I. INTRODUCTION

Motivated by real world porous engineering applications, many correlations between the pressure drop and the geometrical properties of the medium (e.g. porosity, specific area, pore diameter) have been proposed by experimental analysis and posterior fitting [1–4].

Many review papers have been dedicated to the validity of the universality of experimental correlation [3]. Du Plessis [5] and Lacroix [6] correlations evaluated pressure drop correlations within porous foams within a standard error of 30%. Dietrich [1], on the other hand, used over 2500 experimental values reported by over 20 authors to arrive at a correlation that is valid for a range of Reynolds from $10^{-3}$ to $10^{3}$. The geometric parameters needed for both Lacroix and Dietrich correlations are the porosity and the mean particle diameter, however for Du Plessis the tortuosity is involved.

Open cell porous foams are particularly relevant materials for their interesting specific properties of low pressure drop, high porosity and large specific area. In particular, periodic foams are important due to their ease of design for particular problems, increasingly possible with the contemporary advancements in 3-D printing. So, projecting and building foams with a defined structure (e.g. periodic) is becoming a possibility not only to the academia but also a viable option to the industry.

Furthermore, with the increasingly accessible computational power and image techniques (MRI, CT scans) it is now possible to simulate flow in the scale of the structure’s pore and quickly obtain data to analyse the Darcy and inertial regime and replace extremely precise and expensive instruments to measure very low pressure gradients.

Periodic ideal media have been studied as approximations for real porous media [7–9]. Regarding ideal porous media, Krishnan [10] studied the laminar flow and heat transfer in a periodic open-cell structure solving all the spatial and temporal scales with direct numerical simulation (DNS). The results compare reasonably well with the existing experimental and semiempirical models for porosities higher than 94%. Karimian and Straatman [11] tried to characterize a graphitic foam with an ideal structure and simulated flow in the laminar regime concluding the work with developed semi-heuristic models for pressure drop and heat transfer.

Habisreuther [12] developed a random structure where laminar flow was posteriorly simulated as well as on a scanned (MRI) ceramic foam in a Reynolds of 3-373. The pressure drop data from the random structure did not compare well against the data for commercial ceramic porous foams however, the scanned foam results were in good agreement. Tabor [13] used yet another imaging technique, this time to obtain the structural domain of a plastic foam. With a micro-CT scan the authors were able to preform a steady state laminar flow simulation ($Re_s$ (based on strut)< 35) and obtain results where the pressure drop suitably correlates with the flow velocity.

Kuwahara [7] used a periodic array of squares (one of the standard numerical models for porous media) and Large Eddy Simulation, (LES), to analyze macroscopic parameters such as turbulent kinetic energy and pressure drop. The range of Reynolds encompassed the inertial, unsteady and turbulent regime and the porosity was varied for the same geometric shapes. A correlation was obtained for the dimensionless pressure drop at high Reynolds ($Re\geq3000$) which only differed slightly from Ergun’s equations for packed beds. The authors emphasized the need of more numerical experiments to better understand the transition from unsteady to turbulent.

The large porosity values (0.6-0.95) and high surface area along with complex struts with heterogeneous shapes and dimensions make the correlations derived for packed beds erroneous for porous foams. The foams permeability is in the order of $1 \times 10^{-2}$ m$^2$ set by side with $1 \times 10^{-2}$ m$^2$ for packed beds. Analogous to Ergun equations for packed beds other relations might be derived for open cell foams either analytically or experimentally. To some degree of uncertainty these relations detail foam characteristics such as permeability, pressure drop, form/inertia coefficient and tortuosity as functions of other parameters.
of the foam, easier to obtain, such as porosity and specific surface area.

Several researchers have tried to tackled this issue, yet, by virtue of the problem geometric complexity, there is no accord concerning the definition of structural characteristics. Researchers reported theoretical geometrical models that simplify the real foams geometry, and facilitate relations for the morphological characteristics. Lu [14] proposed a cubic cell model which is also the model considered in this work. Other cell models are also possible; Bhattacharya [15] modelled the unit cell as a dodecahedron while Fourie and DuPlessis idealized a tetrakaidecahedron.

