Neutrino masses in the left-right symmetric model with flavour symmetries

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In this thesis, after reviewing the SM, we will describe the general features of left-right symmetric models (LRSMs). Afterwards, we focus on a particular LRSM, in which flavon fields are added and a $A_4 \times Z_2$ discrete flavour symmetry is imposed. Several vacuum configurations for the flavon fields are considered and tested in the light of the most recent neutrino oscillation data. The possibility of having spontaneous charge-parity symmetry violation (SCPV) is also investigated. We conclude that SCPV is indeed possible for some flavon vacuum alignments, and we present the corresponding predictions for several physical parameters which could be tested by ongoing and future neutrino experiments.

I. INTRODUCTION

The advent of Quantum Mechanics (QM) in the turn of the twentieth century opened an entire new field of study in physics that was just as much controversial as strange. Quickly, QM gave way to new, more robust, theories that provided a more suitable description of reality, and soon, the study of elementary particles and their interactions (particle physics) became one of the main areas in physics. The most important landmark happened in the second half of the twentieth century, with the formulation of the Standard Model (SM) of particle physics.

The reason why the SM represents such an important milestone in physics is due to its many successes - until recently, most experiments agreed with high precision with SM predictions. The last major achievement in the field has been the discovery of the Higgs boson, also predicted by the SM.

Despite all successes of the electroweak theory, there are still some questions to which the SM does not provide a satisfactory answer. Namely, the discovery of neutrino oscillations (awarded the Nobel prize in 2015) brought forth an important dilemma for the SM: neutrinos are massive, contrary to the model's prediction. Nevertheless, the SM is an excellent basilar theory upon which many extensions can be constructed in order to solve some (or all) of the SM shortcomings. For instance, the addition of right-handed neutrinos solves the problem of neutrino masses, incorporating the possibility of neutrino oscillations in the theory. However, aesthetically, there is also a deep chirality asymmetry in the SM that has little foundation: left- and right-handed particles are not treated equally. Still, this could be just due to different energy scales in which the parity and the electroweak symmetry are broken. This idea is in the basis of left-right symmetric models (LRSMs).

First, we will go through the well known SM. Afterwards, the rich phenomenology of a LRSM will be studied. The mass spectrum and mixing of gauge and scalar bosons will be determined. Furthermore, charged currents and fermionic mixing will also be covered. In order to increase the level of predictability in the fermion mass and mixing sector, we impose flavour (horizontal) symmetries. As such, the narrower focus of the work will be the study of flavour symmetries in a minimal LRSM (MLRM) and the study of lepton mixing in light of recent neutrino oscillation data.

II. THE STANDARD MODEL

The SM is a field theory described by fields and a gauge group. While these two ingredients are intertwined, they serve different purposes: while the field content establishes which particles exist and are able to interact, the gauge group defines how they interact. Ultimately, changing the particle content allows for more (or different) particles, whereas changing the gauge group forces a new perception as to how the fundamental forces of nature act (at least at a higher energy).

A. Electroweak Sector of the SM

The SM gauge group is $SU(3)_c \times SU(2)_L \times U(1)_Y$, where the subscripts are for colour, left-handed, and hypercharge, respectively. Throughout this document, we will not address the $SU(3)_c$ symmetry, dealing only with the electroweak sector of the theory. There are three families of fermions (quarks, Q, and leptons, l), where left-handed fields are placed in $SU(2)_L$ doublets, whereas the right-handed ones are singlets. The theory also features a $SU(2)_L$ Higgs doublet, ϕ. In summary, the fields and their transformations properties are:

$$
\begin{aligned}
&Q_{iL} = \begin{pmatrix} u_i \\ d_i \end{pmatrix}_L : (2, 1/3), \qquad \begin{matrix} u_{iR} : & (1, 4/3), \\ d_{iR} : & (1, -2/3), \end{matrix} \\
&l_{iL} = \begin{pmatrix} \nu_i \\ e_i \end{pmatrix}_L : (2, -1), \qquad l_{iR} : (1, -2), \\
&\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} : (2, -1).
\end{aligned} \tag{1}
$$

where the subscripts L and R refer to left and right-handed respectively, and $i = 1, 2, 3$ denote the three families. The numbers in parenthesis, $(\dim_L, Y)$, denote the transformation properties under the gauge group. Namely, $\dim_L$ is the dimension of the object with respect to the $SU(2)_L$ symmetry, and Y is the hypercharge. The electric charge of a particle (Q) can be computed through the relation

$$Q = \frac{1}{2}(Y + 2T_3) \tag{2}$$

where T_3 is the third component of isospin.

In order to ensure local invariance under the $SU(2)_L \times U(1)_Y$ group, the ordinary derivative (∂_μ) must be replaced by the covariant derivative (D_μ). For a generic field Ψ:

$$\partial_\mu \Psi \to D_\mu \Psi = \left[\partial_\mu + i\frac{g}{2} W_\mu^a \tau^a + i\frac{g'}{2} B_\mu\right] \Psi, \tag{3}$$

where four gauge bosons must be introduced, namely $W_\mu^{a=1,2,3}$ and B_μ for the $SU(2)$ and $U(1)$ symmetries respectively. In (3), g and g' are the coupling constants for $SU(2)_L$ and $U(1)_Y$, and τ^a are the Pauli matrices.

B. The Scalar Potential and Spontaneous Symmetry Breaking

Empirically, it is known that the vacuum has no charge or spin because of its charge neutrality and isotropy. Thus, while it is not possible for fermions or non-scalar bosons to have a non-vanishing vacuum expectation value (VEV), this may not be true for neutral scalar fields. For a general scalar field, the VEV corresponds to the minimal energy configuration of the field and, as such, is computed through the minimization of the scalar potential, $V(\phi)$:

$$V(\phi) = \mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2. \tag{4}$$

The only possibility where the VEV is non-zero and the potential is bounded from below is $\mu^2 < 0$ and $\lambda > 0$. In this case, the scalar field can be described by its VEV along with oscillations around it (the physical Higgs particle h): $\phi = v + h(x)$, where v is the resulting VEV,

$$v = e^{i\theta} \sqrt{\frac{-\mu^2}{\lambda}}, \tag{5}$$

in which θ is a phase that can be taken zero. This is how Spontaneous Symmetry Breaking (SSB) occurs, the potential is symmetric under the gauge group but the vacuum is not. After SSB, bilinear (mass) terms arise in the Lagrangian.

In particular, for neutral gauge boson masses, we need to diagonalise the mass matrix

$$M_0^2 = \frac{1}{2} v^2 \begin{bmatrix} g^2 & -gg' \\ -gg' & g'^2 \end{bmatrix}, \tag{6}$$

given in the basis (W_μ^3, B_μ). Its eigenvectors are:

$$\begin{aligned} A_\mu &= \sin\theta_W W_\mu^3 + \cos\theta_W B_\mu, \\ Z_\mu &= \cos\theta_W W_\mu^3 - \sin\theta_W B_\mu, \end{aligned} \tag{7}$$

which correspond to the physical gauge bosons. The angle θ_W, known as Weinberg or weak angle, is determined by enforcing that A_μ is the eigenstate of mass zero:

