Sensitivity and Stability Analysis of Quasi-Satellite Orbits

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Abstract

Small bodies — such as Phobos — have assumed a particular interest in scientific communities. They are believed to preserve their formation conditions, which can be insightful to understand the history of the universe. This work tackles the specific challenges that arise from orbiting the Martian moon, Phobos. To begin with, a framework for the mission is established. Then, an exploration of the phase space is performed in order to find sufficiently stable QSOs. Both 2D and 3D orbits are explored. A generic method is developed to asses the sensitivity to injection errors on the spacecraft's initial conditions. Finally, in a step beyond current literature, a strategy to fly-by the moon is implemented. QSO re-insertion is also included. The outcome of this work covers the entire procedure proposed by ESA's CDF Study Report, for Phobos Close Proximity Operations.

Keywords: Quasi-Satellite Orbit, Mars-Phobos System, Stability and Sensitivity Analysis, Phobos Fly-by

1. Introduction

The Martian moon Phobos is a top candidate for future space exploration fleets: it is believed to be a captured asteroid, making it an interesting destination for a scientific mission. These celestial bodies are supposed to preserve their formation conditions, so they can be used as a detailed record to understand the evolution of the Solar system.

The objective of this work is to study the insertion in an orbit about Phobos and to analyze the stability of such an orbit, in the context of ESA's Phobos Sample Return mission [1]. Complementary study about mission design constraints (such as illumination periods and eclipses) is also performed. The feasibility of a fly-by maneuver is also discussed, as a strategy to reach high latitudes.

1.1. Phobos

Small bodies — such as Phobos — have a significant interest in the scientific world. However they present a highly perturbed and extreme environments. Typically, uncontrolled orbits are unstable, and might even escape or impact in short periods of time. Even in the case of actively controlled orbits, the chaotic nature of the motion in these environments can create fundamental difficulties for navigation.

Phobos and Deimos, both natural satellites of Mars, are easier to access and explore, when compared to other small bodies of the solar system. Understanding the physical and chemical state of these moons provides vital information about the origin and evolution of the Martian planetary system and eventually the Solar system.

Many challenges arise when approaching Phobos.

Phobos mass is considerably small, when compared to the mass of Mars. Nevertheless, as shown in previous studies [2], it is not negligible. This difference in the mass of both bodies makes it impossible to navigate about Phobos in a Keplerian-type way, as its region of influence ends below the surface. Besides, in the context of the 3BP, the Lagrange points of Phobos are considerably close to the surface (about 3 km above Phobos' surface [4]).

Also, Phobos orbits about Mars in a slightly eccentric orbit. Even though the moon's eccentricity is small, it makes the dynamical system time-periodic, i.e. a S/C orbiting Phobos experiences a periodic perturbation, which is due to Phobos' non-constant orbital velocity around Mars. In fact, in the proximity of the moon, the eccentricity becomes the dominant term and so it cannot be overlooked [2, 4].

Finally, we must take into account the non-spherical gravitational potential of both Mars and Phobos, even though the latter as an irregular gravitational field, relatively unknown.

2. What is a QSO?

The 3BP describes the motion of three bodies in space, subjected to each others' gravitational attraction. It is a rather complex problem and has been studied extensively.

In the context of the 3BP, a special type of orbit can be defined: the Distant Retrograde Orbits (DRO), also known as Quasi-Synchronous Orbits or Quasi-Satellite Orbits (QSO). These orbits are relatively less well known, when compared to others co-orbital motion of small bodies associated with a planet, such as the Trojan orbits near the Lagrange points and the horseshoe orbits, along the planet's motion.

Imagine an asteroid moving around the Sun, having approximately the same mean motion and mean longitude as a planet, but a different eccentricity. The small perturbation due to the planet's presence slightly affects the motion of the asteroid. This is especially noticed when there is a 1:1 resonance, meaning that their orbital periods are identical. For an observer located at the planet's surface, the asteroid describes a satellite-like retrograde motion. When the distance between the planet and the asteroid is large enough so that it is not a bound satellite, this motion is called a DRO.

2.1. Mars-Phobos System

QSOs assume particular interest for orbiting Phobos, as it is impossible to circumnavigate Phobos in a Keplerian orbit. It is convenient to look at the Mars-Phobos system in the context of the 3BP, making Mars the massive primary, Phobos the other primary and the S/C the third body.

Figure 1 represents the orbital motion of a quasi-satellite orbit, in the Mars-Phobos system.

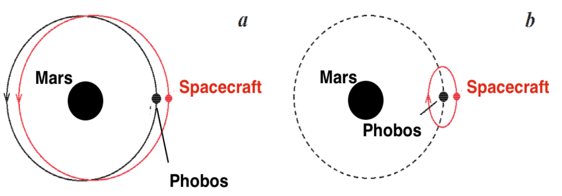

Figure 1: The orbital motion of a quasi-satellite and Phobos, its host.

The QSOs are defined beyond the Lagrange points, as seen from the synodic reference frame, usually used in the 3BP context. The origin of the synodic reference frame is located in the center of the moon, the x axis is always pointing in the Mars-Phobos direction, the z axis points in the direction of the angular momentum and the y axis completes the orthogonal right-handed system. Note that the $x - y$ plane coincides with Phobos' orbital plane around Mars. Such reference frame is identified, in two dimensions, as x and y, in Figure 2.

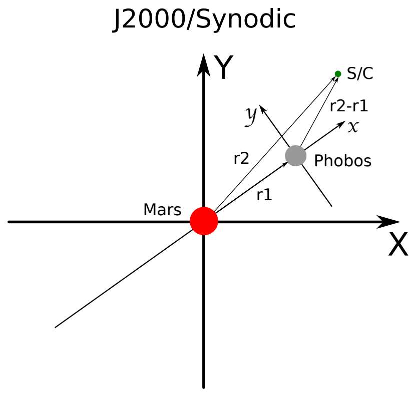

Figure 2: Representation of the inertial coordinate system (J2000 - X, