High-scale neutrino mass degeneracy in the two-Higgs doublet model

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The discovery of neutrino oscillations, to which the 2015 Physics Nobel Prize has been awarded, provided the strongest evidence for physics beyond the Standard Model of Particle Physics. To oscillate, neutrinos must be massive and mix, something not accounted for in the Standard Model. Several possibilities have been put forward in order to explain how neutrino masses are generated, as well as the observed low-energy neutrino mass and mixing spectrum. In the present work we revisit the scenario of high-scale neutrino mass degeneracy in the Standard Model and explore the possibility of having this scenario in the two-Higgs doublet model. We investigate whether radiative corrections (governed by Renormalisation Group Equations) are able to lift the high-scale degeneracy and reproduce the low-energy mass and mixing pattern.

Keywords: Standard Model; Two-Higgs doublet model; Neutrino masses; Neutrino mass degeneracy; Renormalisation group.

I. INTRODUCTION

Our current understanding of the subatomic world lies on the Standard Model (SM) of Particle Physics, a mathematical description of strong and electroweak interactions of elementary particles. The last missing piece predicted by the SM, the Higgs boson, theorized by Peter Higgs and others [1] in the mid-sixties, was finally discovered in 2012, at CERN. However, and despite its self-consistency and remarkable agreement with experiment, the SM presents several theoretical deficiencies by leaving several phenomena unexplained. These open problems are the primary motivation to look for beyond the SM (BSM) scenarios.

The scalar particle observed at CERN appears to be the SM-Higgs boson, but there is no fundamental reason to assume that this boson may not be part of an extended scalar sector, yet to be fully observed. Thus, one natural direction is to consider extensions of the SM scalar sector, being the simplest the two-Higgs doublet model (2HDM) [2, 3]. There are several motivations for 2HDMs, such as supersymmetry, possible additional sources of CP violation or dark matter candidates [2].

The discovery of neutrino oscillations opened up a new challenging problem: neutrinos have mass, which is a strong and clear evidence for physics BSM. The neutrino was first postulated by Pauli in 1930 to rescue the principles of energy and momentum conservation in radioactive β decays, being experimentally detected in 1956. Neutrinos were first thought to be massless but, in the last decades, several experiments revealed that, in fact, neutrinos have non-vanishing but slightly different masses, allowing neutrino mixing.

The SM in the strict sense does not account for the possibility of neutrino masses. However, if ones allows for lepton-number violation, and regards the SM as an effective low-energy theory, we must consider higher-dimensional operators which give rise to neutrino masses. With the SM field content, the lowest-dimensional of such operators has dimension five and is known as Weinberg operator [4].

The slight difference in neutrino masses, revealed by the smallness of mass-squared differences measured in neutrino oscillations experiments [6], may suggest a high-scale neutrino mass degeneracy scenario [7, 8]. Radiative corrections due to the evolution of the Weinberg operator from the high to low energy scales can, in principle, be responsible for lifting the degeneracy and reproducing the experimental mass and mixing parameters [7, 9]. The running of physical parameters is defined by the Renormalisation Group Equations (RGEs) [10], which control the way physical quantities change with energy. In this work, we study the possible scenario of high-scale neutrino mass degeneracy: we revisit it in the SM [7] and explore for the first time the possibility of considering it in the 2HDM [9].

The present document is organized as follows. After a brief review of the electroweak sector of the SM, in Section II, we present the extended model with the addition of one Higgs doublet, the 2HDM, in Section III, with relevant aspects for our work. In Section IV, we discuss effective neutrino masses, presenting the dimension-five Weinberg operator and its quantum corrections. In Section V, we present our main results on the high-scale neutrino mass degeneracy in the SM and, in Section VI, this possibility is explored in the 2HDM. Finally, we gather our main conclusions in Section VII.

II. THE STANDARD MODEL

The SM is a relativistic quantum Yang-Mills theory that combines local gauge invariance under the symmetry group $\mathrm{SU}(3)_c \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$ with spontaneous symmetry breaking (SSB). The group $\mathrm{SU}(3)_c$ describes the strong interaction between particles that carry colour charge, while the electroweak sector is described by the subgroup $\mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$. Under the requirement of local gauge invariance, one has to introduce in the theory twelve gauge fields, one for each group generator, namely G^i_μ $(i = 1, .., 8)$, W^j_μ $(j = 1, 2, 3)$ and

B_μ, corresponding to the SU(3), SU(2) and U(1) gauge fields, respectively. Henceforth, we will focus only on the electroweak sector of the SM, for which the gauge-invariant Lagrangian reads:

$$\mathcal{L}_{\rm SM} = \mathcal{L}_{\rm gauge} + \mathcal{L}_{\rm fermions} + \mathcal{L}_{\rm Higgs} + \mathcal{L}_{\rm Yukawa}. \quad (1)$$

The component $\mathcal{L}_{\rm gauge}$ yields the dynamics of the gauge fields and the remaining ones are given by:

$$\begin{aligned}\mathcal{L}_{\rm fermions} &= \overline{q_\alpha}_L i\gamma^\mu D_\mu q_{\alpha L} + \overline{\ell_\alpha}_L i\gamma^\mu D_\mu \ell_{\alpha L} \\ &\quad + \overline{u_\alpha}_R i\gamma^\mu D_\mu u_{\alpha R} + \overline{d_\alpha}_R i\gamma^\mu D_\mu d_{\alpha R} \\ &\quad + \overline{e_\alpha}_R i\gamma^\mu D_\mu e_{\alpha R} \;, \quad (2) \\ \mathcal{L}_{\rm Higgs} &= \left(D^\mu\Phi\right)^\dagger \left(D_\mu\Phi\right) - V_{\rm Higgs}(\Phi) \\ &= \left(D^\mu\Phi\right)^\dagger \left(D_\mu\Phi\right) - \mu^2\Phi^\dagger\Phi - \frac{\lambda}{2}\left(\Phi^\dagger\Phi\right)^2 \;, \quad (3) \\ -\mathcal{L}_{\rm Yukawa} &= Y^u_{\alpha\beta}\overline{q_\alpha}_L\tilde{\Phi}u_{\beta R} + Y^d_{\alpha\beta}\overline{q_\alpha}_L\Phi d_{\beta R} \\ &\quad + Y^e_{\alpha\beta}\overline{\ell_\alpha}_L\Phi e_{\beta R} + \text{H.c.} \;, \quad (4)\end{aligned}$$

in which the index α shoud be summed over the three generations of fermions. Regarding the field content, $\ell_{\alpha L}$ and $q_{\alpha L}$ represent the left-handed lepton and quark doublets; the fields $e_{\alpha R}$, $u_{\alpha R}$ and $d_{\alpha R}$ correspond to the right-handed counterparts of charged leptons, up- and down-type quarks, respectively, which are $\mathrm{SU}(2)_L$ singlets. We note the absence of right-handed neutrinos. Finally, Φ is the Higgs doublet. In the Higgs potential, $V_{\rm Higgs}(\Phi)$, λ is a dimensionless parameter that satisfies $\lambda > 0$, in order for the potential to be bounded from below, and one considers $\mu^2 < 0$ in order to have SSB. The matrices $Y^{u,d,e}$ are the Yukawa coupling matrices and the covariant derivative D_μ is given by:

$$D_\mu = \partial_\mu - ig\frac{\tau^j}{2}W^j_\mu - ig'\frac{Y}{2}B_\mu, \quad (5)$$

where g and g' are the SU(2) and U(1) gauge coupling constants, $\tau^j (j = 1, 2, 3)$ are the Pauli matrices and Y is the weak hypercharge given by $Y = 2(Q - T_3)$, with Q and T_3 being the electric charge and the third component of the weak isospin.

