The Impact of Honor and Shame on Public Goods Dilemmas

A study on the emergence of cooperation

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ABSTRACT

From the beginning of time cooperation between individuals exists and is necessary for the maintenance of populations, but explaining its emergence and maintenance, overcoming the rational egoism that is still foreseen by classical economics, remains an open question. In this work we use the Public Goods Games paradigm, used to study the evolutionary dynamics of cooperators and defectors in a population in which groups of people interact. However human behavior is more complex than just two decisions, and so we model actions as a continuous interval in which an individual can cooperate at different levels. To obtain results computational simulations of multi-agent populations are used, where each agent acts and learns using processes proposed by Evolutionary Game Theory and we study the evolution in different types of social structures represented here through complex networks. Well-mixed structures (all agents interact with equal probability) and heterogeneous interaction networks (scale-free). With this study we propose a model for the emergence of cooperation: honor and shame. We conclude that the structure of the network and the knowledge that an individual has about the degree of others in the network affect learning and that in a general way honor and shame have a positive impact on altruistic behavior, which is more intense when combined.

CCS CONCEPTS

• **Theory of computation** → **Social networks**; **Algorithmic game theory**; **Convergence and learning in games**; **Network games**; • **Computing methodologies** → **Modeling and simulation**; **Network science**; **Cooperation and coordination**;

KEYWORDS

cooperation; public goods games; evolutionary game theory; regular systems; complex systems; modeling

1 INTRODUCTION

Cooperation is the process in which groups of organisms act together for the common benefit, rather than acting in competition for their own benefit. Many species of animals and plants cooperate both inside and outside their species however the mechanisms that create cooperating agents in a system aren't well understood, since apparently cooperation isn't the rational choice. In Economics human behavior can be reduced to a deterministic rule, the Theory of Rational Action [4], this in turn originates in Social Psychology. This theory defines human decision making in a simplistic way, suggesting that individuals seek to satisfy their needs in a rational way by always measuring benefits, subtracting costs, and acting on profit maximization. It is easy to understand the origin of this theory if we think that when faced with a choice, people tend to privilege the least costly one. This theory however has been countered by various economic anthropologists who point to the fact that in traditional societies the choices that people make in relation to the production and exchange of goods follow patterns of reciprocity that differ sharply from what the previous model contrasts. In fact there are many examples where the action taken by the individual is not the option with the lowest associated cost, and may even be the dominant choice in a population. We can think of the division of labor, the creation of social security states, tax payments and international treaties. These are all cases of cooperation, situations in which an individual grants a benefit to another, suffering the cost of this.

This work thus seeks to explain the emergence and maintenance of cooperation in populations of selfish individuals. Understanding how altruism emerges can be the key to solving various current problems, from environmental sustainability to economic wellbeing. It is the answer to how individuals overcome the Tragedy of the Commons, a situation in which agents acting independently and rationally, according to their own interests, behave in contradiction to the best interests of a community, exhausting some common resource [6].

To understand evolution of cooperation **computational simulations** are at the center of this work: we model a population of individuals, studying possibilities that explain how the cooperation emerges and is maintained. Simulations are a broad process that encompasses not only the construction of the model, but all the experimental method that follows, trying to describe the behavior of the system and trying to predict the future behavior, by observing the effects produced by changes in the system. In turn **modeling** a real situation is to simplify it and formalize it. In the context of human behavior, constructing a model implies assuming hypotheses about what we believe to happen in reality, based on some evidence.

This study focuses on the paradigm **Public Goods Game (PGG)**, most commonly used to understand how agents who are reluctant to cooperate can be induced to participate in agreements. In the popular sense a public good describes the set of benefits that are shared by all members of a given community. Public goods games are thus a pattern of Experimental Economics in which cooperators contribute an amount, part of their private possessions, to a pile - the public good. The contributions are then multiplied by one factor (greater than one and less than the number of players, N) and evenly divided by all players, regardless of whether they are cooperators or defectors. Each player also maintains the possessions that did not contribute.

Human behavior however is not so black-and-white that an individual can be classified as a cooperator or defector exclusively. Most commonly people cooperate to some degree, thus we abandon the notion of cooperators or defectors and model cooperation as a continuous distribution in which an individual can cooperate more or less. This point in particular contrasts with the great majority of the analyzes made to date, for this type of games there are no studies in which cooperation is modeled in a continuous space of strategies.

Many institutional structures suggest that punishment and rewards play an important role in stimulating cooperation. From the social point of view shame can be seen as a form of punishment and honor can be seen as being the object of a reward. We thus concentrate on the study of honor and shame as catalysts of cooperation, independently or jointly.

The problem still separates in two respecting the nature of the population - **well mixed and regular** or **structured and heterogeneous**. These structures are used to understand two problems: 1) if honor and shame can be used as a measure to coerce cooperation, for example a state could identify and make available to the public the worst and best contributor to a mandatory tax - which would lead to the sense of honor and shame of the identified subjects. In this example, all individuals participate with the same number of individuals - the total population, and thus are regular; and 2) if honor and shame can explain the spontaneous emergence of cooperation. In this problem heterogeneous networks are the most appropriate. If we consider that the actions taken by the individuals are of the knowledge of all, then honor and shame happen naturally. In this type of network the accumulation of distinct situations in which an individual has the choice to cooperate is used to calculate their success - being that there is heterogeneity in the number of situations that individuals cooperate and in the number of individuals with whom they do it.

In the next Section we will present key concepts and related works, useful to understand the fundamentals and motivation of our proposal. In Section 3, we formalize the public goods game with honor and shame and in the followings we analyze the results and draw some concluding remarks and provide some insights into future work

2 RELATED WORK

2.1 Mechanisms for the emerge cooperation

Nowak summarizes five models for the emergence of cooperation, where natural selection can lead to cooperation [12]. Each of these mechanisms is explained in more detail below.