The main objective of this work is to develop a closure relation for the Darcy-Forchheimer law based on the detailed 3-D Navier-Stokes solution of the laminar flow in a unit cell configuration. Cubic and ellipsoidal unit cells fluid flow is predicted and integral parameters ($\delta, u_D$) as well as Darcy-Forchheimer coefficients are calculated with Star-CCM+® FVM code. A correlation for $k_1$ and $k_2$ is developed with a theoretical basis and is tested against periodic and non-periodic foams and with the existing correlations with good results for the correlation, for both coefficients, proving to be particularly useful given the difficulty of performing experimental tests at such low Reynolds.

II. PRESSURE DROP CORRELATIONS

A wide range of models for porous media analysis are present in the literature. The first was the Darcy law derived from Henry Darcy’s experimental work with respect to groundwater flow [16], that indicated the flow rate was linearly dependent on the hydraulic head, or, neglecting gravitational effects and generalizing for three dimensions,

$$ - \frac{\Delta P}{L} = \frac{\mu u}{K} $$

Creeping flow, which can be defined as a flow where $Re << 1$, is an upper limit of validity for the Darcy’s law. Deviation from the creeping flow regime implies a need for a different relation for pressure drop and superficial velocity given by the Forchheimer equation:

$$ - \frac{\Delta P}{L} = \frac{\mu u}{k_1} + \frac{\rho u^2}{k_2} $$

Ergun’s [17], pioneering empirical relationship used porosity and a geometrical length scale to describe the pressure drop through a porous media

$$ - \frac{\Delta P}{L} = A \frac{(1 - \epsilon)}{\epsilon^3 d_p^2} \mu u + B \frac{(1 - \epsilon)}{\epsilon^3 d_p} u^2, $$

where $\epsilon$ is the porosity. For spherical particles A and B are 150 and 1.75, respectively, depending both on particle geometry as stated by Macdonald [18], who also recommended replacing $\epsilon^3$ by $\epsilon^{3.0}$.

A. Present work correlation formulation

The present work correlation can be formulated as: Darcian drag force:

$$ F_1 = f_{st} A_{st} \frac{\mu u_p}{\delta} $$

non-Darcian drag force:

$$ F_2 = f_p A_p \frac{1}{2} \rho u_p^2 $$

Force balance:

$$ \frac{V}{L} \frac{dP}{dx} = F_1 + F_2 $$

Pressure-drop correlation:

$$ \frac{dP}{dx} = \frac{1}{\alpha_1} \frac{S_f \tau}{\delta^2 d_s} \mu u_D + \frac{1}{\alpha_2} \frac{S_f \rho u^2}{2 \epsilon^3} \rho u_D $$

With, $\delta$ being the characteristic boundary layer thickness. The relation to Darcy-Forchheimer law and expressions for Darcian and non-Darcian permeabilities:

$$ \frac{dP}{dx} = \frac{1}{k_1} \mu u_D + \frac{1}{k_2} \rho u_D^2 $$

The main target is to find suitable expressions for coefficients $\alpha_1$ and $\alpha_2$ based on detailed simulations and measurements of pressure-drop in open-cell porous structures, where structural parameters are known. From preliminary data analysis, expressions for $\alpha_1$ and $\alpha_2$ have been obtained. Proposed non-Darcian coefficient:

$$ \alpha_1 = c_1 \frac{\epsilon^{1.3} d_{p,i}}{\tau} d_{p,i}^{1.1} + d_s $$

$$ \alpha_2 = \begin{cases} c_2 \frac{\epsilon^{0.3} \rho}{\tau}, & \text{if Periodic media} \\ c_3 \frac{\epsilon^{0.3} \rho}{\tau}, & \text{if Random foams} \end{cases} $$

However, further work is required in order to verify the applicability of these expressions to other foam structures. Additionally, if an expression for $\tau$ could be obtained solely based on structural parameters, then the proposed pressure-drop correlation would present a closed formulation.

III. NUMERICAL METHOD

A. Governing Equations

The governing equations for incompressible laminar flow are given as follows. The continuity equation is

$$ \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = 0. $$

while the momentum conservation equation is given by

$$ \frac{\partial (\rho \vec{v})}{\partial t} + \nabla \cdot (\rho \vec{v} \vec{v}) = -\nabla p + \nabla \cdot (\mu \nabla \vec{v}) + \rho g. $$
By volume averaging the equations and taking into account the theorem for the volume average of a gradient, \( \langle \nabla \psi \rangle = \nabla \langle \psi \rangle + \frac{1}{V} \int_{A_s} \psi \mathbf{n} dA \), the equations are, for continuity,

\[
\frac{\partial \langle \rho \rangle}{\partial t} + \nabla \cdot \langle \rho \mathbf{v} \rangle = 0, \tag{16}
\]

and for momentum,

\[
\frac{\partial (\rho \epsilon \langle \mathbf{v} \rangle)}{\partial t} + \nabla \cdot (\rho \epsilon \langle \mathbf{v} \rangle \langle \mathbf{v} \rangle) = -\nabla (\epsilon \langle p \rangle) + \nabla \cdot \epsilon \langle \mu \nabla \mathbf{v} \rangle + \frac{1}{V} \int_{A_s} (-p + \mu \nabla \mathbf{v}) \mathbf{n} dA + \epsilon \rho g. \tag{17}
\]

The hydrodynamic dispersion \( \nabla \cdot (\rho \epsilon \langle \hat{v} \rangle \langle \hat{v} \rangle) \) is neglected for simplicity. This model holds for Stokes and laminar flow regimes, which can be approximately divided into four categories based on Reynolds number [19].

- \( Re < 1 \), Creeping flow
- \( 1 \leq Re < 150 \), Forchheimer
- \( 150 < Re < 300 \), Laminar unsteady
- \( Re > 300 \), Turbulent

The equations are solved by the Star-CCM+ software using a second order implicit temporal discretisation and the convection and diffusion terms are discretised by second order upwind and central differences, respectively. A polyhedral cell topology is used for its accuracy and less diffusivity compared to tetrahedral and finally, three prismatic layers are added to the cylinder walls.

### B. Domain description and validation

A representative elementary volume (REV) of a unit cubic cell is the starting point. Afterwards three different variations, summarised on Table I are considered based on flow direction and strut shape. The cell is designed to have 90% porosity and is idealized to be fully developed flow, the effect of wall boundaries constraining the medium is not considered as the flow is assumed to be periodic. The boundary conditions are periodic pairs of internal interfaces in the faces of the cube being the motion of the fluid caused by a momentum source per unit volume. This arrangement is made in order to simplify changing the flow direction without the need of geometry change. Regarding the solid surface walls the no-slip condition is assumed. Air is considered for all simulations with specific properties of: \( \rho = 1.18 \text{ kg/m}^3 \) and \( \mu = 1.86 \times 10^{-5} \text{Ns/m}^2 \). A computational domain schematic along with the projected areas of 100 and ABC geometry are shown in Figure 1.

In order to evaluate whether asymptotic grid convergence is attained the following quantities are calculated for each mesh: the maximum, Darcy, and pore velocities, the pore velocity in the momentum source direction and the pressure and friction forces. The probability density function is calculated for both the axial velocity as well as velocity magnitude until refining further made the difference negligible. It can be inferred from figure 2, for both regimes, that a mesh comprising 172623 cells had no advantage in being refined further. The volume mesh used can be seen on figure 3.
C. Solution procedure and post-processing