$$\tan\theta_W = \frac{g'}{g}. \tag{8}$$

Defining

$$W_\mu^\pm = \frac{W_\mu^1 \mp i W_\mu^2}{\sqrt{2}}, \tag{9}$$

we find that the full boson mass spectrum is

$$m_h = \frac{1}{2} v \sqrt{\lambda} = \sqrt{-2\mu^2}, \tag{10}$$

$$m_{W_\mu^\pm} = \sqrt{\frac{1}{2} g^2 v^2}, \tag{11}$$

$$m_Z = v \sqrt{\frac{g^2 + g'^2}{2}} = \frac{gv}{\sqrt{2} c_w}, \tag{12}$$

$$m_A = 0. \tag{13}$$

C. Fermion Weak Interactions: Charged and Neutral Currents

The lepton-gauge interaction Lagrangian in the SM is given by

$$\begin{aligned} \mathcal{L}^l_{\text{int}} &= \frac{i}{2} \overline{l_{iL}} \left(i g \not{W}^i \tau^i - i g' \not{B} \right) l_{iL} - g' \overline{l_{iR}} \not{B} l_{iR} = \\ &- \frac{1}{2} \begin{pmatrix} \overline{\nu_{iL}} & \overline{l_{iL}} \end{pmatrix} \begin{bmatrix} -g \not{W}^3 + g' \not{B} & g \left(\not{W}^1 - i \not{W}^2 \right) \\ g \left(\not{W}^1 + i \not{W}^2 \right) & g \not{W}^3 + g' \not{B} \end{bmatrix} \begin{pmatrix} \nu_{iL} \\ l_{iL} \end{pmatrix} \\ &- g' \overline{l_{iR}} \not{B} l_{iR}, \end{aligned} \tag{14}$$

where $\not{D} \equiv \gamma^\mu D_\mu$, and γ^μ are the Dirac matrices. The off-diagonal terms describe the electroweak charged current (CC) Lagrangian, which for leptons is

$$\begin{aligned} \mathcal{L}^l_{\text{CC}} =& \frac{g}{\sqrt{2}} \left(\overline{\nu_{iL}} \gamma^\mu W_\mu^- l_{iL} + \overline{l_{iL}} \gamma^\mu W_\mu^+ \nu_{iL} \right) \\ \equiv& \frac{g}{\sqrt{2}} j^{l,\mu}_{\text{CC}} W_\mu^- + \text{h.c.}. \end{aligned} \tag{15}$$

On the other hand, the diagonal terms describe the neutral currents (NC), since they involve only neutral gauge bosons. Using the definition (7), the NC Lagrangian becomes:

$$\begin{aligned} \mathcal{L}^l_{\text{NC}} = -&\frac{1}{2} \overline{l_{iL}} \left[(g c_W - g' s_W) \gamma^\mu Z_\mu + (g s_W + g' c_W) \gamma^\mu A_\mu \right] l_{iL} \\ &\frac{1}{2} \overline{\nu_{iL}} (g c_W + g' s_W) \gamma^\mu Z_\mu \nu_{iL} \\ &+ g' \overline{l_{iR}} (c_W \gamma^\mu A_\mu - s_W \gamma^\mu Z_\mu) l_{iR}. \end{aligned} \tag{16}$$

D. Fermion Masses and Quark Mixing

SSB allows not only for massless gauge bosons to become massive, but also generates fermion masses. Via SSB, we can extract fermion masses from the Yukawa terms. Namely, we obtain

$$\begin{aligned}
&\mathcal{L}_{\text{Yuk}} \equiv \\
&-Y^d_{ij}\overline{Q}_{iL}\phi d_{jR} - Y^u_{ij}\overline{Q}_{iL}\tilde{\phi} u_{jR} - Y^l_{ij}\overline{l}_{iL}\phi l_{jR} + \text{h.c.} \xrightarrow{SSB} \\
&-\frac{1}{\sqrt{2}}(v+h)\left(Y^d_{ij}\overline{d_{iL}}d_{jL} + Y^u_{ij}\overline{u_{iL}}u_{jL} + Y^l_{ij}\overline{l_{iL}}l_{jL}\right) + \text{h.c.} \\
&= -M^u_{ij}\overline{u_{iL}}u_{jL} - M^d_{ij}\overline{d_{iL}}d_{jL} - M^l_{ij}\overline{l_{iL}}l_{jL} + \text{h.c.} + \mathcal{L}^f_{\text{int}},
\end{aligned} \tag{17}$$

where $M^{u,d,l} = vY^{u,d,l}$ are the mass matrices for fermionic fields, and $\mathcal{L}^f_{\text{int}}$ contains the fermion-Higgs interaction terms. As, in general, $Y^{u,d,l}$ are 3×3 complex matrices, there is no reason to assume that $M^{u,d,l}$ are diagonal. Hence, a rotation is needed to define the mass eigenstates (physical basis):

$$\psi_{iL} \to \left(V^\psi_L\right)_{ij}\psi_{jL} \equiv \psi'_{jL} \quad \psi_{iR} \to \left(V^\psi_R\right)_{ij}\psi_{jR} \equiv \psi'_{jR}, \tag{18}$$

where ψ denotes the fermion fields u, d, and l; and the ψ' denotes the rotated fields. The mass matrices become:

$$\begin{aligned}
{V^u_L}^\dagger M^u V^u_R =& \text{diag}\left(m_u, m_c, m_t\right), \\
{V^d_L}^\dagger M^d V^d_R =& \text{diag}\left(m_d, m_s, m_b\right), \\
{V^l_L}^\dagger M^l V^l_R =& \text{diag}\left(m_e, m_\mu, m_\tau\right),
\end{aligned} \tag{19}$$

where m_ψ denotes the mass of particle ψ. The rotations of up- and down-type quarks will affect the quark-charged current in such a way that the quark-gauge interactions are not diagonal:

$$\begin{aligned}
\mathcal{L}^q_{CC} =& \frac{g}{\sqrt{2}}\overline{u'_{iL}}\left(V^u_L\right)^*_{ji}\gamma^\mu\left(V^d_L\right)_{jk}d'_{kL}W^-_\mu + \text{h.c.} \\
=& \frac{g}{\sqrt{2}}\overline{u'_{iL}}\gamma^\mu\left({V^u_L}^\dagger V^d_L\right)_{ij}d'_{jL}W^-_\mu + \text{h.c.},
\end{aligned} \tag{20}$$

giving rise to the Cabibbo-Kobayashi-Maskawa (CKM) matrix, V_{CKM}:

$$V_{\text{CKM}} = {V^u_L}^\dagger V^d_L, \tag{21}$$

which relates quark mass and weak eigenstates. Leptonic CCs are not affected since the absence of neutrino mass terms in the theory allows us to rotate charged leptons to any basis.

III. THE LEFT-RIGHT SYMMETRIC MODEL

The SM is a theory built to fit experiment. The nature of weak decays motivated the chiral essence of the theory: the phenomenology of left- and right-handed particles is clearly and abundantly different. However, there is no reason for this to be the case, and it would be natural to assume that, at a higher energy, left and right chirality are at equal footing. This is the idea behind left-right symmetric models: to extend the gauge group in such a way that at a higher energy, the theory is parity symmetric and that parity is broken in such a way that a SM-like paradigm can be achieved. Thus, parity violation is no longer an *ad hoc* assumption based on empirical evidence, but rather a feature arising from the vacuum configuration of the theory. Moreover, neutrino masses and mixing are automatically accounted for in LRSMs.

A. Electroweak Sector of the LRSM

The gauge group of the LRSM is $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, where $B-L$ stands for the difference between baryon and lepton numbers. There are several versions of this model in the literature [1, 2] which differ in their scalar content. Usually, motivated by the concept of this theory, a discrete left-right (LR) symmetry is imposed. To do so, it is necessary for the scalar sector to have either a right-handed doublet or triplet to break the $SU(2)_R$ symmetry, and a left-handed counterpart for the symmetry to be realised. However, the field content of the LRSM requires a Higgs bidoublet for Dirac terms to exist. Since the discrete LR symmetry is optional, there is no need for the extra left-handed scalar field. This simplifies the scalar potential and reduces the number of extra parameters and particles. Since most theories that work under the assumption of the discrete LR symmetry also assume a negligible VEV for that field, the underlying physics is very similar in both cases. In this section, we work under the paradigm of no imposed LR discrete symmetry, and we consider the existence of a right-handed scalar triplet resulting in the field content

$$\begin{aligned}
&Q_{iL} \equiv \begin{pmatrix} u_i \\ d_i \end{pmatrix}_L : (2,1,1/3), \quad Q_{iR} \equiv \begin{pmatrix} u_i \\ d_i \end{pmatrix}_R : (1,2,1/3) \\
&l_{iL} \equiv \begin{pmatrix} \nu_i \\ e_i \end{pmatrix}_L : (2,1,-1), \quad l_{iR} \equiv \begin{pmatrix} \nu_i \\ e_i \end{pmatrix}_R : (1,2,-1) \\
&\Phi \equiv \begin{pmatrix} \phi^0_1 & \phi^+_2 \\ \phi^-_1 & \phi^0_2 \end{pmatrix} : (2,2,0), \quad \Delta_R \equiv \begin{pmatrix} \frac{\delta^+_R}{\sqrt{2}} & \delta^{++}_R \\ \delta^0_R & -\frac{\delta^+_R}{\sqrt{2}} \end{pmatrix} : (1,3,2).
\end{aligned} \tag{22}$$

The subscripts L and R denote left- and right-handed. $i = 1, 2, 3$ identifies the fermion family, and the superscripts of the scalar fields are their electric charge. The numbers in parenthesis characterise the transformation properties under the theory's gauge group. ϕ_1 and ϕ_2 are the components of the Higgs bidoublet Φ.

The electric charge can be computed through the hypercharge ($Y = B - L$), where T_3 are the eigenvalues of the $SU(2)$ generators:

$$Q = T_{3L} + T_{3R} + \frac{B-L}{2}. \tag{23}$$

Under gauge transformations, the fermion fields transform as:

$$\Psi'_L = U_L U_Y \Psi_L, \qquad \Psi'_R = U_R U_Y \Psi_R, \tag{24}$$

where $U_{L,R}$ and U_Y are the $SU(2)_{L,R}$ and $U(1)_Y$ transformations, respectively, and $g_{L,R}$ and g' are the couplings of $SU(2)_L$, $SU(2)_R$, and $U(1)_{B-L}$. As before, $\vec{\tau}$ are the $SU(2)$ generators:

$$U_{L,R} = e^{-ig_{L,R}(\vec{\tau}/2)\vec{\theta}(x)}, \qquad U_Y = e^{-ig'(Y/2)\Theta(x)}, \tag{25}$$

where $\vec{\theta}(x)$ and $\Theta(x)$ show the local character of the transformations. To ensure gauge invariance, one introduces seven gauge fields: $(W^{1,2,3}_{L,R})_\mu$, and B_μ. The covariant derivates then read

$$\begin{aligned} D_\mu \Psi'_L &= \left(\partial_\mu - ig_L \tfrac{\vec{\tau}}{2}\vec{W}_{L\mu} - ig'\tfrac{Y}{2}B_\mu\right)\Psi_L, \\ D_\mu \Psi'_R &= \left(\partial_\mu - ig_R \tfrac{\vec{\tau}}{2}\vec{W}_{R\mu} - ig'\tfrac{Y}{2}B_\mu\right)\Psi_R. \end{aligned} \tag{26}$$

We are now able to construct the fermion-gauge interaction Lagrangian

$$\mathcal{L}_f = \overline{\psi}_L \gamma\mu \left(i\partial_\mu + g_L \frac{\tau^a}{2} W^a_{L\mu} + g' \frac{Y}{2} B_\mu\right)\psi_L + (L \to R), \tag{27}$$

and the gauge-gauge interaction Lagrangian

$$\mathcal{L}_g = -\frac{1}{4} W^{\mu\nu}_{Li} W_{Li\mu\nu} - \frac{1}{4} W^{\mu\nu}_{Ri} W_{Ri\mu\nu} - \frac{1}{4} B^{\mu\nu} B_{\mu\nu}, \tag{28}$$

where, denoting the structure constants of the $SU(2)$ group by f_{abc}, we have

$$\begin{aligned} W^{\mu\nu}_{L,Ri} &= \partial^\mu W^\nu_{L,Ri} - \partial^\nu W^\mu_{L,Ri} - g_{L,R} f_{jki} W^\mu_{L,Rj} W^\nu_{L,Rk}, \\ B^{\mu\nu} &= \partial^\mu B^\nu - \partial^\nu B^\mu. \end{aligned} \tag{29}$$

As usual, fermion (Dirac) masses come from the Yukawa Lagrangian after EWSB via the Higgs bidoublet Φ. Since these terms couple left- and right-handed fields to a scalar field, the latter must transform as a left field on the left, and as a right field on the right ($\Phi' = U_L \Phi U_R^\dagger$), hence the bidoublet structure. As such, we can now construct the Dirac Yukawa Lagrangian

$$\begin{aligned} \mathcal{L}_{\text{Dirac}} = &- \overline{l_{iL}} \left(Y^l_{ij}\, \Phi + \tilde{Y}^l_{ij}\, \tilde{\Phi}\right) l_{jR} \\ &- \overline{Q_{il}} \left(Y^q_{ij}\, \Phi + \tilde{Y}^q_{ij}\, \tilde{\Phi}\right) Q_{jR} + \text{h.c.}, \end{aligned} \tag{30}$$

where $\tilde{\Phi}$ also transforms as $(2, 2, 0)$ under the LRSM gauge group, and can be defined as

$$\tilde{\Phi} = \tau_2 \Phi^* \tau_2 = \begin{pmatrix} \phi_2^{0*} & -\phi_1^+ \\ -\phi_2^- & \phi_1^{0*} \end{pmatrix}. \tag{31}$$

In order to build the kinetic terms for scalars, we need to account not only for the bidoublet, but also for the triplet Δ_R. However, since we are using a 2×2 representation for this field, we need to make use of traces to construct singlet-invariant terms. Moreover, it can be easily shown that we do not need to account for $\tilde{\Phi}$ as its terms would overlap with those of Φ (recall the cyclic property of the trace and the fact that any squared Pauli matrix is the identity). As such, the kinetic part of the scalar Lagrangian will be

$$\mathcal{L}^{\text{kin}}_\Phi = \text{Tr}\left[(D_\mu \Delta_R)^\dagger (D^\mu \Delta_R)\right] + \text{Tr}\left[(D_\mu \Phi)^\dagger (D^\mu \Phi)\right], \tag{32}$$

where the covariant derivatives of the scalar fields are

$$\begin{aligned} D_\mu \Phi &= \partial_\mu \Phi - ig_L W^a_{L\mu} \frac{\tau^a}{2} \Phi + ig_R \Phi \frac{\tau^a}{2} W^a_{R\mu}, \\ D_\mu \Delta_R &= \partial_\mu \Delta_R - ig_R \left[\frac{\tau^a}{2} W^a_{R\mu}, \Delta_R\right] - ig' B_\mu \Delta_R, \end{aligned} \tag{33}$$

in which $[A, B] = AB - BA$.

B. Spontaneous Symmetry Breaking

The most general potential one can write in the LRSM can be divided into Φ-Φ interactions (V_Φ), Δ_R-Δ_R interactions (V_{Δ_R}), and crossed terms (V_{Φ,Δ_R}):

$$V(\Phi, \Delta_R) = V_\Phi(\Phi) + V_{\Delta_R}(\Delta_R) + V_{\Phi,\Delta_R}(\Phi, \Delta_R). \tag{34}$$

Explicitly, V_Φ can be written as

$$\begin{aligned} V_\Phi = &- \mu_1^2 \text{Tr}[\Phi^\dagger \Phi] - \mu_2^2 \text{Tr}[\Phi^\dagger \tilde{\Phi}] - \mu_2^{2*} \text{Tr}[\tilde{\Phi}^\dagger \Phi] \\ &+ \lambda_1 \text{Tr}[\Phi^\dagger \Phi]^2 + \lambda_2 \text{Tr}[\Phi^\dagger \tilde{\Phi}]^2 + \lambda_2^* \text{Tr}[\tilde{\Phi}^\dagger \Phi]^2 \\ &+ \lambda_3 \text{Tr}[\Phi^\dagger \tilde{\Phi}]\ \text{Tr}[\tilde{\Phi}^\dagger \Phi] + \lambda_4 \text{Tr}[\Phi^\dagger \Phi]\ \text{Tr}[\Phi^\dagger \tilde{\Phi}] \\ &+ \lambda_4^* \text{Tr}[\Phi^\dagger \Phi]\ \text{Tr}[\tilde{\Phi}^\dagger \Phi], \end{aligned} \tag{35}$$

while V_{Δ_R} and $V_{\Delta_R,\Phi}$ are given by:

$$\begin{aligned} V_{\Delta_R} = &-\mu_3^2 \text{Tr}[\Delta_R^\dagger \Delta_R] + \alpha_1 \text{Tr}[\Delta_R^\dagger \Delta_R]^2 \\ &+ \alpha_2 \text{Tr}[\Delta_R^\dagger \Delta_R^\dagger]\ \text{Tr}[\Delta_R \Delta_R], \end{aligned} \tag{36}$$

$$\begin{aligned} V_{\Delta_R,\Phi} = &\rho_1 \text{Tr}[\Phi^\dagger \Phi]\ \text{Tr}[\Delta_R^\dagger \Delta_R] + \rho_2 \text{Tr}[\Phi^\dagger \Phi \Delta_R \Delta_R^\dagger] \\ &+ \rho_3 \text{Tr}[\Phi^\dagger \tilde{\Phi}]\ \text{Tr}[\Delta_R^\dagger \Delta_R] + \rho_3^* \text{Tr}[\tilde{\Phi}^\dagger \Phi]\ \text{Tr}[\Delta_R^\dagger \Delta_R], \end{aligned} \tag{37}$$

respectively. In the above equations, μ_i have dimensions of mass while λ_i, α_i, and ρ_i are dimensionless coefficients. Although there are other invariants of dimension four one may construct with this given scalar sector, it can be shown that they reduce to linear combinations of the terms included in $V(\Phi)$. Hence, as all of the coefficients of the potential are free parameters, it is possible to ignore all redundant terms and restrict ourselves to those presented above.

We must now assign a VEV to all neutral scalar fields, namely:

$$\Phi = \begin{pmatrix} \phi_1^0 & \phi_2^+ \\ \phi_1^- & \phi_2^0 \end{pmatrix} \xrightarrow{\text{SSB}} \langle\Phi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} v_1 & 0 \\ 0 & e^{i\theta}v_2 \end{pmatrix}, \tag{38}$$

and

$$\Delta_R = \begin{pmatrix} \delta_R^+/\sqrt{2} & \delta_R^{++} \\ \delta_R^0 & -\delta_R^+/\sqrt{2} \end{pmatrix} \xrightarrow{\text{SSB}} \langle\Delta_R\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 \\ v_R & 0 \end{pmatrix}. \tag{39}$$

As Δ_R is connected to unseen physics and is the field responsible for breaking the new $SU(2)_R$ symmetry, its VEV must be high enough to ensure that its effects at low energies are small. Therefore, we will consider the limit $v_R \gg v_1, v_2$. We introduce three quantities which will be useful later on:

$$v = \sqrt{v_1^2 + v_2^2}, \quad \tan\beta = \frac{v_1}{v_2}, \quad \epsilon = \frac{v}{v_R}. \tag{40}$$

The vacuum configuration can be determined by minimizing the scalar potential given in (35)-(37). For simplicity, we will consider $\theta = 0$, leading to the minimization conditions:

$$\frac{\partial V}{\partial v_1} = 0; \qquad \frac{\partial V}{\partial v_2} = 0; \qquad \frac{\partial V}{\partial v_R} = 0. \tag{41}$$

After computing the VEV, all scalar fields may be described by a physical particle oscillating around the vacuum. Inserting this into the Lagrangian, and keeping only the bilinear terms, we get the boson mass terms. The complex neutral scalar fields are defined as the normalised sum of the real and imaginary components,

$$\phi^0 = \frac{1}{\sqrt{2}}\left(\phi^{0r} + i\phi^{0i}\right), \tag{42}$$

where ϕ is general for ϕ_1^0, ϕ_2^0 and δ_R^0. There will be mixing between the singly-charged particles, the real components, and the imaginary components. The doubly-charged particle in this model is already physical. The exact form of the mixing matrices can be seen in the thesis. By diagonalising them, one obtains the scalar boson mass spectrum:

$$\begin{aligned}
m^2_{\delta_R^{++}} &= \frac{1}{2}\rho_2 v^2\cos 2\beta + 2\alpha_2 v_R^2, \\
m^2_{H^\pm} &= \frac{1}{8}\rho_2 \sec 2\beta\left(v^2\cos 4\beta + v^2 + 4v_R^2\right), \\
m^2_{A^0} &= v^2\left(\lambda_3 - 2\lambda_2\right) + \frac{1}{4}\rho_2 v_R^2\sec(2\beta), \\
m^2_{h^0} &\simeq \frac{v^2}{2}\left[4\lambda_4\sin 2\beta + 2\left(\lambda_1+\lambda_2\right) + \lambda_3 - \cos 4\beta\left(2\lambda_2+\lambda_3\right)\right], \\
m^2_{H_1^0} &\simeq \frac{1}{4}v_R^2\sec 2\beta\left[4\epsilon^2\cos^3 2\beta\left(2\lambda_2+\lambda_3\right) + \rho_2\right], \\
m^2_{H_2^0} &\simeq \alpha_1 v_R^2.
\end{aligned} \tag{43}$$

We also find two charged Goldstone bosons, $G^\pm_{1,2}$, and a neutral one, G_1^0. Denoting the $h^0 - H_1^0$ mixing angle as α, the scalar fields can be written in terms of the physical fields as

$$\begin{aligned}
\phi_1^\pm &= \frac{\sqrt{2}H^\pm\sin\beta}{\sqrt{2+\epsilon^2\cos^2(2\beta)}} + G_1^\pm\cos\beta - \frac{\epsilon G_2^\pm\sin\beta\cos(2\beta)}{\sqrt{2+\epsilon^2\cos^2(2\beta)}}, \\
\phi_2^\pm &= \frac{\sqrt{2}H^\pm\cos\beta}{\sqrt{2+\epsilon^2\cos^2(2\beta)}} - G_1^\pm\sin\beta - \frac{\epsilon G_2^\pm\cos\beta\cos(2\beta)}{\sqrt{2+\epsilon^2\cos^2(2\beta)}}, \\
\delta_R^\pm &= \frac{\epsilon H^\pm\cos(2\beta)}{\sqrt{2+\epsilon^2\cos(2\beta)}} + \frac{\sqrt{2}G_2^\pm}{\sqrt{2+\epsilon^2\cos(2\beta)}}, \\
\phi_1^0 &\simeq -h^0\sin\alpha + H_1^0\cos\alpha + i\left(A^0\sin\beta + G_1^0\cos\beta\right), \\
\phi_2^0 &\simeq h^0\cos\alpha + H_1^0\sin\alpha - i\left(A^0\cos\beta - G_1^0\sin\beta\right), \\
\delta_R^0 &\simeq H_2^0 + iG_2^0.
\end{aligned} \tag{44}$$

For gauge bosons, we find

$$\begin{aligned}
\mathcal{L}_M^g = &\left(W_L^{+\mu} \;\; W_R^{+\mu}\right)\tilde{M}_W^2\begin{pmatrix} W_{L\mu}^- \\ W_{R\mu}^- \end{pmatrix} + \text{h.c.} + \\
&+\tfrac{1}{2}\left(W_{3L}^\mu \;\; W_{3R}^\mu \;\; B^\mu\right)\tilde{M}_0^2\begin{pmatrix} W_{3L}^\mu \\ W_{3R}^\mu \\ B^\mu \end{pmatrix}
\end{aligned} \tag{45}$$

where we have chosen to rotate the $W^{1,2}$ fields into the charge eigenstates

$$W_{L,R}^{\mu\pm} = \frac{W_{L,R}^{1\mu} \mp W_{L,R}^{2\mu}}{\sqrt{2}}. \tag{46}$$