The process of Electroweak Symmetry Breaking (EWSB) is achieved in a consistent way by introducing the complex Higgs scalar doublet Φ [1]. Upon SSB, the Higgs field acquires a vacuum expectation value (VEV), v, which we write as:

$$\langle\Phi\rangle \equiv \begin{pmatrix} 0 \\ v \end{pmatrix} \quad , \quad v^2 = -\frac{\mu^2}{\lambda} > 0. \quad (6)$$

It is possible to parametrise fluctuations around the vacuum by:

$$\Phi = \begin{pmatrix} 0 \\ v + \dfrac{h}{\sqrt{2}} \end{pmatrix}, \quad (7)$$

with h being a real scalar field. Inserting this parametrisation in Eq. (4) and keeping only quadratic or bilinear terms in the fields, it is possible to identify the gauge bosons and scalar field masses:

$$m_Z^2 = \frac{1}{2}v^2(g + g'^2) \;,\; m_W^2 = \frac{1}{2}v^2 g^2 \;,\; m_h^2 = -2\mu^2. \quad (8)$$

The physical fields $W^\pm_\mu$, Z_μ and A_μ are related to W^j_μ and B_μ by:

$$W^\pm_\mu = \frac{W^1_\mu \mp iW^2_\mu}{\sqrt{2}}, \quad (9)$$

$$W^3_\mu = \sin\theta_W A_\mu + \cos\theta_W Z_\mu, \quad (10)$$

$$B_\mu = \cos\theta_W A_\mu - \sin\theta_W Z_\mu, \quad (11)$$

with θ_W being the weak mixing angle for which the following relations with the SU(2) and U(1) coupling constants hold:

$$\sin\theta_W = \frac{g'}{\sqrt{g^2 + g'^2}} \;,\; \cos\theta_W = \frac{g}{\sqrt{g^2 + g'^2}}. \quad (12)$$

After SSB, one boson remains massless, the photon, and there is a residual U(1) symmetry, related with electric charge conservation.

After EWSB, fermion masses arise from the Yukawa part of the Lagrangian $\mathcal{L}_{\rm Yukawa}$, being the mass matrices given by:

$$M^{u,d,e}_{\alpha\beta} = vY^{u,d,e}_{\alpha\beta}. \quad (13)$$

In the SM, fermion masses are of Dirac type and, due to the absence of righ-handed neutrinos, no Dirac neutrino masses are generated. Thus, neutrinos are massless in the SM. In general, the mass matrices arising from EWSB are not diagonal, so one must 'rotate' the fields to bring them to the physical (diagonal) basis. The mass matrices can be diagonalised through bi-unitary transformations:

$$\begin{aligned} V^{u\dagger}_L M^u V^u_R &= \mathrm{diag}(m_u, m_c, m_t) \equiv D^u, \\ V^{d\dagger}_L M^d V^d_R &= \mathrm{diag}(m_d, m_s, m_b) \equiv D^d, \quad (14) \\ V^{e\dagger}_L M^e V^e_R &= \mathrm{diag}(m_e, m_\mu, m_\tau) \equiv D^e, \end{aligned}$$

where $V^{u,d,e}_{L,R}$ are 3×3 unitary matrices and m_α are real and non-negative masses. Thus, applying the transformations:

$$\begin{aligned} u_{\alpha L} &\to (V^u_L)_{\alpha\beta}\, u'_{\beta L} \;,\; u_{\alpha R} \to (V^u_R)_{\alpha\beta}\, u'_{\beta R}, \\ d_{\alpha L} &\to \left(V^d_L\right)_{\alpha\beta} d'_{\beta L} \;,\; d_{\alpha R} \to \left(V^d_R\right)_{\alpha\beta} d'_{\beta R}, \quad (15) \\ e_{\alpha L} &\to (V^e_L)_{\alpha\beta}\, e'_{\beta L} \;,\; e_{\alpha R} \to (V^e_R)_{\alpha\beta}\, e'_{\beta R}, \end{aligned}$$

corresponds to changing from the weak to the mass basis, denoted as the primed basis.

From the weak interactions that arise from Eq. (3), we obtain that charged currents mix left-handed components of the up- and down-type quarks. With the previous transformations, there will be a misalignment in these interaction terms:

$$\begin{aligned}\mathcal{L}^{\mathrm{CC}}_{\mathrm{quarks}} &= \frac{g}{\sqrt{2}}\overline{u_\alpha}_L\gamma^\mu d_{\alpha L}W^+_\mu + \mathrm{H.c.}\\ &= \frac{g}{\sqrt{2}}\overline{u'_\alpha}_L\gamma^\mu \left(V^{u\dagger}_L V^d_L\right)_{\alpha\beta} d'_{\beta L} W^+_\mu + \mathrm{H.c.} \quad .\end{aligned} \tag{16}$$

From this equation we see that, in the quark case, a non-diagonal unitary matrix arises $V_{\mathrm{CKM}} \equiv V^{u\dagger}_L V^d_L$. This matrix is the Cabibbo-Kobayashi-Maskawa (CKM) matrix, which describes quark mixing and is responsible for all CP-violating phenomena in flavour changing processes in the SM. In the lepton case, the charged-current terms are diagonal, since one is free to perform a rotation of the neutrino fields in flavour space (due to the absence of neutrino masses).

III. BEYOND THE SM: THE 2HDM

In the 2HDM, the most general renormalisable, *i.e.* quartic, scalar potential may be written as [2]:

$$\begin{aligned}V_{\mathrm{2HDM}} = &\, m^2_{11}\Phi^\dagger_1\Phi_1 + m^2_{22}\Phi^\dagger_2\Phi_2 - \left(m^2_{12}\Phi^\dagger_1\Phi_2 + \mathrm{H.c.}\right)\\ &+ \frac{1}{2}\lambda_1\left(\Phi^\dagger_1\Phi_1\right)^2 + \frac{1}{2}\lambda_2\left(\Phi^\dagger_2\Phi_2\right)^2\\ &+ \lambda_3\left(\Phi^\dagger_1\Phi_1\right)\left(\Phi^\dagger_2\Phi_2\right) + \lambda_4\left(\Phi^\dagger_1\Phi_2\right)\left(\Phi^\dagger_2\Phi_1\right)\\ &+ \left[\frac{1}{2}\lambda_5\left(\Phi^\dagger_1\Phi_2\right)^2 + \lambda_6\left(\Phi^\dagger_1\Phi_1\right)\left(\Phi^\dagger_1\Phi_2\right) + \mathrm{H.c.}\right]\\ &+ \left[\lambda_7\left(\Phi^\dagger_2\Phi_2\right)\left(\Phi^\dagger_1\Phi_2\right) + \mathrm{H.c.}\right],\end{aligned} \tag{17}$$

where m^2_{ij} have mass-squared dimensions and the quartic couplings λ_k are dimensionless. By hermiticity, m^2_{11}, m^2_{22}, and $\lambda_{1,2,3,4}$ are real, while m^2_{12} and $\lambda_{5,6,7}$ are, in general, complex. There is an alternative notation for the 2HDM scalar potential more suitable for the analysis of symmetries in the scalar sector, namely [2]:

$$V_{\mathrm{2HDM}} = \sum_{a,b=1}^{2}\mu_{ab}\Phi^\dagger_a\Phi_b + \frac{1}{2}\sum_{a,b,c,d=1}^{2}\lambda_{abcd}\left(\Phi^\dagger_a\Phi_b\right)\left(\Phi^\dagger_c\Phi_d\right), \tag{18}$$

where, by definition, $\lambda_{abcd} = \lambda_{cdab}$; and, by hermiticity, $\mu_{ab} = \mu^*_{ba}$ and $\lambda_{abcd} = \lambda^*_{badc}$.

A. Z_2-constrained 2HDM

In the 2HDM, regarding the Yukawa interactions, both scalar doublets can couple to both up- and down-type fermions. In general, the Yukawa matrices $Y^{u,d,e}_{1,2}$ will not be simultaneously diagonalisable and, so, there is the possibility of flavour changing neutral currents (FCNC) at tree level. Glashow and Weinberg, and independently Paschos, showed that such currents can be avoided in theories with multiple Higgs doublets if all fermions with the same quantum numbers couple to the same Higgs doublet [11]. In the 2HDM, this is achieved by introducing discrete or continuous symmetries. One option is to impose a discrete Z_2 symmetry in order to supress FCNC [11]:

$$Z_2: \quad \Phi_1 \to \Phi_1, \quad \Phi_2 \to -\Phi_2\,, \tag{19}$$

giving rise to a Type I 2HDM, with all fermions coupling to Φ_1, enforcing $Y^{u,d,e}_2 = 0$, in the Yukawa interactions. The discrete symmetry also eliminates all terms odd in the doublets. In the second notation for the scalar potential, we directly see that it enforces the vanishing of the couplings odd in the doublet indices, which corresponds to $m^2_{12} = \lambda_6 = \lambda_7 = 0$, in the first notation. We take the remaining five quartic couplings real, assuming CP conservation.

We are interested in a version of the Z_2-constrained 2HDM in which $v_1 \neq 0$ and $v_2 = 0$ - inert 2HDM [12]. One can show that this configuration corresponds to the global minimum of $V^{Z_2}_{\mathrm{2HDM}}$ if and only if [12, 13]:

$$\begin{aligned}\lambda_1 \geq 0 \quad &, \quad \lambda_2 \geq 0,\\ \lambda_3 \geq -\sqrt{\lambda_1\lambda_2} \quad &, \quad \lambda_3 + \lambda_4 - |\lambda_5| \geq -\sqrt{\lambda_1\lambda_2},\end{aligned} \tag{20}$$

as long as all Higgs-boson masses squared are positive.

B. SSB and particle content

With the above considerations, the constrained scalar potential has a single stationary condition given by:

$$m^2_{11} = -\lambda_1 v^2 < 0. \tag{21}$$

The requirement that one mass squared, here m^2_{11}, is negative allows SSB, in complete analogy to the SM. After SSB, the SM scalar structure predicts the existence of a single massive scalar boson. At this point, the same approach can be performed in the 2HDM, through a parametrisation of the doublets around the normal vaccum state, which allows to obtain scalar mass matrices. There are eight degrees of freedom in the two doublets:

$$\Phi_1 = \begin{pmatrix} G^+ \\ v + \dfrac{h + iG^0}{\sqrt{2}}\end{pmatrix} \quad , \quad \Phi_2 = \begin{pmatrix} H^+ \\ \dfrac{H + iA}{\sqrt{2}}\end{pmatrix}, \tag{22}$$

in which G^+ and G^0 correspond to Goldstone bosons, H^+ is a charged scalar, A is a pseudoscalar, h and H are CP-even neutral scalars. The physical masses are given by [12]:

$$\begin{aligned} m_{H^\pm}^2 &= m_{22}^2 + \lambda_3 v^2, \\ m_A^2 &= m_{22}^2 + (\lambda_3 + \lambda_4 - \lambda_5) v^2, \\ m_H^2 &= m_{22}^2 + \lambda_{345} v^2 \\ m_h^2 &= m_{11}^2 + 3\lambda_1 v^2 = 2\lambda_1 v^2. \end{aligned} \tag{23}$$

C. Alignment and decoupling limits

It is possible to adjust m_h^2 to coincide exactly with the SM-one, by making $m_h^2 = (125\ \text{GeV})^2$. Moreover, in order to have h emulating the SM-like Higgs boson not only in mass but also in couplings, we choose to impose the alignment limit [14]. We get the values $\beta = 0$ and $\alpha = -\pi/2$ for the angles associated with the rotations that diagonalise the neutral scalar and the charged and pseudoscalar mass matrices, respectively.

We also want to ensure the decoupling limit [9], in which one of the Higgs neutral scalars is kept light, while the remaining ones acquire masses much larger than the electroweak scale. The neutral scalar h is already the SM-like Higgs boson, so a last necessary condition for the decoupling limit is $m_{22}^2 \gg v^2$, which leads to $m_{H^\pm}^2, m_A^2, m_H^2 \gg v^2$.

IV. EFFECTIVE NEUTRINO MASSES

The SM does not account for neutrino masses, due to the absence of right-handed neutrinos (no Dirac mass terms) and to lepton number conservation. The hypothesis of generating Dirac neutrino masses, by adding right-handed neutrinos is, however, quite unnatural since extremely small Yukawa couplings are required.

One way of accounting for neutrino masses is to construct a mass term with left-handed neutrino fields only. These are Majorana mass terms:

$$\mathcal{L}_{\text{Majorana}} = -\frac{1}{2} m \left(\overline{\nu_L^C} \nu_L \right) + \text{H.c.} \ , \tag{24}$$

in which the charge-conjugated field is given by $\nu^C \equiv C\overline{\nu}^T$, with C the charge conjugation matrix. The particles associated to a Majorana field are their own antiparticles. Therefore, one loses the freedom to rephase the neutrino field, due to the fact that $\nu = \nu^C$ and, so, lepton number is violated in Eq. (24). A Majorana mass term is forbidden in the SM since there are no fields with the required quantum numbers to construct an invariant term of such type. So, how can one introduce such mass terms?

A. The Weinberg operator

If one considers the SM as an effective low-energy theory, higher-dimensional operators must be taken into account. It has been proved that the symmetries of the SM allow for a unique dimension-five effective operator, which leads to Majorana neutrino masses after EWSB. This operator is often referred to as the Weinberg operator [4], and it is a combination of four SM fields, two lepton fields and two Higgs fields. In a SM scalar extension with n_H Higgs doublets, Φ_a, the possible Weinberg operators can be written as [10]:

$$\mathcal{O}_\kappa^{ab} = \kappa_{\alpha\beta}^{ab} (\ell_{\alpha L}^i)^T C^{-1} (\varepsilon^{ij} \Phi_{aj}) \ell_{\beta L}^k (\varepsilon^{kl} \Phi_{bl}), \tag{25}$$

in which, as usual, α, β are flavour indices; i, j, k, l are SU(2) indices; ε is the antisymmetric two-dimensional tensor with $\varepsilon^{12} = 1$, and $a, b = 1, 2, ..., n_H$. The matrices κ^{ab} have dimension $d = -1$ and satisfy $\kappa^{ab} = \kappa^{ba\,T}$. It is straightforward to see that the following Lagrangian:

$$\mathcal{L}_\kappa = \frac{1}{4} \sum_{a,b=1}^{n_H} \kappa_{\alpha\beta}^{ab} (\ell_{\alpha L}^i)^T C^{-1} (\varepsilon^{ij} \Phi_{aj}) \ell_{\beta L}^k (\varepsilon^{kl} \Phi_{bl}) + \text{H.c.}, \tag{26}$$

gives rise to Majorana neutrino mass terms:

$$\mathcal{L}_{\text{Majorana}} = -\frac{1}{2} \overline{\nu_{\alpha L}^C} \left(\sum_{a,b=1}^{n_H} \frac{1}{2} \kappa_{\alpha\beta}^{ab} v_a v_b \right) \nu_{\beta L} + \text{H.c.} \ , \tag{27}$$

after EWSB. In the above equation, we assumed $\langle \phi_a^0 \rangle = v_a$ and the relation $\overline{\nu_L^C} = -\nu_L^T C^{-1}$ was used. Comparison with Eq. (24) leads to the following result for the Majorana neutrino mass matrix $\mathcal{M}$:

$$\mathcal{M}_{\alpha\beta} = \sum_{a,b=1}^{n_H} \frac{1}{2} \kappa_{\alpha\beta}^{ab} v_a v_b. \tag{28}$$

Notice that we have not specified the high-energy theory which gives rise to the effective Weinberg operator. The most common way to introduce such effective terms is to consider the existence of heavy fields, with masses of order Λ, the new physics scale, decoupled from the theory at low-energies. The best known scenario of such high-energy theories accommodate the seesaw mechanism, in which the Weinberg operator arises after integrating out massive states from tree-level interactions in which they are exchanged. Such interactions reduce, at low-energy, to a four-point interaction of the form $\ell\ell\Phi\Phi$.

B. Lepton mixing

The lepton mass Lagrangian with massive Majorana neutrinos is:

$$\mathcal{L}_{\text{mass}} = -\overline{e_\alpha}_L M_{\alpha\beta}^e e_{\beta R} - \frac{1}{2} \overline{\nu_{\alpha L}^C} \mathcal{M}_{\alpha\beta} \nu_{\beta L} + \text{H.c.} \quad , \tag{29}$$

in which M^e and $\mathcal{M}$ are the charged lepton and neutrino mass matrices, respectively. The matrix M^e is completely arbitrary and can be diagonalised as in Eq. (14).

	Normal ordering		Inverted ordering	
	Best fit $\pm 1\sigma$	3σ range	Best fit $\pm 1\sigma$	3σ range
$\sin^2\theta_{12}/10^{-1}$	$3.21^{+0.18}_{-0.16}$	$2.73 \to 3.79$	$3.21^{+0.18}_{-0.16}$	$2.73 \to 3.79$
$\theta_{12}/^\circ$	$34.5^{+1.1}_{-1.0}$	$31.5 \to 38.0$	$34.5^{+1.1}_{-1.0}$	$31.5 \to 38.0$
$\sin^2\theta_{23}/10^{-1}$	$4.30^{+0.20}_{-0.18}$	$3.84 \to 6.35$	$5.96^{+0.17}_{-0.18}$	$3.88 \to 6.38$
$\theta_{23}/^\circ$	41.0 ± 1.1	$38.3 \to 52.8$	50.5 ± 1.0	$38.5 \to 53.0$
$\sin^2\theta_{13}/10^{-2}$	$2.155^{+0.090}_{-0.075}$	$1.89 \to 2.39$	$2.140^{+0.082}_{-0.085}$	$1.89 \to 2.39$
$\theta_{13}/^\circ$	$8.44^{+0.18}_{-0.15}$	$7.9 \to 8.9$	$8.41^{+0.16}_{-0.17}$	$7.9 \to 8.9$
$\delta/^\circ$	252^{+56}_{-36}	$0 \to 360$	259^{+47}_{-41}	$0 \to 31$ & $142 \to 360$
$\Delta m^2_{21}/10^{-5}$ eV^2	7.56 ± 0.19	$7.05 \to 8.14$	7.56 ± 0.19	$7.05 \to 8.14$
$\lvert\Delta m^2_{31}\rvert/10^{-3}$ eV^2	2.55 ± 0.04	$2.43 \to 2.67$	2.49 ± 0.04	$2.37 \to 2.61$

TABLE I. Gloabl fit results for mixing angles, Dirac phase and neutrino mass-squared differences, for both orderings, considering 1σ and 3σ deviations.

Regarding neutrinos, $\mathcal{M}$ is a complex 3×3 matrix and can be diagonalised through the transformation:

$$\begin{aligned} \nu_{\alpha L} = &\ (U_L^\nu)_{\alpha\beta}\, \nu'_{\beta L} \\ &\Rightarrow \quad U_L^{\nu T} \mathcal{M} U_L^\nu = \mathrm{diag}(m_1, m_2, m_3), \end{aligned} \tag{30}$$

in which m_i $(i = 1, 2, 3)$ are the neutrino masses. As in the case of quarks, this rotation produces a misalignment in the charged current interactions, namely:

$$\begin{aligned} \mathcal{L}^{\mathrm{CC}}_{\mathrm{leptons}} &= \frac{g}{\sqrt{2}} \overline{e_\alpha}_L \gamma^\mu \nu_{\alpha L} W_\mu^- + \mathrm{H.c.} \\ &= \frac{g}{\sqrt{2}} \overline{e'_\beta}_L \gamma^\mu \left(V_L^{e\dagger} U_L^\nu\right)_{\beta\rho} \nu'_{\rho L} W_\mu^- + \mathrm{H.c.}\ , \end{aligned} \tag{31}$$

where the matrix $U_{\mathrm{PMNS}} \equiv V_L^{e\dagger} U_L^\nu$ is the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix, which describes lepton mixing.

In this case, due to the Majorana character of neutrinos, one loses the freedom to rephase the three neutrino fields, which implies that not as many phases can be removed from lepton mixing matrix as in the quark case. One gets three mixing angles and three physical phases, one of Dirac type and two Majorana phases. The PMNS matrix can thus be parametrised as [5]:

$$U_{\mathrm{PMNS}} = U_\delta \mathrm{diag}(1, e^{i\alpha_1}, e^{i\alpha_2}), \tag{32}$$

in which α_1 and α_2 are the Majorana phases and:

$$\begin{aligned} U_\delta = &\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \cdot \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \\ &\cdot \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \end{aligned} \tag{33}$$

with δ the Dirac phase, $c_{ij} \equiv \cos\theta_{ij}$ and $s_{ij} \equiv \sin\theta_{ij}$ and the three mixing angles θ_{ij} being θ_{12}, θ_{13} and θ_{23}.

The evolution in time of quantum superpositions of massive neutrino states depends on the values of their different masses. Thus, neutrinos produced in specific flavour states can oscillate between flavours, when propagating freely during a period of time. This phenomenon, known as neutrino oscillations, breaks individual lepton numbers L_{e_α}, being the only experimental evidence of lepton mixing. Since they rely on quantum interference, neutrino oscillations are only sensitive to the mass-squared differences $\Delta m^2_{ij} \equiv m_i^2 - m_j^2$. Δm^2_{21} has a positive value but the sign of Δm^2_{32} is still unkown, indicating that there are two possible orderings of the neutrino mass spectrum [6]:

- Normal ordering (NO):

$$m_1 < m_2 < m_3, \tag{34}$$

- Inverted ordering (IO):

$$m_3 < m_1 < m_2. \tag{35}$$

For both orderings, $\Delta m^2_{21} \ll |\Delta m^2_{32}| \simeq |\Delta m^2_{31}|$. In Table I, we summarise the experimental results from a global fit to neutrino oscillation data [6].

C. Renormalisation group evolution of neutrino mass operators

Low-energy neutrino masses and mixing are most likely the result of some physics which decouple at a high scale Λ. Therefore, it is crucial to consider the running of physical parameters with energy, which is controlled by Renormalisation Group Equations (RGEs). Under the assumption that the SM is an effective theory valid up to the high-energy scale Λ, we are interested in the

RGEs for the matrices κ^{ab} from Λ down to the electroweak scale (commonly taken as the Z-boson mass m_Z). This is justified by the fact that neutrino mass and mixing parameters are measured at low energies. The RGEs for neutrino mass operators are first-order differential equations which describe the evolution of couplings with $t = \ln(\mu/\mu_0)$ (μ and μ_0 are the renormalisation and the low-energy scales).

Let us consider a multi-Higgs extension of the SM, in which the Higgs potential, V_{Higgs}, has the form:

$$V_{\text{Higgs}} \supset \frac{1}{2} \sum_{a,b,c,d=1}^{n_H} \lambda_{abcd} (\Phi_a^\dagger \Phi_b)(\Phi_c^\dagger \Phi_d). \tag{36}$$

In this framework, one-loop quantum corrections lead to the following RGEs for the neutrino mass effective operators [10]:

$$\begin{aligned} 16\pi^2 \frac{d\kappa^{ab}}{dt} = & \sum_{c=1}^{n_H} \left\{ \frac{1}{2} \left[\kappa^{ab} Y_c^{e\dagger} Y_c^e + \left(Y_c^{e\dagger} Y_c^e \right)^T \kappa^{ab} \right] \right. \\ & +2 \left[\kappa^{cb} Y_a^{e\dagger} Y_c^e - (\kappa^{ac} + \kappa^{ca}) Y_b^{e\dagger} Y_c^e \right] \\ & +2 \left[\left(Y_b^{e\dagger} Y_c^e \right)^T \kappa^{ac} - \left(Y_a^{e\dagger} Y_c^e \right)^T (\kappa^{cb} + \kappa^{bc}) \right] \\ & + \left[3\mathrm{Tr} \left(Y_c^{u\dagger} Y_a^u + Y_c^d Y_a^{d\dagger} \right) + \mathrm{Tr} \left(Y_c^e Y_a^{e\dagger} \right) \right] \kappa^{cb} \\ & \left. + \left[3\mathrm{Tr} \left(Y_c^{u\dagger} Y_b^u + Y_c^d Y_b^{d\dagger} \right) + \mathrm{Tr} \left(Y_c^e Y_b^{e\dagger} \right) \right] \kappa^{ac} \right\} \\ & -3g^2 \kappa^{ab} + 2 \sum_{c,d=1}^{n_H} \lambda_{cadb} \kappa^{cd}. \end{aligned} \tag{37}$$

The full renormalisation group evolution of the effective neutrino mass operators is only complete when considering the RGEs for the other parameters appearing in Eq. (37), namely, the quartic Higgs couplings λ_{abcd}, Yukawa matrices $Y_a^{d,u,e}$ and gauge coupling constants [2, 10].

V. NO-GO FOR EXACT HIGH-SCALE NEUTRINO MASS DEGENERACY

The smallness of neutrino mass-squared differences may suggest an exact high-scale neutrino mass degeneracy, which has to be lifted. In the limit of exact mass degeneracy, the neutrino mass matrix, $\mathcal{M}_0$, is diagonalised by a unitary matrix U_0. Defining m as the common degenerate mass, one has [7, 8]:

$$U_0^T \mathcal{M}_0 U_0 = m \ \mathrm{diag}(1,1,1), \tag{38}$$

from which follows [7, 8]:

$$\mathcal{M}_0 = m \ U_0^* U_0^\dagger \equiv m \ V, \tag{39}$$

in which V is a symmetric unitary matrix. The freedom in defining the phases of the charged leptons can be used to consider the third row and the third column of V real. The matrix V is parametrised in terms of two angles and one phase [8] and can, thus, be written as [7]:

$$V = \begin{pmatrix} c_\Omega & s_\Omega & 0 \\ -s_\Omega & c_\Omega & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e^{2i\alpha} & 0 & 0 \\ 0 & c_{2\psi} & -s_{2\psi} \\ 0 & -s_{2\psi} & -c_{2\psi} \end{pmatrix} \begin{pmatrix} c_\Omega & -s_\Omega & 0 \\ s_\Omega & c_\Omega & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{40}$$

In the above parametrisation, U_0 has a vanishing entry in the (31) position.

In the SM, there is a single Higgs doublet in the scalar sector, *i.e.* $n_H = 1$, and so there is an unique effective neutrino mass operator, κ. Assuming a high-scale exact mass degeneracy scenario, the Majorana mass matrix at the scale Λ is given by:

$$\mathcal{M}(\Lambda) = \frac{1}{2} \kappa(\Lambda) v^2 = \mathcal{M}_0. \tag{41}$$

We are now interested in seeing how radiative corrections modify $\kappa(\Lambda)$ as one evolves down to the low scale μ_0. For $n_H = 1$, Eq. (37) simply reads:

$$\begin{aligned} 16\pi^2 \frac{d\kappa}{dt} = & -\frac{3}{2} \left[\kappa \left(Y^{e\dagger} Y^e \right) + \left(Y^{e\dagger} Y^e \right)^T \kappa \right] \\ & + 2 \left[3\mathrm{Tr} \left(Y^{u\dagger} Y^u + Y^d Y^{d\dagger} \right) + \mathrm{Tr} \left(Y^e Y^{e\dagger} \right) \right] \kappa \\ & - 3g^2 \kappa + 2\lambda\kappa. \end{aligned} \tag{42}$$

This equation can be directly used for $\mathcal{M}$. Neglecting the electron and muon Yukawa couplings, the low-energy mass matrix is given by [7]:

$$\mathcal{M}(\mu_0) \propto \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 + \varepsilon_\tau \end{pmatrix} V \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 + \varepsilon_\tau \end{pmatrix}, \tag{43}$$

in which $\varepsilon_\tau = \frac{3}{2} \frac{y_\tau^2}{16\pi^2} \ln\left(\frac{\Lambda}{\mu_0}\right) \ll 1$. Here we have used a leading-log approximate limit in which the renormalisation group evolution of the charged lepton Yukawa couplings is neglected.

A. Neutrino masses at low energies

The eigenvalues of the hermitian matrix $\mathcal{H}_F = \mathcal{M}(\mu_0)^\dagger \mathcal{M}(\mu_0)$ are, at leading order in ε_τ, given by:

$$\begin{aligned} m_a^2 &= m'^2, \\ m_b^2 &= m'^2 \left[1 + 4\varepsilon_\tau \sin^2\psi + \mathcal{O}(\varepsilon_\tau^2) \right], \\ m_c^2 &= m'^2 \left[1 + 4\varepsilon_\tau \cos^2\psi + \mathcal{O}(\varepsilon_\tau^2) \right], \end{aligned} \tag{44}$$

in which m' stands for a corrected common neutrino mass. Given the results of Table I we are interested in solving $\Delta m_{21}^2/\Delta m_{31}^2 = \pm 0.03$. Taking into account the results of Eq. (44) and considering $\psi \in [0, 2\pi]$, we get the following cases:

1. Normal ordering: $m_1^2 < m_2^2 < m_3^2$

- $m_a^2 < m_b^2 < m_c^2$:

$$\begin{aligned} \frac{\Delta m_{21}^2}{\Delta m_{31}^2} &= \frac{m_b^2 - m_a^2}{m_c^2 - m_a^2} \simeq \tan^2\psi \\ &\Rightarrow \psi \simeq 0.05\pi \vee \psi \simeq 0.95\pi \\ &\vee \psi \simeq 1.05\pi \vee \psi \simeq 1.95\pi \end{aligned} \tag{45}$$

- $m_a^2 < m_c^2 < m_b^2$:

$$\begin{aligned} \frac{\Delta m_{21}^2}{\Delta m_{31}^2} &= \frac{m_c^2 - m_a^2}{m_b^2 - m_a^2} \simeq \cot^2\psi \\ &\Rightarrow \psi \simeq 0.45\pi \vee \psi \simeq 0.55\pi \\ &\vee \psi \simeq 1.45\pi \vee \psi \simeq 1.55\pi \end{aligned} \tag{46}$$

2. Inverted ordering: $m_3^2 < m_1^2 < m_2^2$

- $m_a^2 < m_b^2 < m_c^2$:

$$\begin{aligned} \frac{\Delta m_{21}^2}{\Delta m_{31}^2} &= \frac{m_c^2 - m_b^2}{m_a^2 - m_b^2} \simeq -\frac{\cos(2\psi)}{\sin^2\psi} \\ &\Rightarrow \psi \simeq 0.25\pi \vee \psi \simeq 0.75\pi \\ &\vee \psi \simeq 1.25\pi \vee \psi \simeq 1.75\pi \end{aligned} \tag{47}$$

- $m_a^2 < m_c^2 < m_b^2$:

$$\begin{aligned} \frac{\Delta m_{21}^2}{\Delta m_{31}^2} &= \frac{m_b^2 - m_c^2}{m_a^2 - m_c^2} \simeq \frac{\cos(2\psi)}{\cos^2\psi} \\ &\Rightarrow \psi \simeq 0.25\pi \vee \psi \simeq 0.75\pi \\ &\vee \psi \simeq 1.25\pi \vee \psi \simeq 1.75\pi \end{aligned} \tag{48}$$

We see that, in the interval $\psi \in [0, 2\pi]$, each one of the normal ordering equations has four solutions, and both set of solutions are distinct. However, in the inverted ordering scenario, both equations give similar results, since we have $\sin^2\psi \simeq \cos^2\psi$. It is clear the π-periodicity of all the obtained expressions.

In order to test the validity of Eqs. (44)-(48) we diagonalise $\mathcal{M}(\mu_0)$ numerically and obtain the set of values (α, Ω, ψ) for which the ratio $\Delta m_{21}^2/\Delta m_{31}^2$ is within the experimental range. Our numerical approach confirms the results obtained in Eqs. (45)-(48). For both mass orderings, the scatter plots indicate a clear dependence on a single parameter, namely ψ. In the interval $\psi \in [0, 2\pi]$, we obtain exactly the eight different values for ψ, in normal ordering, and four, in inverted, confirming our analytical results.

We have performed several identical analysis for different energy scales, namely $\Lambda = 10^5$, 10^{10} and 10^{15} GeV. We have found that there is no significative difference between all the results, which was expected due to the logarithmic behaviour of Λ/μ_0 present in the perturbative parameter ε_τ.

B. Mixing angles at low energies

The eigenvectors of the hermitian matrix $\mathcal{H}_F = \mathcal{M}(\mu_0)^\dagger \mathcal{M}(\mu_0)$ are given to leading order in ε_τ by the matrix:

$$U \simeq \begin{pmatrix} -\cos\Omega & -\cos\psi\sin\Omega + \mathcal{O}(\varepsilon_\tau^2) & \sin\psi\sin\Omega + \mathcal{O}(\varepsilon_\tau^2) \\ \sin\Omega & -\cos\psi\cos\Omega + \mathcal{O}(\varepsilon_\tau^2) & \sin\psi\cos\Omega + \mathcal{O}(\varepsilon_\tau^2) \\ 0 & \sin\psi & \cos\psi \end{pmatrix}, \tag{49}$$

in which the columns are the eigenvectors associated with the eigenvalues $m_{a,b,c}^2$. This matrix is the low-energy lepton mixing matrix, up to a diagonal phase matrix that assures that neutrino masses are real and positive. Let us address the mentioned cases in the normal ordering scheme, considering the mixing angle θ_{23}:

1. Normal ordering

- $m_a^2 < m_b^2 < m_c^2$:

$$\theta_{23} = \arctan(|\tan\psi\cos\Omega|) \;,\; \tan^2\psi \simeq 0.03 \;; \tag{50}$$

- $m_a^2 < m_c^2 < m_b^2$:

$$\theta_{23} = \arctan(|\cot\psi\cos\Omega|) \;,\; \cot^2\psi \simeq 0.03 \;. \tag{51}$$

Given the values of ψ and the allowed ranges for θ_{23} (see Table I), it is straightforward to conclude that the above equations have no solution. Therefore, in the NO scheme one cannot reproduce the observed neutrino mixing pattern.

2. Inverted ordering

Here, the first column of the matrix U in Eq. (49) should be identified with the third column of the leptonic mixing matrix. Since $U_{33} = 0$, we immediately obtain $\theta_{23} = 90°$, which is not allowed by data. Moreover, the fact that $\cos^2\psi \simeq \sin^2\psi$ in inverted ordering implies $\theta_{12} \simeq 45°$. Both results are inconsistent with the experimental 3σ range, and thus we conclude that no inverted ordering case can reproduce the low-energy mixing spectrum.

For each set (α, Ω, ψ) compatible with the experimentally allowed values for $\Delta m_{21}^2/\Delta m_{31}^2$ at 3σ, we have numerically obtained the corresponding mixing angles. Since the vanishing entry in the eigenvector matrix do not change under RGE running, we do not perform this analysis for the inverted ordering scenario, directly concluding that there is not any set of values for which the mass-squared differences and the mixing angles are compatible with the data. For normal ordering, the results are presented in Fig. 1, also confirming mixing angles are not consistent with experimental data.

C. Full renormalisation group evolution

The full renormalisation group evolution of the neutrino mass matrix down to m_Z is only complete when

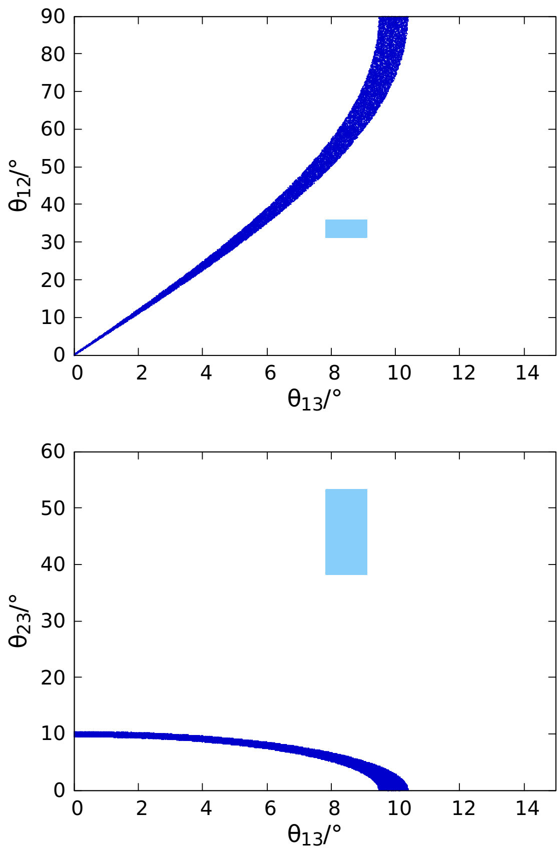

FIG. 1. Regions in the neutrino mixing planes (dark blue) obtained for the values of (α, Ω, ψ) which lead to $\Delta m^2_{21}/\Delta m^2_{31}$ compatible with experimental data at 3σ, considering $\Lambda = 10^{14}$ GeV and $m = 1$ eV, in the normal ordering scheme. The experimental 3σ range for the mixing angles is presented in light blue.

considering the remaining RGEs for Yukawa couplings, quartic scalar parameter and gauge coupling constants [2, 10]. Starting with the low-energy Yukawa, quartic scalar and gauge couplings (taken at the electroweak scale), we have performed a bottom-up RGE running, from which we got the high-energy couplings. Taking the high-energy neutrino mass matrix parametrised as in Eq. (40), it is possible to run the RGEs down to the electroweak scale, for different (α, Ω, ψ). Then, it is possible to numerically calculate the eigenvalues and eigenvectors of the low-energy neutrino mass matrix, which gives us information on the low-energy mass and mixing parameters.

Regarding low-energy masses, the full set of RGEs predicts an identical dependence on the parameter ψ of mass-squared difference ratios, just like in the leading-log approximation. The obtained results only suggest a slight influence of the parameter Ω in the normal ordering case, since the straight lines appear to curve in the (Ω, ψ) plot, which did not appear in the leading-log case. However, this is not a significative result. Concerning low-energy mixing angles, we have obtained similar results to the previous ones for both mass orderings. In normal ordering scenario, the regions in the neutrino mixing planes are not compatible with the data. In inverted ordering scheme, we have again obtained $\theta_{12} \simeq 45°$ and $\theta_{23} \simeq 90°$. Therefore, we conclude that the results of the full renormalisation group running follow the same pattern as those in the leading-log approximation, most likely due to the fact that the charged lepton Yukawa couplings do not change significantly with the RGE running.

VI. NO NO-GO FOR EXACT HIGH-SCALE NEUTRINO MASS DEGENERACY IN THE 2HDM?

With the minimal scalar sector of the SM, exact high-scale neutrino mass degeneracy is not viable. However, in SM scalar extensions, the possibility of high-scale exactly degenerate neutrinos reproducing the low-energy mass and mixing pattern is still open. We explore it in the context of the 2HDM, *i.e.* considering $n_H = 2$. More specifically we draw our study in a Z_2-constrained 2HDM (see Section III).