In order to obtain the integral parameters of \( k_1 \) and \( k_2 \) when high velocity flow is present in the porous media one has to fit the calculated pressure drop and velocities with the Forcheimamer equation and so to have the points equally spaced and not lose resolution on the Darcy regime the pressure source terms imposed are spaced equally in a logarithmic scale and plotted against the logarithm of the obtained velocities. The fit is then made to minimize the sum of the square of the difference between the estimated and simulated value. The solution procedure for the simulations was completed as follows:

- Imposing a momentum source in every cell of the domain
- Obtaining the pressure and shear force on the structure
- Obtaining characteristic velocities in the domain: maximum, Darcy, pore, and axial pore velocity

The pore velocity, \( u_p \), is then calculated as a volume average quantity and can be written in discrete form by:

\[
u_p = \frac{\sum N i u_{mag,i} Vol_i}{\sum N i Vol_i}.
\]  

Regarding the axial pore velocity, \( u_{pi} \), the formula is identical, however, the velocity considered is the velocity component in the direction of the flow. As for the Darcy velocity it is calculated by means of a surface integral of the velocity on all the REV faces, calculating the volumetric flux for each direction (pair of faces).

\[
u_D = \sum_i u_{axial,i} A_i n_i
\]

With the volumetric flux for each face the Darcy velocity is computed calculating the magnitude of the vector as

\[
u_D = \sqrt{u_D^2 + u_D^2 + u_D^2}.
\]

IV. RESULTS

A. Porosity effects

The Dimensionless Pressure Gradient (DPG) equation can be obtained simply dividing the Forcheimamer equation by \( \rho \) and \( u_D^2 \) as:

\[dp^* = \frac{d}{k_1 Re_D} + \frac{1}{k_2}.\]

The DPG is plotted on figure 4 against Reynolds number for the aligned and angled directions. As expected, with an increase in porosity the pressure drop decreases. This means that increasing porosity one increases permeability (see figure 5). The curve topology is conserved throughout the increase in porosity being dependent only on the geometry and the assumption that \( k_1 \) does not depend on \( S_F \) still holds for every porosity. To account for the effects of porosity on the transition to the inertial regime the Forcheimamer number is used. From the definition of Forcheimamer number, \( Fo = u_D k_2 / k_1 \), it can be said that \( u_D = \frac{u_D^2 Fo}{k_1} \) and hence the transition to Forcheimamer regime based on this definition is given by the equation

\[Re_{tr} = D^* Fo \frac{k_2}{k_1}.
\]

Table II lists the results for the ABC geometry. It can be seen that the transition Reynolds number decreases with the increased porosity. To gain a better understanding of this phenomena the evolution of the fraction of pressure to total force is plotted on figure 6 for both 100 and ABC. It is possible to see that the fraction of pressure force is constant, under Stokes flow, for each porosity and that the departure from this state is first made for the higher porosities, confirming the results obtained using equation (22). The pressure force ratio increases as the porosity decreases and this happens not only in ratio but also in magnitude, for both forces, as would be expected from the increases in projected and specific surface area. Regarding \( k_2 \), the values are plotted on figure 7.
TABLE II: Transition Reynolds for ABC geometry and different porosities

<table>
<thead>
<tr>
<th>Porosity</th>
<th>$Re_{tr}$</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.6</td>
<td>3.58</td>
<td>0.49</td>
</tr>
<tr>
<td>0.7</td>
<td>2.79</td>
<td>0.41</td>
</tr>
<tr>
<td>0.8</td>
<td>1.88</td>
<td>0.32</td>
</tr>
<tr>
<td>0.9</td>
<td>1.31</td>
<td>0.22</td>
</tr>
</tbody>
</table>

TABLE III: Pressure and friction forces for $Re = 29$ for 0.9 porosity

<table>
<thead>
<tr>
<th>Geometry</th>
<th>$F_P$</th>
<th>$F_V$</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>1.20</td>
<td>0.60</td>
</tr>
<tr>
<td>AAA</td>
<td>1.22</td>
<td>0.59</td>
</tr>
</tbody>
</table>

FIG. 6: Pressure to total force ratio for geometries 100 (Top) and ABC (Bottom)

The effect of porosity is analogous to the Darcian permeability and $k_2$ increases as the porosity increases. The angled directions are homogeneous with no major difference between modified and non modified cylinder struts. The same cannot be said about the aligned directions where the ellipsoidal shaped struts have considerably lower permeabilities, particularly for higher porosities. It would be expected that the ellipsoidal shape would promote pressure forces and hence it might be reasonable to argue that these would take more importance as Reynolds increase and the inertial regime is in order. Comparing the forces acting on the structure at $Re = 29$ table III is presented. It is clear that for the same velocity the pressure gradient is higher for the ellipsoidal structure and hence the permeability will have to be lower. Proposed by Duda [20] equation (23) was found and takes into account the velocities calculated. Figure 8 plots the tortuosity against the porosity for all arrangements studied. It is obvious that aligning the flow with the structure tends to lower tortuosity and that as porosity decreases tortuosity increases. The former is explained with the increase in $S_p$ while the latter can be explained due to the higher fluid constraints which make it impossible for streamlines not to curve.

\[ \tau \leq \frac{<U>}{<U>_x}, \]  

where $<>$ denotes a spatial average over the pore space.

B. Foam anisotropy and geometric parameters

In order to study anisotropy two angles are considered, $0^\circ$ and $45^\circ$. The pairs of arrangements studied for a porosity of 0.6 are presented and the dimensionless pressure gradient (DPG) is plotted in figure 9. Firstly, the anisotropy in the 1-series foam influences the inertial term by noticeably deviating the curve from the linear regime. This can be attributed to the increase in projected area making the term not negligible for the same velocity. The effect of the inertial coefficient, $k_2$, only becomes notorious when the velocity turns the first term on the RHS low enough to become of the same order of magnitude of $1/k_2$. This happens when $u_D \approx 3 \times 10^{-4}, Re \approx 9$ for the arrangements at $45^\circ$. The geometric parameters are present in the accompanying dissertation.

One can also conclude that modifying the strut shapes translates the curve up meaning the permeability is lower.
Additionally, the curves 100 and 111 coincide for the lower Reynolds number because their specific surface is the same and the velocity of the flow is still not considerable. Consequently, the flow is constrained by the same obstacles and it is not until later (when the projected area starts to become meaningful, due to higher $u_2^2$) that the flow will feel this change. This effect is evident since for the same pressure gradient the velocities obtained are the same and so the intrinsic permeability, $k_1$, is the same for both arrangements.

In the ABC and AAA arrangements, the specific surface is greater and therefore for the same imposed pressure gradient the velocity will be lower which translates into a decrease in $k_1$ that explains the translation of the curve upwards, since for low velocities the second term on the RHS is negligible. The equality of $k_1$ for both ABC and AAA and the negligibility of the constant term for low velocities is also the reason why the curves overlap in this regime.

Conclusions about the possible onset of unsteadiness can also be drawn. For both geometries when the flow is angled, hints of unsteadiness happen at lower imposed dimensionless pressure gradients. Comparing different struts for the same type of flow one can see that, as expected, the ellipse shaped struts anticipate transition for both angled and aligned flows.

Figure 9 sheds some light on the hypothesis of a transition criterion based on the root of permeability or Reynolds number alone. It is often used as a rule of thumb that the transition to the inertial regime happens at $Re_D \approx 10$, however, from figure 9 this is only the case for the angled flow arrangements. For aligned flow this seems not the case as no influence of quadratic effects can be detected from the plots.