Kin Selection states that the more genetically related individuals are the more likely the co-operation will emerge. The idea behind this mechanism is that cooperation can override natural selection if the donor and recipient of an altruistic act are genetically related so as to protect the propagation of their genes. This explanation is convincing, yet insufficient. Indeed, is easy to observe cooperation among people that are not related genetically.

Direct reciprocity is based on the idea that cooperation emerges with greater probability, the greater the likelihood of playing again with the same player. This mechanism assumes the existence of repeated encounters between two individuals using the Prisoner's Dilemma. In each round an individual has the choice of cooperating or defecting, and therefore an individual can cooperate in one round, due to the expectation that the other cooperate in the next round and thus achieving greater profits. Several strategies have been studied to play this game. From Tit-for-tat (TFT) [18], generous-tit-for-tat strategy or Win-Stay Lose-Shift (WSLS) [13], the number of possible strategies is infinite, but one rule is general: direct reciprocity can lead to the emergence of cooperation although is insufficient explaining it since cooperation often exists without repeated encounters.

Indirect reciprocity is based on the idea of reputation [14]. Cooperation establishes a good reputation that in the future will be rewarded by other individuals. Thus if an individual has a better reputation it leads to more individuals cooperating with him, and the more they cooperate the better their reputation. This model is quite satisfactory in human population, but in animals it has less potential since it requires a high cognitive degree: it is necessary to memorize the previous interactions and to monitor the changes in the population, and also a language is needed to share the reputation information.

Group Selection is based on the idea that individuals tend to cooperate more within their environment and defect when interacting with individuals from other groups. The theory behind this mechanism is that natural selection exists not only in the individual but also in groups, and that a group of cooperators may be more successful than one of defectors. In this mechanism a population is subdivided into groups, in which cooperators help only within their group and defectors do not help anyone. Groups made of cooperators grow faster than groups where there are only defectors, and in mixed groups defectors reproduce more. Thus, the selection at the lowest level, within the group, favors the defectors and at the highest level, between groups, favors the cooperators.

Network Reciprocity focuses on the fact that real populations are not well mixed, but have spatial structures or social networks implying that some individuals interact more often than others. An approach to capture this effect is the evolutionary graph theory, in which individuals occupy the vertices of a graph. The edges determine who interacts with whom. In this mechanism a cooperator pays a cost and for each neighbor receives a benefit. Defectors are costless, and their neighbors do not receive benefits and thus network reciprocity can foster co-operation: co-workers can dominate the population by generating mutually supportive clusters. Santos et al. [16] show that highly connected nodes (hubs) are those that transform more rapidly into cooperation. This is because, under this contribution model, the fitness of a cooperator increases with its connectivity. Consequently, heterogeneity confers a natural advantage to the hubs. The authors also explain how a Defector (D) alone in a large hub can be taken by a Cooperator (C): Ds are victims of their own success in generating other Ds in their neighborhood, reducing their fitness. Consequently, they become vulnerable to nearby Cs. Once invaded by a C, a hub will remain C, because by putting Cs in their neighborhoods, they increase their fitness. The role of Cs is therefore crucial in spreading the cooperator's strategy through these networks and in mastering the hubs they are in.

Dieckmann and Kun [9] show how cooperation can be maintained through **resource heterogeneity**. Cooperation can then emerge even in populations where the temptation to defect (corresponding to relatively low gains in promoting the public good) is so strong that actors would act in a wholly selfish way if their resources were evenly distributed. Cooperators in rich places improve the returns of neighboring cooperators in poor places and thus allow cooperation to spread in a heterogeneous environment. The work in question also shows that resource heterogeneity can hinder cooperation when the temptation to defect is significantly reduced. When defectors are unable to survive in the homogeneous environment, they can not be expelled from the population in the heterogeneous environment thanks to the defectors rich in clusters.

2.2 Games

In a PGG in which the multiplier factor M is less than the size of the population N and in which there are both Cooperators (C) and Defectors (D) C are always at a disadvantage. This is because if k C contribute c, Ds ends with a profit equivalent to kMc/N and Cs gets $kMc/N - c$. But in reality, this Nash equilibrium is not always found in experiments. People tend to add something to the pot. The average contribution usually depends on the multiplication factor, and the higher factors produce larger proportions in the contribution [5]. Tests with a large group, about 40 individuals and a very low multiplication factor, 1.03 show that the population tends so that nobody contributes anything after some iterations of the game. However, with the same size group and a multiplication factor of 1.3, the average initial donation level contributed to the pot is about 50% [7]. Iterated public goods games involve the same group of individuals who play the basic game during a series of rounds. The typical result is a decreasing proportion of public contribution, in each game. When contributors note that not all players contribute as much as they tend to reduce the amount they share in the next round [10, 11].

The option of **punishing defectors and rewarding the greatest contributions** after a round of the Public Goods Game has been the subject of many studies. The results strongly suggest that not rewarding is not seen as a punishment, and that rewards do not replace punishment. Instead, the rewards are used in a completely different way: as a means to reinforce cooperation and greater profits. On the other hand, a 2007 study found that rewards alone could not sustain long-term cooperation [17]. Many studies, therefore, emphasize the combination of punishment and rewards. The combination seems to produce a higher level of cooperation and rewards for games iterated in heterogeneous groups [1, 17], as well as in homogeneous groups.

Sigmund et al [19] also study the effects of reputation along with Punishment and Reward and conclude that it is considerably more effective with punishment than reward.