The mass matrices of (45) are:

$$\tilde{M}_W^2 = \frac{1}{4}\begin{pmatrix} g_L^2 v^2 & -g_L g_R v^2\sin 2\beta e^{i\theta} \\ -g_L g_R v^2\sin 2\beta e^{-i\theta} & g_R^2\left(v^2 + 2v_R^2\right) \end{pmatrix} \tag{47}$$

and

$$\tilde{M}_0^2 = \frac{1}{2}\begin{pmatrix} \frac{1}{2}g_L^2 v^2 & -\frac{1}{2}g_L g_R v^2 & 0 \\ -\frac{1}{2}g_L g_R v^2 & \frac{1}{2}g_R^2\left(v^2+4v_R^2\right) & -2g_R g' v_R^2 \\ 0 & -2g_R g' v_R^2 & 2g'^2 v_R^2 \end{pmatrix}. \tag{48}$$

Although in most cases the phase θ is assumed to be zero, it was intentionally left general here to show its irrelevant nature as far as gauge boson masses are concerned.

These mass matrices given above can be diagonalised through unitary rotations, defining mixing angles in such a way that

$$\begin{pmatrix} W_L^{\pm\mu} \\ W_R^{\pm\mu} \end{pmatrix} = \begin{pmatrix} \cos\zeta & -\sin\zeta \\ \sin\zeta & \cos\zeta \end{pmatrix} \begin{pmatrix} W^{\pm\mu} \\ W'^{\pm\mu} \end{pmatrix} \tag{49}$$

where

$$\tan 2\zeta = \frac{2 g_L g_R \epsilon^2}{2 g_R^2 + (g_R^2 - g_L^2)\epsilon^2}. \tag{50}$$

The mixing between W and W' can be neglected in most cases, i.e., one can safely assume $W^{\pm} \sim W_L^{\pm}$ and $W'^{\pm} \sim W_R^{\pm}$. Finding the physical neutral gauge boson states requires two mixing angles. To a good approximation,

$$\begin{pmatrix} W_L^{3\mu} \\ W_R^{3\mu} \\ B^{\mu} \end{pmatrix} = \begin{pmatrix} c_W & \mathcal{O}(\epsilon^2) & s_W \\ -s_W\cos\varphi & -\sin\varphi & c_W\cos\varphi \\ -s_W\sin\varphi & \cos\varphi & c_W\sin\varphi \end{pmatrix} \begin{pmatrix} Z^{\mu} \\ Z'^{\mu} \\ A^{\mu} \end{pmatrix}, \tag{51}$$

where $c_W \equiv \cos\theta_W$ and $s_W \equiv \sin\theta_W$, being θ_W given by:

$$\cos\theta_W = g_L \sqrt{\frac{g_R^2 + g'^2}{g_L^2\left(g_R^2 + g'^2\right) + g_R^2 g'^2}}. \tag{52}$$

As for φ, we have

$$\cos\varphi = \frac{g'}{\sqrt{g_R^2 + g'^2}} = \frac{g_L}{g_R}\tan\theta_W. \tag{53}$$

The matrix entry $(1,2)$ is computed at order $\mathcal{O}(\epsilon^2)$, being

$$\frac{\epsilon^2}{4}\cot\theta_W\cos\varphi\sin^3\varphi \ll 1, \tag{54}$$

since $\epsilon \ll 1$. From here, we conclude that Z-Z' mixing is controlled by the quantity

$$\tan\xi = \frac{\cos\varphi\sin^3\varphi}{s_W}\frac{\epsilon^2}{4} \ll 1. \tag{55}$$

As for gauge boson masses, we have:

$$\begin{aligned} M_W &\simeq \frac{1}{2} g_L v \left(1 - \frac{\epsilon^2}{4}\sin^2 2\beta\right), \\ M_{W'} &\simeq \frac{\sqrt{2}}{2} g_R v_R \left(1 + \frac{\epsilon^2}{4}\right), \\ M_Z &\simeq \frac{1}{2} v \sqrt{g_L^2 + \frac{g_R^2 g'^2}{g_R^2 + g'^2}} \simeq \frac{M_W}{c_W} \\ M_{Z'} &\simeq v_R\sqrt{g_R^2 + g'^2} \simeq \frac{\sqrt{2}}{\sin\varphi} M_{W'}, \end{aligned} \tag{56}$$

where $M_W = M_Z c_W$ holds at first order of ϵ, as well as $\sqrt{2}M_{W'} = M_{Z'}\sin\varphi$. Furthermore, and as it should, the photon, A^{μ}, remains massless.

C. Fermion Masses and the Neutrino Seesaw Mechanism

The most general quark Yukawa Lagrangian in the LRSM is

$$\mathcal{L}_{\text{Yuk}}^{Q} = -\overline{Q}_L \left(Y_q \Phi + \tilde{Y}_q \tilde{\Phi}\right) Q_R + \text{h.c.}, \tag{57}$$

where Y_q and $\tilde{Y}_q$ are general complex Yukawa matrices. After SSB, the quark mass lagrangian becomes

$$\mathcal{L}_M^{q} = -\overline{U}'_L M_u U'_R - \overline{D}'_L M_d D'_R + \text{h.c.}, \tag{58}$$

where M_u and M_d are mass matrices for the quarks and $U'_{L,R}$ and $D'_{L,R}$ are three-dimensional vectors composed by the weak eigenstates of the quark fields. The above matrices are diagonalised by biunitary transformations

$$U'_{L,R} = V_{L,R}^{u} U_{L,R}, \qquad D'_{L,R} = V_{L,R}^{d} D_{L,R}, \tag{59}$$

such that

$$\begin{aligned} V_L^{u\dagger} M_u V_R^{u} &= \text{diag}\left(m_u, m_c, m_t\right) \\ V_L^{d\dagger} M_d V_R^{d} &= \text{diag}\left(m_d, m_s, m_b\right). \end{aligned} \tag{60}$$

The misalignment between V^u and V^d is in the origin of quark mixing and the CKM matrix. If the phase θ in the Higgs bidoublet VEV is nonzero, the CKM matrices for the left and right-handed quarks will not coincide, as seen in [4].

In the case of leptons, we have the Yukawa Lagrangian:

$$\mathcal{L}_{\text{Yuk}}^{l} = -\bar{l}_L \left(Y_l \Phi + \tilde{Y}_l \tilde{\Phi}\right) l_R + \text{h.c.} - \overline{l_R^c} Y_R \Delta_R l_R, \tag{61}$$

where the superscript c refers to charge conjugation. As for quarks, we can define the charged-lepton and Dirac neutrino mass matrices, M_l and m_D:

$$M_l = \frac{1}{\sqrt{2}}\left(v_1 Y_l + v_2 \tilde{Y}_l\right), \quad m_D = \frac{1}{\sqrt{2}}\left(v_2 Y_l + v_1 \tilde{Y}_l\right). \tag{62}$$

There is also a right-handed neutrino mass term $M_R = v_R Y_R = M_R^T$. Finally, the neutrino mass Lagrangian can be written as

$$\mathcal{L}_M^{\nu} = -\frac{1}{2}\left(\overline{n}'^c_L M_\nu n'_R + \overline{n}'^c_R M_\nu^* n'_L\right) \tag{63}$$

where

$$n'_R = \begin{pmatrix} \nu'^c_R \\ \nu'_R \end{pmatrix}, \quad n'_L = \begin{pmatrix} \nu'_L & \nu'^{\,c}_L \end{pmatrix}, \quad \nu'^c_{L,R} = C\overline{\nu'}^T_{L,R} \tag{64}$$

The full mass matrix is then:

$$M_\nu = \begin{pmatrix} 0 & m_D \\ m_D^T & M_R \end{pmatrix}. \tag{65}$$

In the LRSM, the VEV hierarchy $v_R \gg v$ automatically implies $M_R \gg m_D$. This is the basis of the type-I seesaw mechanism, which is a natural way to explain the smallness of neutrino masses. In this case, the effective neutrino mass matrix is given by $M_{\text{light}} = -m_D M_R^{-1} m_D^T$.