The PDF's for the full range of Reynolds are plotted on figure 10. It is apparent from the PDF for 100 geometry that the curves converge as the Reynolds increase, as can be seen by the negligible difference between $Re = 30$ and $Re = 60$. After $Re \approx 60$ a change in regime from steady to unsteady seems to occur for 100 geometry while for a single cylinder it occurs typically at $Re \approx 50$. This can be explained by several factors. The results of a delayed transition to unsteadiness are concordant with both Kevlahan [22] and Price [23] results from simulation and experiments with arrays of cylinders where unsteadiness occurs around $Re \approx 100$ for both of them. Kevlahan also tackled the angled flow problem and the results are concordant with this work’s qualitative analysis, the transition to unsteadiness occurs at $Re \approx 60$, earlier than for aligned flow. This might be explained by the obstruction of the free flow lane as depicted on figure 11. It is expected that the values are different from the ones obtained in this work since a steady solver is used, however, qualitatively the conclusions are sound. Regarding the AAA PDF, the flow is only steady until $Re \approx 30$. The difference can be attributed not to the structure projected area but to the geometry of the ellipsoidal strut. If the axis is perpendicular to the x-axis transition occurs earlier as studied by Jackson [24], who predicted a critical Reynolds equal to $Re = 39$ for a 2-D ellipse with the major to minor axis ratio used in this work. If the minor axis is perpendicular to the flow the inverse happens.

### C. Correlation validation

The validity of the correlation will now be tested with the results obtained above and results offered by Eric Werzner, PhD [26] and André Calado [27]. This analysis will be done for the permeability and non-Darcian permeability separately, as $k_1$ is independent of $k_2$ and is treated as a constant for the whole range of Reynolds. For low Reynolds number ($Re \leq 3$) the pressure force varies linearly with Reynolds number, and the same can be said for the shear force. It is then clear that only one simulation is needed to calculate $k_1$ which will be used for the whole spectrum of Reynolds and all the uncer-
tainty about the flow is put in the variables $\frac{1}{d_L}$ and $\tau$ of equation (9). In order to test the validity of this coefficient several foams are studied. Calado [27] used a ceramic commercial foam of 10 PPI while Werzner analysed several ceramic foam filters with a range of 10-30 PPI and two different materials, $\text{Al}_2\text{O}_3 - \text{C}$ and $\text{Al}_2\text{O}_3$.

These foams also vary in strut shape factor that can be either 0.9 or 1.1. The geometric parameters can be seen on the accompanying dissertation for the various foams and the foams morphology is presented on figure 12. The exponents are fine tuned to minimise the sum of absolute error of all geometries, however, the constants $c_1$, $c_2$ and $c_3$ are only computed, for both $k_1$ and $k_2$, taking into account the studied geometries and only then applied to the foams of Werzner and Calado to avoid validating a fitted model.

1. Darcy permeability, $k_1$

As reported on section II the exponents of the parameters that are theoretically derived had to be adjusted in order to fit the data obtained. In order to arrive at the proposed coefficients a sensitivity analysis is made of equation (12) and the coefficient $c_1$ is calculated for the range of porous foams with equation (24).

$$c_1 = \frac{k_1}{\frac{\epsilon^3 d_L}{S_v^{2/3}} (\frac{d_L}{L_i})^{1.1}}$$  \hspace{1cm} (24)

The results can be seen on figure 13. The coefficients fit nicely to a constant curve of value $c_1 = 0.461$. This value is obtained by averaging only the periodic foam coefficients but demonstrate to be well suited to the range of foams tested. Table IV shows the correlation maximum and average absolute error. The previous analysis is important in order to obtain knowledge on the geometrical and exact parameter dependencies of the permeability.

However, while average absolute errors of 7% are appealing it is important to remember that they are obtained with previous knowledge of tortuosity. To obtain a practical correlation with no dependency of simulation to find tortuosity a constant parameter has been found. The tortuosity parameter present in equation (9) from the theoretical basis is incorporated into equation (12) in order to form the parameter $f_\beta = \frac{\tau}{\frac{\epsilon}{d_L}}$. This parameter is found to be extremely well correlated to porosity and with an equation of the form of equation (25) with $a = 0.029807$ and $b = 3.401212$.

$$f_\beta = a\epsilon^{b\epsilon}$$  \hspace{1cm} (25)