Another mechanics studied that allows a stimulus to cooperation is the introduction of a minimum limit of cooperators. This is a Game known as **Stag-Hunt**, which originates the problems of collective action [3, 8]. An interesting example is the "three in a boat, two must row" [20, 22] model, a generalization of the three-player game where contributions of two out of three players are needed for success. The results show that if two others paddle, there is an incentive to defect, but if only one row exists an incentive to contribute. Pacheco et al. [15] show that in normal conditions exist two stable equilibrium situations, in which the fraction of cooperators stabilizes, one in which the selection favor cooperation and another in which is favor selfishness.

2.3 Summary of related work

The cooperation mechanisms have been a lot of study. However, the context of this study will focus only on public good games as they fit into a number of real cooperation situations - tax payments, creation of social security states or public health services, international treaties or voluntary attempts to reduce consumption of fossil fuels can all be modeled as individuals who choose to incur a cost for the group to gain a common advantage. In this study we propose Honor and Shame as catalysts for cooperation in a multi-player context in which cooperation can be seen as continuous strategies modeled between [0, 1] - an option closer to reality, in which people often do not cooperate or defect exclusively.

3 MODEL

In this section, we propose the model for cooperation, with honor and shame, profit, fitness and learning of the Public Goods Game of N-People and infinite strategies and variants for regular unstructured populations or heterogeneous structured populations and threshold.

Offer

In the context of this study, the player i contribution, $x(i)$ is modeled as a real in the continuous space $\in [0, 1]$, thus being a model closer to reality, in which individuals can invest more or less..

The **maximum contribution** is modeled as a fixed value per game and homogeneous in the population.

We assume that the player's **initial offer** $x_0(i)$ follows an adaptation of the Gaussian distribution, centered on the desired initial average offer (IAO).

$$x_0(i) = r * \sigma + \mu \tag{1}$$

where r is a randomly generated decimal from a Gaussian distribution with mean 0 and standard deviation 1, μ is the IAO and σ is the standard deviation given by

$$\sigma = \frac{min(\mu - x_{Min}, x_{Max} - \mu) + \theta}{3} \tag{2}$$

where x_{Min} is the minimum bid and x_{Max} is the maximum, corresponding to 0 and 1 in our model. This way we guarantee that there is a maximum variation within the interval $[x_{Min}, x_{Max}]$ according to the 3-sigma rule, that is, according to the specifications of the Gaussian distribution we guarantee that 99.7% of generated offers contained in the interval [0,1].

Offers outside this range are converted to either x_{Min} or x_{Max}, if they are below or above that range respectively. Note that the closer μ is to $\frac{x_{Min}+x_{Max}}{2}$ the greater the range of bids.

Finally, θ has been added as an error that guarantees variation of values in extreme cases where $\mu = x_{Min}$ or $\mu = x_{Max}$. In our model we define $\theta = 0.1$.

Population

The scope of this study includes the creation and analysis of a regular unstructured population (RUP) and a heterogeneous structured population (HSP)

In RUP agents interact in groups of the same size, in which these groups are always formed in a random way, from game to game, being therefore an unstructured population. Thus in each game, N elements are selected according to a uniform distribution, in other words all nodes have the same probability of interacting with each other. As we saw earlier, a regular network is a network where each vertex has the same number of neighbors and therefore this network is also regular. In our model the size of the group N was defined as $N = 6$ for the HSPs.

The HSP was modeled as a scale-free network. The Barabasi-Albert algorithm [2] was chosen and is based on two general concepts: growth and preferential annexation. The growth process starts with an arbitrary network of n_0 nodes. New nodes are added to the network, one by one, and each of these new nodes makes connections to existing c nodes, where c is a template variable. The probability of the new node s connecting to another node i any already present in the network is proportional to the number of connections that the node i has. Formally, the probability p_i that the new node s connects to a vertex i already present is:

$$p_i = \frac{k_i}{\sum_j k_j} \tag{3}$$

where k_i is the degree of the node i and the sum in the denominator, performed on all nodes already existing in the network, is the normalization of probability. This connection selection process is then repeated for each of the c connections that the new node will make.

With this mechanism, nodes with a high degree tend to rapidly accumulate even more connections, while low-level nodes containing only a few connections are very likely to be chosen as the destination for a new connection. Thus, the new nodes have a "preference" for joining us already strongly connected. Note also that there is a minimum value for the degree of a node, being equal to the parameter c and that the average degree $< k >$ of the network can be calculated analytically, $being < k >= 2c$.

In our model we define $n_0 = 10$ thus starting with a complete regular network of size 10. The number of nodes added to new node was defined as $c = 4$ thus guaranteeing minimum degree, $k_{Min} = 4$ e average grade $< k >= 8$.

Honor and shame

Several models could be used to implement honor and shame, since they are abstract values that signify an emotional but also a social impact and are seen as a reduction or addition to an individual's fitness. We chose to define honor and shame as a percentage relative to the maximum contribution, to fit the values under study and show the impact of their variation. This relationship would be especially interesting in the study of resource heterogeneity, in which a proportional relation of shame and honor could be maintained with the possessions that an individual has.

In this way the profit of a player i can be affected by an increase or decrease of $hx_{Max}(i)$ and $vx_{Max}(i)$, respectively, where h and v are the honor and shame factors and $x_{Max}(i)$ is the maximum contribution to the player i.

Threshold

The threshold can be modeled as a percentage of the maximum contribution of all players or as an absolute value. In this model we have chosen to set the threshold as a value relative to the maximum contribution of the group, where T is the percentage, and t the threshold, and $x_{max}(i)$ the maximum contribution of the player i

$$t = T \sum_{i=0}^{N} x_{max}(i) \tag{4}$$

Profit

Profit is modeled the same way regardless of the type of population. The game metaphor is usually defined by its reward matrix, and that's where we'll start.

For the most common version of the game, the two strategies are defined as

- Cooperate: contribute x, $x > 0$
- Defect: contribute 0

where x is a real number contained in]0,1].