The fact that charged-leptons and neutrinos cannot be simultaneously diagonalised brings forth the non-trivial lepton mixing which was nonexistent in the SM: the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) mixing matrix, defined as

$$\begin{aligned}\mathcal{L}_{CC} &\to \tfrac{g_L}{\sqrt{2}}\overline{\nu_{\alpha L}}\gamma^\mu \left(U^{\nu\dagger}U^l\right)_{\alpha i} l_{iL}W_\mu^- + \text{h.c.} \equiv \\ &\equiv \tfrac{g_L}{\sqrt{2}}\overline{\nu_{\alpha L}}\gamma^\mu \left(U^\dagger_{\text{PMNS}}\right)_{\alpha i} l_{iL}W_\mu^- + \text{h.c.},\end{aligned} \tag{66}$$

where

$$\begin{aligned}U^{\nu T} M_{\text{light}} U^\nu &= \text{diag}(m_1, m_2, m_3), \\ U^{l\dagger} M_l U^l &= \text{diag}(m_e, m_\mu, m_\tau).\end{aligned} \tag{67}$$

In general, U_{PMNS} can be parametrised as

$$U_{\text{PMNS}} = U_\delta \text{diag}\left(e^{i\alpha_1}, e^{i\alpha_2}, 1\right), \tag{68}$$

where $\alpha_{1,2}$ are Majorana phases and U_δ is

$$\begin{aligned}U_\delta = &\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23}\end{pmatrix}\begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 0 & 1 \\ -s_{13}e^{i\delta} & 0 & c_{13}\end{pmatrix} \\ &\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1\end{pmatrix}\end{aligned} \tag{69}$$

where the notation $c_{ij} \equiv \cos\theta_{ij}$ and $s_{ij} \equiv \sin\theta_{ij}$ was used and δ is the Dirac CP-violation phase. The three angles θ_{ij} are the mixing angles between the three generations of leptons.

IV. LEFT-RIGHT SYMMETRIC FLAVOUR MODELS

We now study LRSMs, where both left- and right-handed leptons are placed in a triplet representation of the discrete group A_4. The simplest model (compatible with current neutrino data) one can build under the concept explained above has the field content shown in TABLE I. The flavour symmetry is $A_4 \times Z_2$. Both left- and right-handed lepton are placed in A_4 triplets; two flavons, ψ^l and ψ^ν, are introduced, one of which is non-trivially charged under the Z_2 symmetry; one A_4 singlet is added to counteract unwanted consequences of the Z_2 symmetry (the vanishing of the dimension-four term). The scalar fields Φ and Δ_R are singlets under A_4, but may be charged under Z_2.

Symmetry	l_L	l_R	Φ	Δ_L	Δ_R	ψ^l	ψ^ν	ξ
$SU(2)_L$	2	1	2	3	1	1	1	1
$SU(2)_R$	1	2	2	1	3	1	1	1
$U(1)_{B-L}$	−1	−1	0	2	2	0	0	0
A_4	3	3	1	1	1	3	3	1
Z_2	0	0	1	0	0	1	0	1

TABLE I: Field content of the LR flavour model and symmetry assignments.

Given the field content, the Yukawa Lagrangian reads[1]

$$\begin{aligned}\mathcal{L}_{\text{Yuk}} =& \overline{l_L}\left(Y_\xi \xi + Y_{l1}\psi^l + Y_{l2}\psi^l\right)\Phi l_R + \\ &+\overline{l_L}\left(\tilde{Y}_\xi \xi + \tilde{Y}_{l1}\psi^l + \tilde{Y}_{l2}\psi^l\right)\tilde{\Phi} l_R + \\ &+\overline{l_R^c}\left(Y_R^0 + Y_R^\nu \psi^\nu\right) i\tau_2 \Delta_R l_R,\end{aligned} \tag{70}$$

where $\overline{l_R^c}$ transforms as l_R^T and τ_2 is a Pauli matrix, and the Yukawa matrices have the following structure, given by the A_4 tensor product in the basis of [6]:

$$\begin{aligned}&Y_l = (Y_l^\xi + Y_{l1} + Y_{l2}) = \\ &\begin{pmatrix} y_l^0 + 2y_l^1\psi_1^l & -(y_l^1+y_l^2)\psi_3^l & -(y_l^1-y_l^2)\psi_2^l \\ -(y_l^1+y_l^2)\psi_2^l & y_l^0 - (y_l^1-y_l^2)\psi_1^l & 2y_l^1\psi_3^l \\ -(y_l^1-y_l^2)\psi_3^l & 2y_l^1\psi_2^l & y_l^0 - (y_l^1+y_l^2)\psi_1^l\end{pmatrix},\end{aligned} \tag{71}$$

$$\begin{aligned}&\tilde{Y}_l = (\tilde{Y}_l^\xi + \tilde{Y}_{l1} + \tilde{Y}_{l2}) = \\ &\begin{pmatrix} \tilde{y}_l^0 + 2\tilde{y}_l^1\psi_1^l & -(\tilde{y}_l^1+\tilde{y}_l^2)\psi_3^l & -(\tilde{y}_l^1-\tilde{y}_l^2)\psi_2^l \\ -(\tilde{y}_l^1+\tilde{y}_l^2)\psi_2^l & \tilde{y}_l^0 - (\tilde{y}_l^1-\tilde{y}_l^2)\psi_1^l & 2\tilde{y}_l^1\psi_3^l \\ -(\tilde{y}_l^1-\tilde{y}_l^2)\psi_3^l & 2\tilde{y}_l^1\psi_2^l & \tilde{y}_l^0 - (\tilde{y}_l^1+\tilde{y}_l^2)\psi_1^l\end{pmatrix},\end{aligned} \tag{72}$$

$$\begin{aligned}&Y_R = (Y_R^0 + Y_R^\nu) = \\ &\begin{pmatrix} y_R^0 + 2y_R\psi_1^\nu & -y_R\psi_3^\nu & -y_R\psi_2^\nu \\ -y_R\psi_3^\nu & 2y_R\psi_2^\nu & y_R^0 - y_R\psi_1^\nu \\ -y_R\psi_2^\nu & y_R^0 - y_R\psi_1^\nu & 2y_R\psi_3^\nu\end{pmatrix}.\end{aligned}$$

As usual, the mass matrices are defined by

$$M_l = v_1 Y_l + v_2\tilde{Y}_l, \qquad m_D = v_1\tilde{Y}_l + v_2 Y_l, \tag{73}$$

$$M_R = v_R Y_R. \tag{74}$$

The flavon alignment of [3] reads, in our basis, $\psi^l \propto (1,0,0)$ and $\psi^\nu \propto (1,\omega,\omega^2)$, where $\omega = e^{2i\pi/3}$. This leads to a diagonal charged-lepton mass matrix:

$$M_l = \text{diag}\left(a+2b, a+c-b, a-b-c\right) \tag{75}$$

---

[1] Although flavon fields are scalars, we consider them to be dimensionless since we are normalizing them by an overall energy scale.