The curve is fitted to periodic porous media (see figure 14) with a correlation coefficient $R = 0.998$ although it can be seen that the results of non periodic foams are also well predicted by the equation. The parameter is used in detriment of tortuosity on the periodic and non periodic foams. The Darcy permeability can be calculated with equation (26).

$$k_1 = \frac{c_1 \epsilon^2 d_s f_\beta}{S_v}$$  \hspace{1cm} (26)

Table V lists the average of absolute errors as well as the predicted values of other literature correlations. A bar graph is presented on figure 15 with a limited y-axis (Lacroix predictions for Calado foams $k_1$ are severely off). The periodic geometries Darcian permeabilities are subject of good predictions with the present correlation.

<table>
<thead>
<tr>
<th>Correlation variation</th>
<th>Max. error [%]</th>
<th>Avg. abs. error [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Theoretical</td>
<td>80</td>
<td>31</td>
</tr>
<tr>
<td>Sens. Analysis</td>
<td>17</td>
<td>7</td>
</tr>
<tr>
<td>$f_\beta$</td>
<td>31</td>
<td>9</td>
</tr>
</tbody>
</table>

**TABLE IV: $k_1$ Correlation variations error**

<table>
<thead>
<tr>
<th>Regime</th>
<th>Correlation</th>
<th>DuPlessis</th>
<th>Dietrich</th>
<th>Lacroix</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_1$ (All)</td>
<td>9</td>
<td>129</td>
<td>24</td>
<td>757</td>
</tr>
<tr>
<td>$k_1$ (Peri)</td>
<td>7</td>
<td>224</td>
<td>30</td>
<td>75</td>
</tr>
<tr>
<td>$k_2$ (All)</td>
<td>31</td>
<td>115</td>
<td>50</td>
<td>33</td>
</tr>
<tr>
<td>$k_2$ (Peri)</td>
<td>25</td>
<td>63</td>
<td>45</td>
<td>20</td>
</tr>
</tbody>
</table>

**TABLE V: Correlations average of absolute error for both regimes for all foams or just periodic (angled for $k_2$)**
2. Non-Darcy permeability, \( k_2 \)

The Forchheimer equation will require both Darcy and non-Darcy permeability in order to compute pressure drop. In order to invert the equation and arrive at \( k_2 \) both the results of \( k_1 \) from simulation will be used to avoid error propagation. The non-Darcian permeability will then be computed from two simulations, one in the Darcy regime and one in the inertial and not from a fitting of a more extensive collection of points. This should mean that \( k_2 \) is treated as dependent of Reynolds number.

The procedure to fine tune \( k_2 \) is identical to the Darcian counterpart and a sensitivity analysis is performed. The \( c_2 \) coefficient is calculated for the range of porous foams with equation (27).

\[
c_2 = \frac{k_{2,\text{sim}}}{2 \cdot \frac{\mu}{\rho} \cdot \frac{1}{\epsilon}} \quad (27)
\]

Figure 16 shows the plotted coefficients and it is apparent that the coefficients are roughly arranged in two groups, periodic and non periodic foams. The coefficient for periodic foams is \( c_2 = 1.49 \) and for the non periodic \( c_3 = 0.96 \).

If this coefficients are used to calculate \( k_2 \) with equation (28) one obtains the error bar graphs presented in figure 18 comparing with the literature correlations and the average error can be seen on table V. The y-axis here is also limited (predictions of Dietrich for Calado foams \( k_1 \) are severely off).

\[
k_2 = c_2 \frac{\mu^2 \cdot \rho^2 \cdot \epsilon^4}{S_p} \quad (28)
\]

As for \( k_1 \) to achieve a closed formulation a tortuosity equation would need to be found. To achieve a usable correlation for periodic porous media the tortuosity angled foams is related with the fraction of specific projected area to porosity with a second order polynomial function with great success. Figure 17 shows the fitting of the form of equation (29) with a correlation coefficient \( R = 0.992 \) and coefficients \( a_2 = 0.02095 \), \( a_1 = 0.12244 \) and \( a_0 = 0.97039 \).