Player i's profit , $l(i)$, can be defined as the result of the Public Good Game plus the remainder of the initial value that the player had available to contribute, as can be seen in the equation

$$l(i) = \frac{M \sum_j^N x_j}{N} + (1 - x_i), 1 < M < N \tag{5}$$

where N is the total of the players, C the total of contributors, x_i is the player i's contribution and M is the multiplier factor (greater than 1 and less than N).

Adapting the usual 2x2 payoff matrix for N-players we get the following matrix, where we can see a player's profit depending on the cooperators number

	$N-1$	$N-2$	...	0
Cooperate	$\frac{M \sum_{j=1}^{N} x_j}{N}$	$\frac{M \sum_{j=1}^{N-1} x_j}{N}$	...	$\frac{M x_i}{N}$
Defect	$\frac{M \sum_{j \neq i}^{N} x_j}{N} + (1 - x_i)$	$\frac{M \sum_{j \neq i}^{N-1} x_j}{N} + (1 - x_i)$	...	x_i

From the previous table it is possible to draw a conclusion: a player will choose to Defect because it gives him the greater profit, but when all the players opt for the same strategy they will fall in the equilibrium in which all Defect, obtaining as profit the value they initially had.

Adapting the previous model to the **game with threshold** we can model the fitness of the player as the following equation

$$l(i) = \begin{cases} \frac{M \sum_j^N x_j}{N} + (1 - x_i), & \text{if } \sum_j^N x_j \geq t \\ 1 - x_i, & \text{otherwise} \end{cases} \tag{6}$$

where t is the threshold, x_j is the player's contribution j, N the total players, M the multiplier factor and $1 < M < N$.

In this model, an incentive to contribute is expected.

As we have seen before, honor and shame will be modeled as a percentage of a player's maximum contribution. This way the models for a player's profit can be modeled as the previously modeled profit plus the honor if it is one of the players that contributed the maximum and decreased of the shame if contributed to the lower supply. In the game with threshold we will study two variants, one in which we apply honor and shame always, and another in which we apply honor and shame exclusively depending if the threshold is achieved or not, respectively.

Fitness

To obtain the fitness of an individual in a **regular unstructured network** this individual will be put to play many dilemmas of public good and then the average of this results is calculated. In each of these games are selected N random players, with $N << Z$, being Z the size of the population.

In this way the fitness of an individual i can be modeled as the average of the profit obtained in J games

$$f(i) = \frac{\sum_{j=0}^{J} l_j(i)}{J} \tag{7}$$

where $l_j(i)$ is the player's profit i for the game j.

In **heterogeneous structured networks** the fitness of an individual is seen as the sum of the profit of their games, rather than the average, so as not to eliminate the effect of network heterogeneity.

$$f(i) = \sum_{j=0}^{J} l_j(i) \tag{8}$$

In this model an individual i with k_i plays $k_i + 1$ games, k_i games centered on each of its neighbors, and a game centered on i. That is, an individual plays a game with his neighbors, and also plays the games of his neighbors, in which he plays the role of neighbor, as can be seen in the figure 1. In each of these games, N will then be k_i, being i the individual center of the game in question.

We also consider that for the average degree of a network $< k >$, the average number of games $< J >$ is $< J >=< k > +1$ and the average number of individuals in each game $< N >$ is $< N >=< k > +1$.

Learning

To model learning we use the Fermi function [21] which gives the probability that B will replace A

$$p = \frac{1}{1 + e^{\beta(f_B - f_A)}} \tag{9}$$

Such a process is called pairwise comparison. When using this update rule, imitation will occur with probability proportional to the difference between the fitness of both individuals $(f_B - f_A)$.

If agent A and agent B have the same fitness, imitation will occur with probability 0.5, meaning that fitness will have no influence on this imitation process. The β parameter will determine the strength

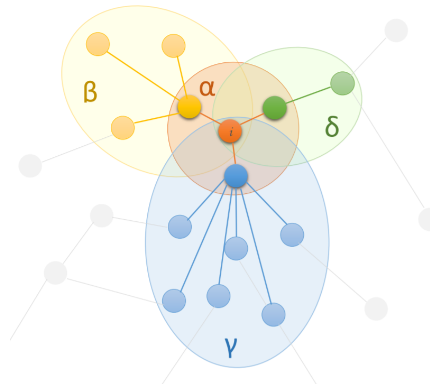

Figure 1: Neighborhood and focus of HSP for a player i, with degree $k_i = 3$. $k_i + 1 = 4$ games are obtained, where corresponding players are identified by distinct color groups α, where i is the center and β, γ, δ centered on its neighbors.

of the selection. Again, if this value is 0, the imitation will occur randomly and with probability equal to 0.5. The bigger the β the more deterministic the learning is.

In our model β is defined so it is adequate to each population, based on the maximum possible difference between the fitness of two individuals.

We chose a β that guaranteed an imitation probability of approximately 95%, for the simple version of the game in order to offer some leeway to the maximum fitness variation with the application of threshold and honor and shame factors.

In order to be able to explore the range of infinite strategies and to allow a margin of error in learning defined the new strategy x_{t+1} as

$$x_{t+1} = x_t + (y * \epsilon), y \in \{-1, 0, -1\} \tag{10}$$

where x_t is the strategy to imitate, ϵ the error parameter, and y generated from a random uniform distribution, in other words, all values of y are equally likely.

We further define that both individuals A and B in game are chosen randomly in the network, according to a uniform distribution of size Z, regardless of population type.

In **RUP** we define $\beta = 1$, which allows an imitation probability $p \approx 95\%$ for the maximum fitness difference in the PGG. The selected β led to $p \approx 99.3\%$ in PGG with honor and shame, $p \approx 97\%$ in PGG with threshold, and $p \approx 99.5\%$ in PGG with honor, shame and threshold for the maximum difference between fitness.

In **HSP** we assume that an individual does not know the absolute maximum fitness difference between themselves and the other individual to possibly imitate but only has knowledge of the maximum fitness difference for the average degree of population.

We have chosen this model of learning because it offers a greater similarity to reality: in a population an individual is not aware of the real number of connections that another has, especially if he does not know it (not connected in the network) or how much the neighbors of this have. Realistically, one knows only an average of the degree that an individual in the population has, and therefore only an approximate notion of the maximum fitness difference for the average degree of the population.

After analysis we reached $\beta = 0.1$ which allows an imitation probability of $p \approx 95\%$ for $< k >$'s maximum fitness difference in the PGG. This β guarantees a $p \approx 98.5\%$ in PGG with honor and shame, $p \approx 96.4\%$ in PGG with threshold, and $p \approx 99.3\%$ in PGG with honor, shame and threshold for the maximum fitness difference for the networks' average degree.

Summary

We summarize in the following tables the parameters of the model for the two population structures and some measures.

Table 1: Summary of parameters

	RUP	HSP
Offers		
Initial average offer (IAO) - variable	∈ [0,1]	∈ [0,1]
Offer creation error θ	0.1	0.1
Structure		
Initial network size n_0	-	10
Nodes added c	-	4
Population size Z	600	900
Learning		
Learning error ϵ	0.05	0.05
Learning intensity β	1	0.1
Generations G	50000	500000
Game		
Number of player in each group N	6	-
Number of games J	3000	-
Multiplier factor	3	3
Threshold	0.5	0.5

Table 2: Summary of emerging measures in heterogeneous structured networks

Measure	Value
Average degree $< k >$	8
Minimum degree k_{Min}	4
Players in each group N	$k(j) + 1$, j is a neighbor of i
Games J	$k(i) + 1$, for the subject i

4 REGULAR UNSTRUCTURED POPULATIONS (RUP)

4.1 Honor and Shame in Public Goods Games

In figure 2 we can see the impact on cooperation when both factors are applied together, for the extreme case when the initial average offer is 0 on the left and for IAO = 0.5 on the right.

On the ordinate we have the honor and in the abscissa the shame. The colors, a scale of green to red, represent the intensity of the average cooperation obtained at the end of the simulations, green being the cooperation null and red the maximum cooperation. These graphs, called color plots, were obtained through various simulations combining different values of honor and shame. We start by fixing $h = 0$ and get the average population supply at the end of G iterations for different values of shame, specifically for $v \in [0, 0.1, ..., 0.9, 1]$ and we repeat this step for $h \in [0, 0.1, ..., 0.9, 1]$, so we need 127 simulations to build a graph. The graphs were obtained from randomly chosen particular simulations. All color plots follow the same configurations.

The results show that when combined, the two factors are more effective at stimulating cooperation in a totally selfish society, than separate. They also show a slight superior impact of honor when the initial average supply is close to zero and shame when there is already some cooperation in the population. We may also note that the higher the IAO, the more impact honor and shame had on cooperation in the same time interval, revealing a tendency for maximum cooperation.

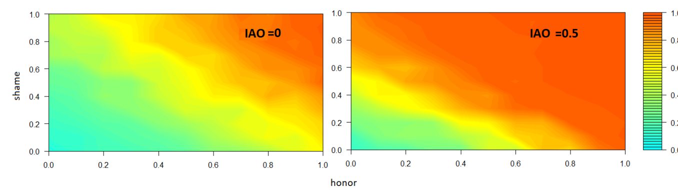

Figure 2: Honor and shame impact for $IAO = 0$ on the left and $IAO = 0.5$ on the right. We can see how honor and shame combined lead to the increase in average supply and when IAO = 0 honor has more impact, and IAO = 0.5 shame has more impact.

4.2 Honor and Shame in public goods games with threshold

In this section we start by analyzing coordination games without the impact of honor and shame. As we have previously seen without any external stimulus, the selfish nature of the individual leads to cooperation disappearing in games of public goods. However, in the threshold model this is not always true. The fact that below the threshold the profit of the game is 0 leads to an increase in the benefit of cooperating when the average offer is very close to the limit, and also to an increase in the cost of not cooperating when above the threshold, offering a stimulus to cooperation. This can be seen in image 3 in which we fix $t = 0.5$ in order to analyze the evolution of cooperation over time to different initial states.

The results point to the existence of two equilibrium points, one that tends to lack of cooperation and another that tends to the threshold.

We start by analyzing the model in which we apply simultaneously h and v, regardless of whether the threshold is reached.

The results in the figure 4 for the extreme case where we set IAO = 0, far from the threshold, show that honor and shame have a similar impact to stimulating cooperation and that combined are more to do so.

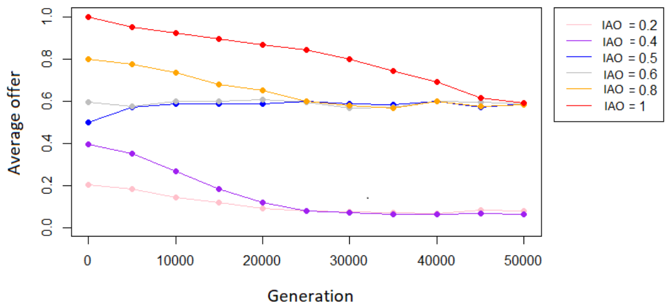

Figure 3: Threshold impact with $t = 0.5$. The results point to the existence of two equilibrium points, one that tends to lack of cooperation and another that tends to t.

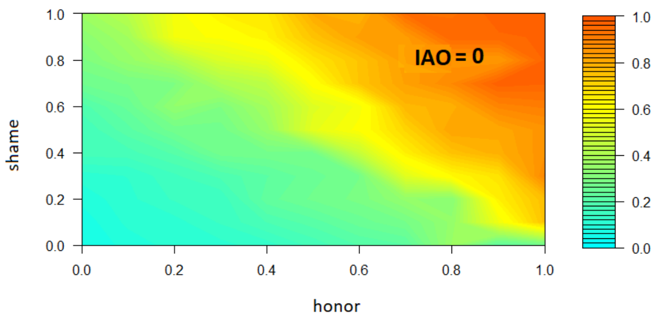

Figure 4: Honor and shame impact for IAO = 0, with $t = 0.5$. We can see that h and v are more effective together.

In fig.5 we can see the results when we are close to reaching the threshold, IAO = 0.4 on the left, and after reaching it, IAO = 0.5 on the right. We can see when we are close to reaching the threshold the cooperation is already near maximum, except for very small values of honor and shame. Above the threshold the results are quite similar, differing only in the area where honor and shame are not able to stimulate cooperation and therefore the final average offer is equal to the threshold.

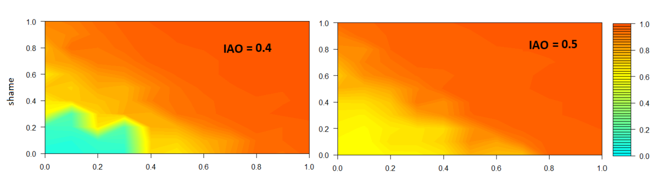

Figure 5: Honor and shame impact with $t = 0.5$ and IAO = 0.4 on the left and IAO = 0.5 on the right. When we are close to the threshold the cooperation is almost total, only small values of h and v do not stimulate cooperation.

Finally we study the model in which shame is applied if the threshold isn't met and honor if it is.

Looking at fig. 6 we see that for IAO = 0, far from the threshold, shame is ineffective at stimulating cooperation alone. This happens due to two aspects of the model: 1) the high homogeneity of network offerings leading to slower learning evolution, since the standard deviation is only the error $\theta = 0.1$ and therefore about 99,7 of the offers are in the range [0,0.1] and 2) the individuals' small fitness variation, because we are so below the threshold the profit of all individuals is 0 and therefore has little influence on learning.

We also see that when the initial average offer is below the threshold honor is not able to stimulate cooperation. This is caused by the specification of the model: since the IAO is below the threshold the profit of the groups does not reach the minimum threshold of cooperation and therefore honor is not attributed and only shame is.

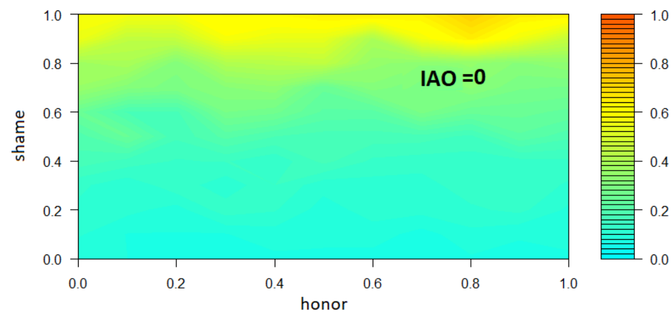

Figure 6: Honor and shame impact for IAO = 0, with $t = 0.5$. Honor does not appear to have an impact and shame is ineffective at stimulating cooperation when the IAO is far from the threshold.

As we approach the threshold and there is more variation in population offerings the shame factor allows for cooperation and the honor factor leads to higher offerings. Above this value shame ceases to have weight since it is not attributed. This can be seen in the fig. 7, which presents the simulation with IAO = 0.4 on the left and IAO = 0.5 on the right showing the impact of honor and shame when standing near and after reaching the threshold.

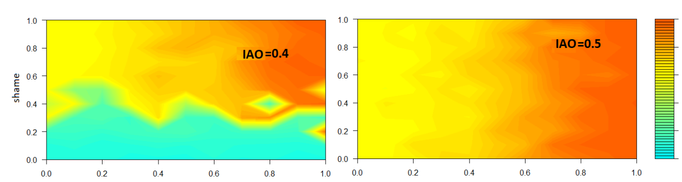

Figure 7: Honor and shame impact, with $t = 0.5$ and IAO = 0.4 on the left and IAO= 0.5 on the right. When $OMI < t$ honor is not enough to stimulate cooperation but guarantees higher values of cooperation and when $IAO \geq t$ shame ceases to have impact on cooperation.

As with uncoordinated games, the results show that it is more effective to apply honor and shame together. The study of the two models of attribution of honor and shame leads us to conclude that it is more advantageous to attribute the two factors independently of reaching the threshold, since it requires lesser combinations of honor and shame, and especially in the case where IAO = 0 the first model presents much superior results of cooperation.

5 HETEROGENEOUS STRUCTURED POPULATIONS (HSP)

5.1 Honor and Shame in Public Goods Games

From the analysis of simulations in Public Goods Games with Honor and Shame we can conclude that both have an impact on cooperation, which is more pronounced when combined, and exists independently of the IAO. However, it differs greatly from the results in regular unstructured networks: honor and shame offer a rather concise stimulus between several randomly selected particular simulations, given the heterogeneity of the network and the fixed β calculated for the average degree of the network.

We can see the extreme case where the IMO is 0 in fig. 8 and conclude that although honor and shame stimulate cooperation, this stimulus is much smaller than for them values in RUP and that never reaches a maximum average offer of 1.0, once again by the weight that the number of neighbors has in the imitation. These color charts follow the same settings as in the previous chapter, regarding axes, colors and the method of obtaining them.

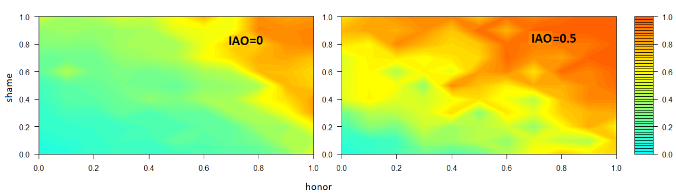

Figure 8: Honor and shame impact on cooperation for IAO = 0 on the left and IAO = 0.5 on the right. h and v offer a stimulus to cooperation, although much lower than for the same IAO values in RUP.

5.2 Honor and Shame in public goods games with threshold

Once again we begin by analyzing the initial state of the coordination game without the application of honor and shame. These results are less linear than in the RUP, since the heterogeneous structure of the population creates a higher margin of error in learning. This feature can be seen in more detail in fig. 9, where it can be observed that this error leads to very different evolutions in the simulations.

First we analyze the model in which we simultaneously apply h and v, regardless of whether the threshold is reached. The results show a much more pronounced impact on the final cooperation, when compared to the same non-threshold game, revealing a positive impact of applying a threshold in cooperation (fig.10). It also shows the strongest impact of shame, showing that for IAO = 0 and combinations of $\{h, v\} = \{0, 0.8\}$ it is possible to stimulate cooperation, contrary to the honor that with no value of shame can

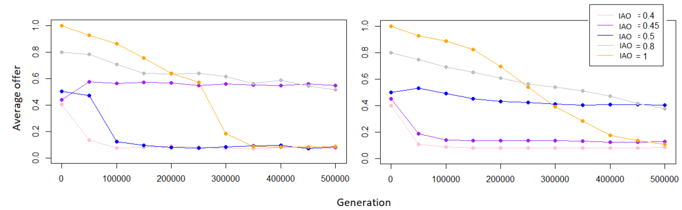

Figure 9: Threshold impact for $t = 0.5$ for a particular simulation chosen randomly on the left and on the average of 10 simulations on the right. The results show significant variation between a randomly selected simulation and the average result obtained in the set of several simulations. t does not provide a coherent stimulus to cooperation.

not stimulate cooperation. However it is visible how honor leads to higher values of cooperation.

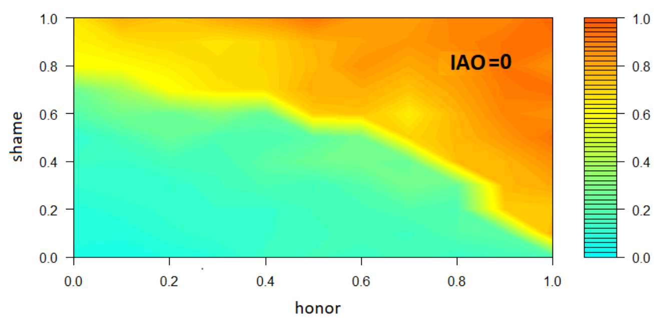

Figure 10: Honor and shame impact with t=0.5 and $IAO = 0$. Both the h and v factors stimulate cooperation, being more effective together. v reveals a greater impact at stimulating the emergence of cooperation and h guarantees higher values of cooperation. The combination of honor and shame in games with threshold leads to higher values of altruistic behavior than in games without threshold.

We can see in fig. 11 how cooperation evolves when we move from $IAO < t$ to the left, to $IAO \geq t$ to the right. We can see that cooperation is less strong than under the same conditions in RUP, in which the graphs presented the greater part of the colored area in red, corresponding to the maximum cooperation. It is also noticeable that unlike the RUPs, when the IAO = t and h and v do not offer stimulus to the cooperation, cooperation tends to 0 as opposed to tending to t.

We then analyze the results for the model in which we apply shame only if the threshold is not reached and honor if is. The study shows that when $IAO < t$, honor alone is not enough to stimulate cooperation as defined in the model, as we can see in fig. 12 that presents the extreme case where IAO = 0. However we can see that honor encourages higher values of cooperation. Another interesting fact to note in this case is the existence of a transition to cooperation

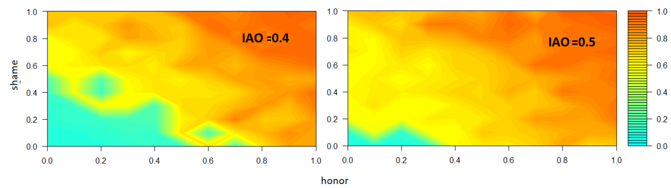

Figure 11: Honor and shame impact with $t = 0.5, IAO = 0.4$ on the left and $IAO = 0.5$ on the right. The cooperation obtained is less intense than in RUP. When IAO = t and h and v do not offer stimulus to cooperation, cooperation tends to 0 as opposed to tending to t, as in RUP.

when shame corresponds to 0.7, even when IAO = 0, showing once again a stimulus superior to the cooperation in HEP than for the same conditions in RUP.

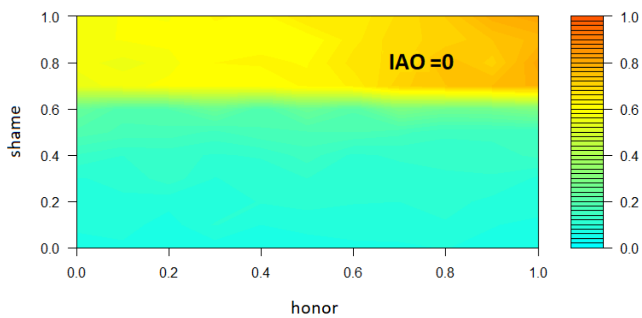

Figure 12: Honor and shame impact with t=0.5 and $IAO = 0$. When $IAO < t$, honor alone is not enough to stimulate cooperation, but it encourages higher values of cooperation. Higher values of cooperation in HEP are achieved when OMI = 0 than in RUP.