In the most general 2HDM, there are four possible dimension-five neutrino mass operators, namely, $\kappa^{11}\ell\ell\Phi_1\Phi_1$, $\kappa^{12}\ell\ell\Phi_1\Phi_2$, $\kappa^{21}\ell\ell\Phi_2\Phi_1$ and $\kappa^{22}\ell\ell\Phi_2\Phi_2$, which, after EWSB, contribute to Majorana neutrino masses. However, in the inert 2HDM configuration, the Majorana neutrino mass matrix solely depends on κ^{11} [9]. Assuming high-scale mass degeneracy, $\mathcal{M}(\Lambda)$ is given by:

$$\mathcal{M}(\Lambda) = \frac{1}{2}\kappa^{11}(\Lambda)v^2 = \mathcal{M}_0, \tag{52}$$

which fixes the form of $\kappa^{11}(\Lambda)$. In the considered 2HDM type, the RGE for κ^{11} reads:

$$\begin{aligned}16\pi^2\frac{d\kappa^{11}}{dt} = &-\frac{3}{2}\left[\kappa^{11}\left(Y^{e\dagger}Y^e\right) + \left(Y^{e\dagger}Y^e\right)^T\kappa^{11}\right] \\ &+2\left[3\mathrm{Tr}\left(Y^{u\dagger}Y^u + Y^dY^{d\dagger}\right) + \mathrm{Tr}\left(Y^eY^{e\dagger}\right)\right]\kappa^{11} \\ &-3g^2\kappa^{11} + 2\lambda_1\kappa^{11} + 2\lambda_5\kappa^{22}.\end{aligned} \tag{53}$$

The first terms are equivalent to the SM ones. However, the last term does not have any direct correspondence in the SM and introduces new qualitative and quantitative features. In a leading-log approximation, quantum effects introduce corrections to the low-energy neutrino mass operator of the form [9]:

$$\kappa^{11}(\mu_0) \simeq \kappa^{11}(\Lambda) - \frac{1}{16\pi^2}\beta_{\kappa^{11}}(\Lambda)\ln\left(\frac{\Lambda}{\mu_0}\right). \tag{54}$$

In this approximation, the low-scale mass matrix reads:

$$\begin{aligned}\mathcal{M}(\mu_0) \simeq \frac{1}{2}v^2\Bigg[&-\varepsilon_1\kappa^{11}(\Lambda) + \begin{pmatrix}1&0&0\\0&1&0\\0&0&I_\tau\end{pmatrix}\kappa^{11}(\Lambda)\begin{pmatrix}1&0&0\\0&1&0\\0&0&I_\tau\end{pmatrix} \\ &-\varepsilon_\lambda\kappa^{22}(\Lambda)\Bigg],\end{aligned} \tag{55}$$

in which:

$$\varepsilon_1 = \frac{1}{16\pi^2}\ln\left(\frac{\Lambda}{\mu_0}\right)\left\{2\left[3\mathrm{Tr}\left(Y^{u\dagger}Y^u + Y^dY^{d\dagger}\right)\right.\right.$$
$$\left.\left. + \ \mathrm{Tr}\left(Y^eY^{e\dagger}\right)\right] - 3g^2 + 2\lambda_1\right\},$$
$$\varepsilon_\lambda = \frac{1}{16\pi^2}\ln\left(\frac{\Lambda}{\mu_0}\right)2\lambda_5. \tag{56}$$

Assuming that the radiative evolution does not introduce new sources of CP violation, the low-energy leptonic mixing matrix U can be given by [15]:

$$U = U_0 O \ , \tag{57}$$

in which U_0 is the degenerate mixing matrix and the matrix O is an orthogonal matrix, parametrised as:

$$O = \begin{pmatrix} c_{\phi_1} & s_{\phi_1} & 0 \\ -s_{\phi_1} & c_{\phi_1} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c_{\phi_3} & 0 & s_{\phi_3} \\ 0 & 1 & 0 \\ -s_{\phi_3} & 0 & c_{\phi_3} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{\phi_2} & s_{\phi_2} \\ 0 & -s_{\phi_2} & c_{\phi_1} \end{pmatrix}. \tag{58}$$

U still diagonalises the mass matrix $\mathcal{M}_0$ and can be understood as a perturbation of the exact degeneracy limit. Under this assumption, we wish to fix the form of $\kappa^{22}(\Lambda)$ in order to have the correct neutrino masses differences. Multiplying Eq. (55) on the left (right) by U^T (U) and solving for $\kappa^{22}(\Lambda)$, we get:

$$\kappa^{22}(\Lambda) = -\frac{1}{\varepsilon_\lambda}\left[\frac{2m}{v^2}U^*\left(\frac{D_\nu}{m} + \varepsilon_1\mathbb{1}\right)U^\dagger - P_\tau\kappa^{11}(\Lambda)P_\tau\right]. \tag{59}$$

in which we have identified $P_\tau = \mathrm{diag}(1, 1, 1 + \varepsilon_\tau)$. D_ν is the diagonal neutrino mass matrix, with real non-negative masses, that can be adjusted to accommodate the correct mass-squared differences.

Choosing the first row and the first column of the matrix V to be real, it follows [8, 15]:

$$V = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_\phi & s_\phi \\ 0 & s_\phi & -c_\phi \end{pmatrix} \begin{pmatrix} c_\theta & s_\theta & 0 \\ s_\theta & -c_\theta & 0 \\ 0 & 0 & e^{i\alpha} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_\phi & s_\phi \\ 0 & s_\phi & -c_\phi \end{pmatrix}, \tag{60}$$

for which U_0 has a vanishing entry in the (13) position. This choice is based on the fact that θ_{13} is known to be a small angle. The perturbation with O is able to give rise to a non-vanishing value in the (13) position and, so, $U = U_0O$ may reproduce the required mixing angles. Considering $U = U_0O$, one gets the following expressions for $\sin^2\theta_{ij}$:

$$\sin^2\theta_{13} = \cos^2\frac{\theta}{2}\left(O_{13}\right)^2 + \sin^2\frac{\theta}{2}\left(O_{23}\right)^2, \tag{61}$$

$$\sin^2\theta_{12} = \frac{1}{1-\sin^2\theta_{13}}\left[\cos^2\frac{\theta}{2}\left(O_{12}\right)^2 + \sin^2\frac{\theta}{2}\left(O_{22}\right)^2\right], \tag{62}$$

$$\sin^2\theta_{23} = \frac{1}{1-\sin^2\theta_{13}}\left[\left(\cos\phi\sin\frac{\theta}{2}O_{13} + \sin\phi\cos\frac{\alpha}{2}O_{33}\right)^2\right.$$
$$\left. + \left(\cos\phi\cos\frac{\theta}{2}O_{23} + \sin\phi\sin\frac{\alpha}{2}O_{33}\right)^2\right]. \tag{63}$$

Using these three equations and the experimental ranges for $\sin^2\theta_{ij}$ summarised in Table I, it is possible to obtain numerically the parameter regions in which O is able to perturb the degeneracy limit in such a way that the experimental mixing angles are obtained. The 1σ and 3σ regions are presented in Fig. 2. For inverted ordering, similar regions are obtained.