\[
f_{\tau} = a_2 \left( \frac{S_p}{\epsilon} \right)^2 + a_1 \frac{S_p}{\epsilon} + a_0 \quad (29)
\]

Figure 19 shows a bar graph comparing the correlation with a tortuosity relation and the literature correlations for the angled foams. The averaged values are listed on table V where it is obvious that Lacroix’s correlation as a slight advantage predicting permeability on periodic foams. A possible explanation for the increase in error from \( k_1 \) to \( k_2 \) is the fact that the simulations are not well into the
inertial regime which handicaps the evaluation of the $k_2$ weight. This would justify the better results from correlations derived from higher, experimental Reynolds.

V. CONCLUSIONS

Modelling of pore scale laminar flow was detailed within a representative elementary volume with periodic boundaries. The finite volume method of Star-CCM+ code is adequate to solve pore scale flow details using polyhedral cells. Different variations of the unit cubic cell were modelled in order to access the influence of different geometric parameters on fluid flow. The effects of porosity are studied and it is proven that lowering porosity has a stabilizing effect on unsteadiness. By use of a modified Reynolds number based on Forcheimer characteristic velocity a delay on transition to inertial regime is found. Higher values of porosity lead to higher permeabilities, Darcy and non-Darcy for every geometry and flow arrangement. The effects of anisotropy on transition to inertial regime and onset of unsteadiness were studied. The transition to inertial regime happens at lower Reynolds under angled flow when compared to aligned which disproves the hypothesis of a transition criteria based on Reynolds or Darcy permeability and the relevance of a Forcheimer number criterion was highlighted. It is proven that by modifying the structure orientation the onset of unsteadiness changes from a Reynolds of $Re \approx 20$ (111) to $Re \approx 60$ (100). A theoretical correlation for the Darcian and non-Darcian permeability is developed based on the Navier-Stokes with simplifying assumptions. The correlation is tested against periodic and non-periodic random foams and fine tuned for posterior use. For the Darcy permeability a correlation is developed based solely on geometric parameters with an average absolute error of 9.21%, compared with 23.5% for the best correlation from literature. The non-Darcian permeability correlation has an average of 25% absolute error falling short of Lacroix correlation which has 20% for the same periodic media. The present correlation of $k_1$ surpassed several shortcomings of the existing correlations being more accurate for both periodic and non-periodic porous media.

Nomenclature

- $p$ pressure (Pa)
- $u_D$ superficial velocity (m/s)
- $u_p$ intrinsic pore velocity (m/s)
- $k_1$ Darcian permeability (m$^2$)
- $k_2$ non-Darcian permeability (m)
- $A_{sf}$ solid-fluid surface area (m$^2$)
- $A_p$ projected solid area (m$^2$)
- $F_1$ Darcian drag force (N)
- $F_2$ non-Darcian drag force (N)
- $d_s$ characteristic strut diameter (m)
- $d_{p,i}$ characteristic pore diameter in flow direction (m)
- $f_{sd}$ active fraction of $A_{sf}$
- $f_{st}$ active fraction of $A_p$
- $V_f$ fluid volume (m$^3$)
- $V_T$ total volume (m$^3$)
- $S_v$ specific surface area (m$^{-1}$)
- $S_p$ specific projected area (m$^{-1}$)
- $c_1$ Darcian constant
- $c_2$ non-Darcian constant for periodic media
- $c_3$ non-Darcian constant for non-periodic media

Greek Symbols

- $\alpha_1$ Darcian coefficient
- $\alpha_2$ non-Darcian coefficient
- $\delta$ characteristic boundary layer thickness (m)
- $\beta$ characteristic boundary layer thickness nondimensionalised by strut diameter
- $\varepsilon$ porosity
- $\mu$ dynamic viscosity (Pa.s)
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