equation 4.3.