We can see in fig. 13 how cooperation evolves when we move from $OMI < t$, on the left, to $OMI \geq t$ on the right. For the threshold dependent model, shame loses its impact (since it is not attributed) and it is the honor that goes on to ensure that cooperation is maintained or stimulated.

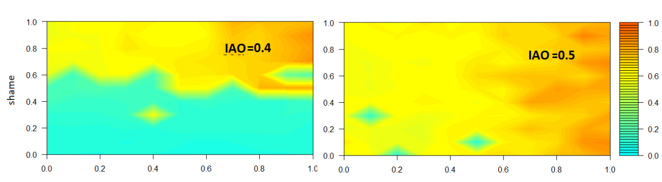

Figure 13: Honor and shame impact with $t = 0.5, IAO = 0.4$ to the left and $IAO = 0.5$ to the right. When it goes from $IAO < t$ to $IAO > t$, shame loses its impact and it is honor that guarantees cooperation. Once again it is clear that the cooperation obtained is less than in RUP for the same conditions.

In the first model, where both factors exist independently of the threshold, the results show that the cooperation reaches higher values for the same number of generations than in the second model, showing once again that together the factors have more impact.

6 CONCLUSIONS

In this section we will summarize the conclusions drawn from our study

Honor and shame have a positive impact on cooperation. There is no doubt about the results obtained in simulation, honor and shame do indeed play an important role in cooperation. Throughout this study we have shown this role on different models and situations and the conclusions have always been the same: when applied together it has more intense consequences. Whether in regular unstructured networks or heterogeneous structured networks this observation can have positive consequences in society. We can then conclude that when an individual's actions are known to all, making him susceptible to honor and shame as he is exposed about his altruism relative to others, then these feelings may explain the emergence of cooperation in human societies, whether naturally in a context of public actions or coerced in the form of measures that expose the actions of individuals.

In games with a minimum objective of cooperation (threshold) is visible a slight upper impact of the shame and in the version without threshold is visible the opposite effect.

Together honor and shame have more impact. We have observed in all simulations that when they exist together these feelings have a much greater impact on cooperation, and in some cases they do not have sufficient weight to encourage altruistic behavior. This conclusion has a very important impact in understanding the mechanisms that stimulate cooperation, in particular it can be used by entities that wish to motivate individuals to cooperate in a system. With this study we conclude that it is always preferable to take measures that expose the worst and best contributors at the same time. We can think of the case of the public list of debtors to social security - causing shame and the lottery of the fate of the Tax Authority - causing honor, which despite being separate entities belong to the same State and complement each other, influencing cooperation in general.

The existence of a minimum objective of cooperation increases the impact of honor and shame. We have seen how in both regular networks and heterogeneous networks the existence of a threshold, below which all individuals have no profit, positively influences cooperation, leading to when applied in conjunction with honor and shame if higher values of altruistic behavior are observed. Thus, if an entity had to choose a model to stimulate cooperation in a context where it would apply honor and shame, it would be more effective to apply a minimum threshold of cooperation to achieve stronger values of cooperation. .

It is important to remember that we are based on a threshold relative to the number of individuals to interact, in the form of the REF equation, so the conclusions are only valid for this model, and that we only studied the cooperation for a threshold that without honor and shame already influenced the cooperation.

The knowledge that an individual has about the structure of the population affects learning. As we mentioned earlier, for the heterogeneous structured population, the learning model implemented is based on the average degree of the network (implying that one individual does not have all the information about the other individuals in the network) and relative to the difference between the accumulated profit of two individuals in the various interactions they have. This model of learning has led to individuals with a higher degree being imitated almost always, since they accumulate more profit from their interactions, regardless of their decisions and the profit they could have on other decisions - much like the metaphor for real life in that "popular" individuals influence other individuals without their understanding of their actual situation. This was an interesting analysis, which led to the insertion of a new form of error in learning.

Heterogeneous structure presents more error than homogeneous structure. We saw how error, or noise in the output graph was superior in heterogeneous structured networks than in regular unstructured networks. We have also seen that a possible cause for this situation was the error inserted in the previous point, since individuals that play with individuals with greater degree affect the evolution of the network. In general, the heterogeneous network structure showed a higher variation in results between a randomly selected simulation and the mean of the results of several simulations.

6.1 Future work

As we have seen, small variations in the models we choose have great impact on the results obtained. Thus, for future work we consider the following variations to the model:

Resource Heterogeneity. In our model all individuals have a modeled amount between [0.1] to offer to the common pot, seen as a percentage of the holdings they had available and the available assets were always 1 for all individuals. However, it would be interesting to study the impact of different amounts available to individuals in the population, as in reality. Thus, we could study a model in which the offers are modeled as a real $\in [0,1] * c_i$, where c_i is an integer that varies from individual to individual and that models the possessions of that subject.

Learning. As we have seen, our model of heterogeneous networks, without honor and shame, has converged to the absence of cooperation, in contrast to the article *Social diversity promotes the emergence of cooperation in public goods games* by Santos et al [16] . To study whether this was due to the modeling of offers as a continuous interval or whether it was due to learning based on the average degree $< k >$ it would be interesting to model learning based on the maximum fitness difference between two individuals according to the rule

$$p = \frac{f(B) - f(A)}{f_{Max}(B) - F_{Max}(A)} \tag{11}$$

where p is the probability that A imitates B.

Evolution. One situation that could be closer to reality in our model is evolution based on the selection of two random individuals. It would be interesting to study a model with heterogeneous structure in which the two chosen individuals were connected in the network and observe the impact of starting at points with different characteristics. Another interesting variant would be to choose two random elements and if they were connected use the previously mentioned learning rule and if they were not use the same studied in the model, modeling the imitation with knowledge about the real conditions of a person we know or someone we do not have real information (imitation of "influencers").

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