The numerical results obtained by the full renormalisation group running are also presented in Fig. 2, in which the RGEs for gauge coupling constants, Yukawa matrices, quartic scalar parameters and the symmetry-allowed matrix κ^{22} [2, 10] are also considered. We consider $\kappa^{22}(\Lambda)$ given by Eq. 59 and see how does it behaves under the RGE running. Moreover, there has to be a balance when choosing the scalar quartic couplings values: on one hand, they have to satisfy the stability conditions, and on the other, we have to prevent quartic couplings to become immensely large at high energies. At low scale, we consider $\lambda_1(\mu_0) = \frac{m_h^2}{2v^2}$, $\lambda_2(\mu_0) \in [0, 1]$, $\lambda_3(\mu_0) \in [-\sqrt{\lambda_1\lambda_2}, 1]$ and $\lambda_4, \lambda_5(\mu_0) \in [-1, 1]$, in which the five parameters have to give a bounded from below potential. At high energies, the obtained five couplings have also to satisfy the bounded from below conditions and one imposes upper bounds, namely $\lambda_1, \lambda_3, \lambda_4(\Lambda) \in [-5, 5]$ and $\lambda_2, \lambda_5(\Lambda) \in [-0.1, 0.1]$. We have chosen smaller values for the couplings λ_2 and λ_5, since these couplings directly influence the running of the matrix κ^{22}. Regarding the obtained numerical results, we see that there is some dispersion of points around the regions we obtained previously, mainly due to the running of the scalar quartic couplings and the matrix κ^{22}. However, we see that they follow the same pattern of the analytical regions, namely their shapes and periodicity.

VII. CONCLUDING REMARKS

In this work we have explored in detail the possibility of exactly degenerate neutrinos at high scale reproducing the low-energy mass and mixing parameteres, within the SM and in the 2HDM. In the SM, the performed analytical and numerical analyses show that the required mass parameters cannot be simultaneously obtained with the low-energy mixing angles. However, this is not the case in the considered 2HDM. We have considered an orthogonal perturbation of the high-scale degenerate mixing matrix which is able to reproduce the low-energy mixing pattern, while fixing the RGE correction in such a way that the required mass differences are also obtained.

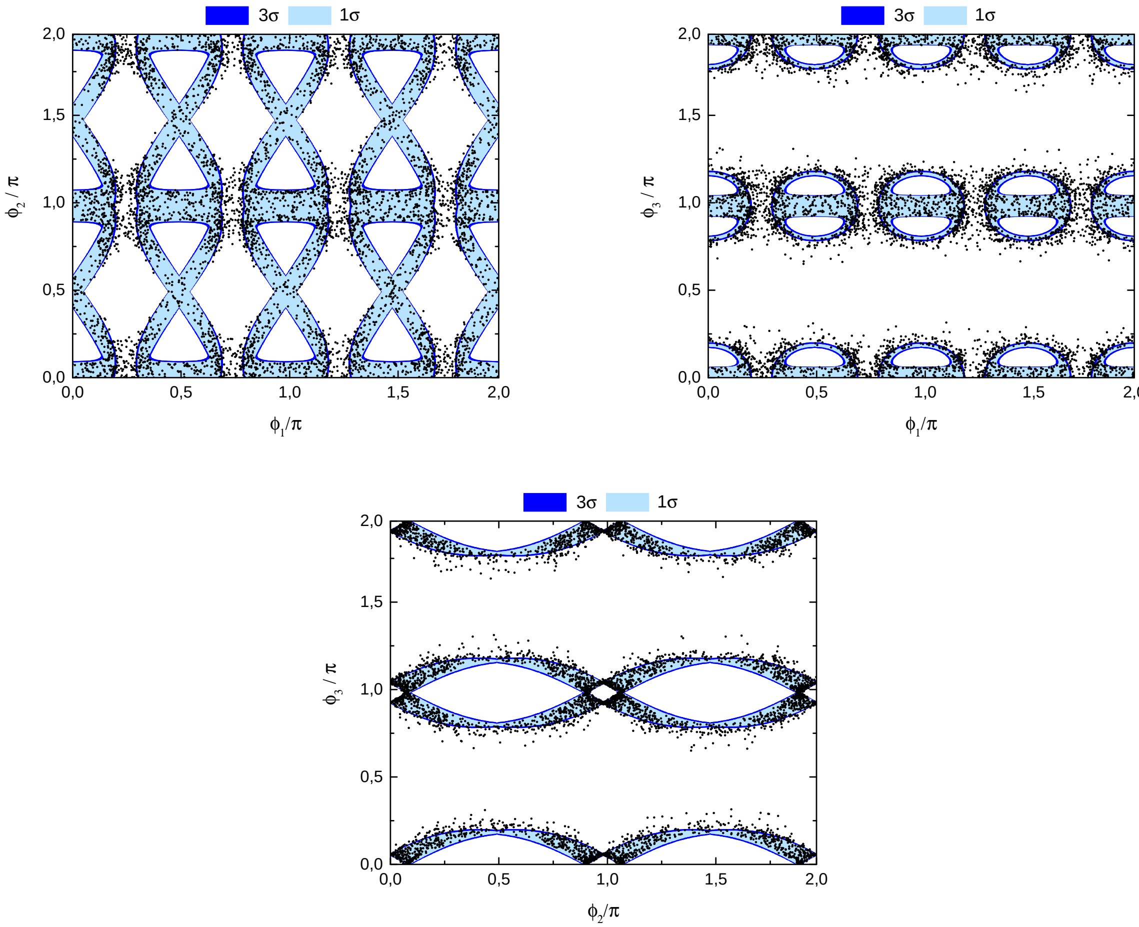

FIG. 2. Numerical results consistent with 3σ experimental range obtained from the full renormalisation group running, in the normal ordering scheme. We have considered $\Lambda = 10^{12}$ GeV, $\mu_0 = m_Z$ and $m = 1$ eV. The numerical results, in black, are presented over the analytical regions, in blue.

[1] P. Higgs. Phys. Rev. Lett., 13, 1964; P. Higgs. Phys. Rev. Lett., 145, 1966; F. Englert and R. Brout. Phys. Rev. Lett., 13, 1964; C. Guralnik, C. Hagen, and T. Kibble. Phys. Rev. Lett., 13, 1964.

[2] G. C. Branco, P. M. Ferreira, L. Lavoura, M. N. Rebelo, M. Sher, and J. P. Silva. *Theory and phenomenology of two-Higgs-doublet models.* 2011. arXiv:1106.0034v3 [hep-ph].

[3] T. D. Lee. Phys. Rev. D, 8, 1973.

[4] S. Weinberg. Phys. Rev. Lett., 43, 1979.

[5] S. Antusch, J. Kersten, M. Lindner, and M. Ratz. *Running neutrino masses, mixings and CP phases: Analytical results and phenomenological consequences.* Nucl. Phys. B, 674, 2003.

[6] P. F. de Salas, D. V. Forero, C. A. Ternes, M. Tortola, and J. W. F. Valle. *Status of neutrino oscillations 2017.* 2017. arXiv:1708.01186v1 [hep-ph].

[7] A. M. Dighe and A. S. Joshipura. *No-go for exactly degenerate neutrinos at high scale?* 2000. arXiv:hep-ph/0010079v2.

[8] G. C. Branco, M. N. Rebelo, and J. I. Silva-Marcos. *Degenerate and quasi degenerate Majorana neutrinos.* Phys. Rev. Lett., 82, 1999.

[9] A. Ibarra and C. Simonetto. *Understanding neutrino properties from decoupling right-handed neutrinos and extra Higgs doublets.* JHEP, 11, 2011.

[10] W. Grimus and L. Lavoura. *Renormalization of the neutrino mass operators in the multi-Higgs-doublet standard model.* Eur. Phys. J. C, 39, 2005.

[11] S. Glashow and S. Weinberg. Phys. Rev D, 15, 1977; E. Paschos. Phys. Rev D, 15, 1977.

[12] R. Barbieri, L. J. Hall, and V. S. Rychkov. Phys. Rev. D, 74, 2006.

[13] K. G. Klimenko. Theor. Math. Phys, 62, 1985.

[14] G. Bhattacharyya and D. Das. *Scalar sector of Two-Higgs-Doublet models: A mini-review.* 2015. arXiv:1507.06424v1 [hep-ph].

[15] G. C. Branco, M. N. Rebelo, J. I. Silva-Marcos, and D. Wegman. *Quasidegeneracy of Majorana neutrinos and the origin of large leptonic mixing.* Phys. Rev. D, 91, 2015.