where $a = v_1 y_{l0} + v_2 \tilde{y}_{l0}$, $b = v_1 y_{l1} + v_2 \tilde{y}_{l1}$, and $c = v_1 y_{l2} + v_2 \tilde{y}_{l2}$. It is possible to verify that charged-lepton masses can be accommodated in this model. Assuming real parameters for simplicity, taking

$$a = -\frac{(m_{l1} + m_{l2} + m_{l3})}{3}, \quad b = \frac{(-2m_{l1} + m_{l2} + m_{l3})}{6}, \\ c = \frac{m_{l3} - m_{l2}}{2}, \tag{76}$$

results in a diagonal squared mass matrix $M_l M_l^\dagger = \text{diag}(m_{l1}^2, m_{l2}^2, m_{l3}^2)$, where m_{li} can be assigned to any of the lepton masses. Since m_D has the same structure as M_l, it will also be diagonal, and can be factorised as $m_D = \lambda\,\text{diag}(1, r_2, r_3)$. The heavy right-handed neutrino mass matrix M_R is

$$M_R = a_R \begin{pmatrix} 2z+1 & -\omega^2 z & -\omega z \\ . & 2\omega z & 1-z \\ . & . & 2\omega^2 z \end{pmatrix}, \tag{77}$$

where $a_R = v_R\, y_{R0}$ and $z = y_R/y_{R0}$. Finally, the light neutrino mass matrix reads:

$$M_{\text{light}} = \frac{m}{3z+1} \begin{pmatrix} z+1 & \omega z r_2 & \omega^2 z r_3 \\ . & \omega^2 \dfrac{z(3z+2) r_2^2}{3z-1} & \dfrac{(z-3z^2+1) r_2 r_3}{1-3z} \\ . & . & \dfrac{\omega z(3z+2) r_3^2}{3z-1} \end{pmatrix} \tag{78}$$

where $m = \lambda^2/a_R$.

In the left (right) plots of FIG. 1 we show our results for several parameter correlations in the case of a Normal Ordering (NO) (Inverted Ordering (IO)) neutrino mass spectrum. The results shown are compatible with the latest neutrino oscillation data obtained by the global analysis of [7]. Different colours correspond to different types of solution labeled in [3].

Analysing the results for NO (left plots), we see there are two types of solution. One (A_N: green dots) results in maximal CP violation, which is currently preferred by experimental data, i.e., $\sin\delta = -1$ is very close to the best-fit point (bfp) for the phase δ. Nevertheless, in the other type of solution (B_N: blue dots), $\sin\delta$ spreads the entire range $[-1, 1]$, but also shows results near $\sin\delta = -1$ for higher values of m_L. We also see that m_L can take any value from around 0.03 to 0.12 eV, while remaining in a region near $\sin\delta = -1$. However, this range is not for a single type of solution, requiring a shift from solution A_N to B_N at around $m_L \approx 0.09$ eV.

While at this point both solutions seem suitable, FIG. 1 (middle left) shows that the results from B_N have a narrow range of results for θ_{23}, which is in fact out of the 1σ experimental range. As such, we conclude that the most suitable type of solution is A_N and, from FIG. 1 (bottom left), we note a clear correlation between θ_{23} and m_L, where the higher values of m_L require proximity to $\theta_{23} = 45^{\text{o}}$. Since, the bfp of θ_{23} lies around 41.5^{o}, the resulting m_L is small in comparison to the entire range found, being $m_L \approx 0.06$ eV. Attending to the bfp values of Δm_{21}^2 and Δm_{31}^2, we see that the heaviest neutrino would be less than twice the mass of the lightest neutrino, resulting in a non-hierarchical solution.

On the other hand, we find three different types of solutions for IO: A_I (green dots), B_I (blue dots), and C_I (orange dots). From FIG. 1 (top right), we see that similarly to NO, the A_I solutions always result in maximal CP violation, and B_I shows a range for $|\sin\delta|$ of approximately $[0.8, 1]$. The new kind of solution, C_I, shares the range for $\sin\delta$ of B_I. We can see that m_L is, in general, smaller than the case of NO, with C_I predicting much smaller masses than the remaining solutions. Again, larger values of m_L are possible in solution B_I, whereas A_I predicts lighter but comparable values than B_I.

Interestingly, contrary to NO, where the bfp of θ_{23} excluded one solution, we see in FIG. 1 (middle right) that all three types of solutions converge at the bfps of $\sin\delta$ and θ_{23}. This convergence means that not only no solution is excluded, but all three are predict values near the bfps. We do not show any plots of θ_{12} nor θ_{12} since these show no correlation with any other observables, and their values spread all over the allowed range. As such, all three solutions predict values near the bfps of $\sin\delta$ and θ_{23} (as well as the remaining neutrino observables). This means that our reading of FIG. 1 (bottom right) has three different predictions for m_L, assuming $\theta_{23} \approx 50^{\text{o}}$: C_I predicts $m_L \approx 0.001$ eV, A_I predicts m_L around 0.03, and B_I near 0.08 eV. Similarly to the NO case, the solutions A_I and B_I predict that all three neutrino masses lie in the same order of magnitude. However, the solution C_I returns that the heaviest neutrinos would have a mass fifty times greater than the lightest neutrino, being a more hierarchical solution.

After reproducing the results of [3], we imposed a more restrictive scenario where all Yukawa coupling are taken to be real. The goal of this restriction is to assess if flavon alignments are capable of generating spontaneous CP violation. The extensive list of possible minima for the two-triplet A_4 potential given in [5] was analysed and, requiring that the parameters of this potential are also real, we find one case ($\psi^l = (1, 1-i\sqrt{3}, 1+i\sqrt{3})$ and $\psi^\nu = (2, 1+\omega, 1+\omega^2)$ in the Altarelli-Feruglio basis) which can accommodate current neutrino oscillation data. In this case, the elements of the mass matrices read:

$$\begin{aligned} M_{11} &= a + 2b \\ M_{12} &= -(1 + \omega - \omega^2)(b + c) \\ M_{13} &= (1 - \omega + \omega^2)(c - b) \\ M_{21} &= -(1 + \omega - \omega^2)(b + c) \\ M_{22} &= a - b + c \\ M_{23} &= 2(1 + \omega - \omega^2) b \\ M_{31} &= (1 + \omega - \omega^2)(c - b) \\ M_{32} &= 2(1 - \omega + \omega^2) b \\ M_{33} &= a - b - c, \end{aligned} \tag{79}$$

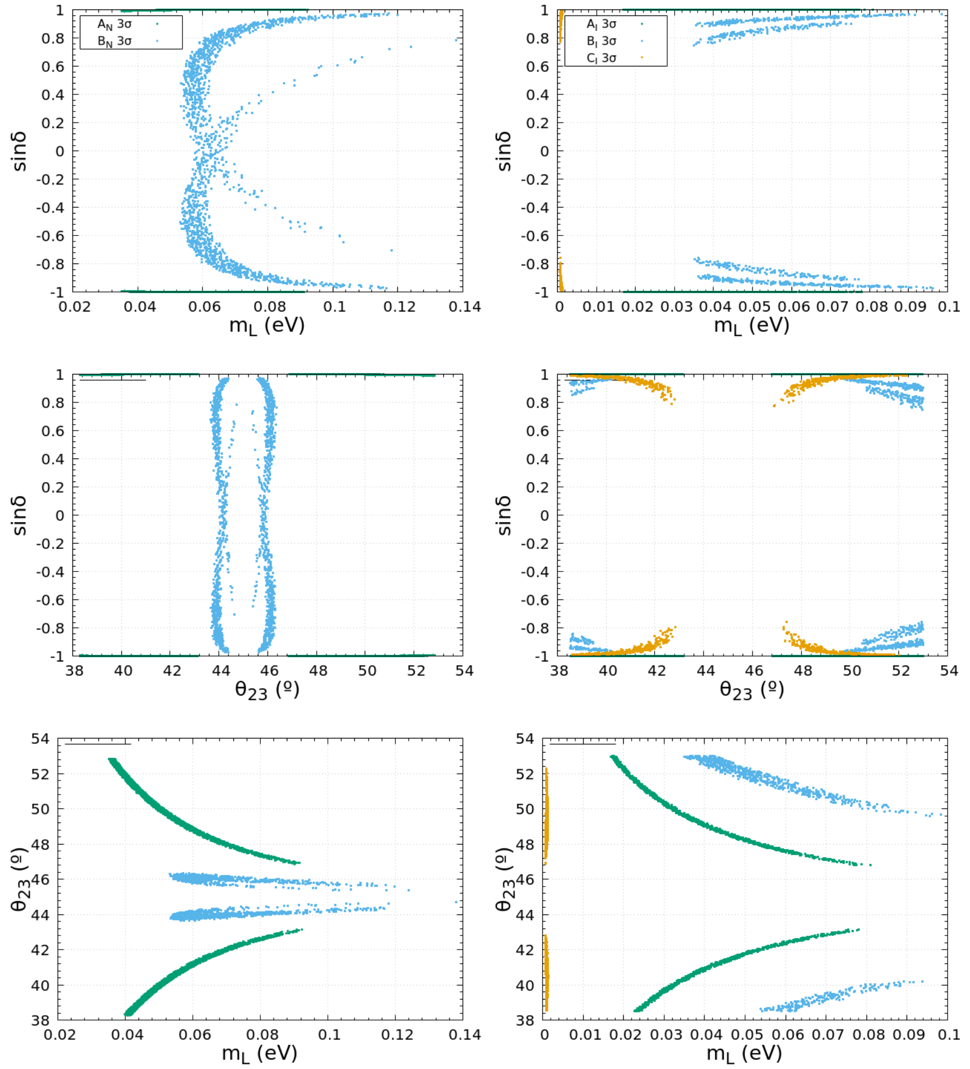

FIG. 1: Results of the $A_4 \times Z_2$ left-right symmetric model described in section IV. The left (right) plots correspond to the case of a NO (IO) neutrino mass spectrum. For our analysis, we have used the neutrino oscillation data from the global analysis of [7]. Different colours correspond to different types of solutions classified in [3].

where M is general for M_l and m_D. For the case of M_l, the parameters are given by:

$$a = v_1 y_l^0 + v_2 \tilde{y}_l^0, \quad b = v_1 y_l^1 + v_2 \tilde{y}_1^l, \quad c = v_1 y_l^2 + v_2 \tilde{y}_l^2, \tag{80}$$

whereas for m_D we have

$$a = v_2 y_l^0 + v_1 \tilde{y}_l^0, \quad b = v_2 y_l^1 + v_1 \tilde{y}_1^l, \quad c = v_2 y_l^2 + v_1 \tilde{y}_l^2. \tag{81}$$

The right-handed neutrino mass matrix is

$$M_R = v_R \begin{pmatrix} 4y_R + y_R^0 & -(y_R^0 + \omega^2 y_R) & -(y_R^0 + \omega y_R) \\ . & 2(y_R^0 + \omega y_R) & y_R^0 - 2y_R \\ . & . & 2(y_R^0 + \omega^2 y_R) \end{pmatrix}. \tag{82}$$

These are all the needed ingredients to compute the neutrino seesaw matrix, which will not be shown here, due to its complicated structure. Although y_R and y_R^0 were taken as two separate parameters, they can amount into a single parameter $z = y_R/y_R^0$ as in the previous case. First, we note that this analysis does not take β (40) into account. However, the requirement that the leptonic (80) and neutrino (81) parameters are free, automatically excludes the case of $\beta = \pi/4$. In other words, $\beta = \pi/4$ implies $v_1 = v_2$ and, therefore, $M_l = m_D$. Since this restriction was not applied, our results are valid for $\beta \neq \pi/4$. Nevertheless, leptonic and neutrino parameters can take remarkably distinct values, since this difference governed by β, and the right-handed scale, v_R. As such, even in the parameter region close to $\beta = \pi/4$ we can have valid results, through increasing v_R until Yukawas couplings are perturbative and are not fine-tuned. Since y_R and y_R^0 can be translated into a single parameter, our parameter count shows three parameters for leptonic

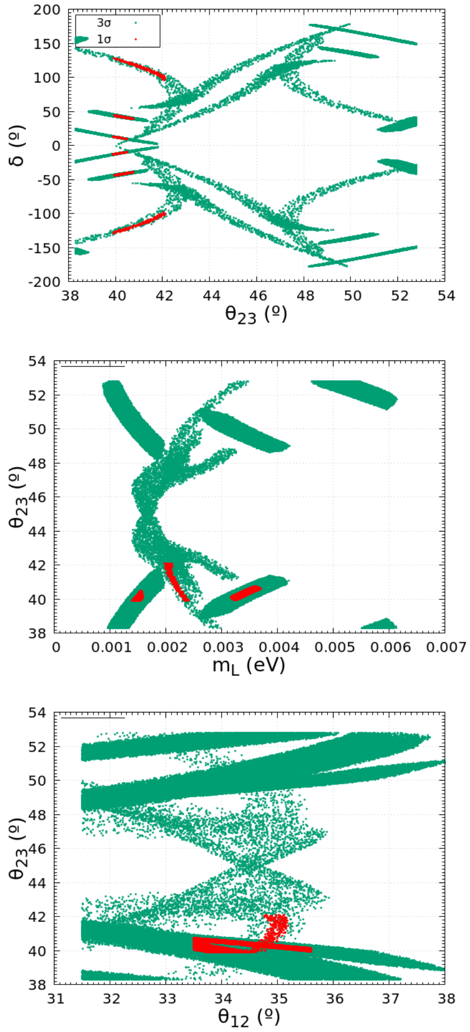

FIG. 2: Correlations for δ and θ_{23} (top), θ_{23} and m_L (mid), and θ_{23} and θ_{12} (bottom). In green (red), are shown the 3 σ (1 σ) results.

masses (making them fully constrained), and four parameters in the neutrino sector (the same as before, except for the imaginary component of z). Since we are fitting 5+1 parameters (three mixing angles, two squared mass differences, and one CP violating phase), we are in the presence of a predictive theory. As such, we take δ as a prediction and do not enforce it to be within experimental range. We show the results in FIG. 2. It is important to note that only the NO of neutrino masses proves valid. FIG. 2 shows the correlation of δ and θ_{23} in the top, of θ_{23} and. m_L, where m_L is the lightest neutrino mass, in the middle, and of θ_{23} and θ_{12} in the bottom.

We can see from FIG. 2 (top) that we are able to place δ at its bfp in one of the solutions. FIG. 2 (bottom) shows it is possible to place θ_{23} and θ_{12} simultaneously inside the 1 σ region from three different curves of this plot. Analysing the found mass range for m_L in FIG. 2 (middle), we see this model predicts a lightest neutrino of at most 0.004 eV, resulting in slightly hierarchical neutrino masses. Nevertheless, the most interesting result is that δ can take non-zero values, with no complex parameters in the Yukawa Lagrangian. This means that the source of CPV must be spontaneous. Although the VEV has an arbitrary phase that arises from a complex parameter in the flavon potential, the imposition that it vanishes leads to a "geometric" SCPV (calculable phases only). That is, although the VEV is not a true GSCPV alignment, imposing that the phase vanishes forces the CPV source to come from the group's structure.

V. CONCLUSIONS

We find that the model put forth by [3] is capable of accommodating current neutrino oscillation data. However, we see that it is possible to build more economical models with the same field content, where all parameters in the Yukawa Lagrangian are real. In this way, we find a flavon alignment in which the model features "geometric" spontaneous CP violation (GSCPV). However, at least in our context of LR symmetric flavour models for the LRSM, we see a definite preference for NO over IO, since the model only proves valid for the first. In sum, we discover a flavon alignment which is able to provide SGCPV in the paradigm of a predictive theory.

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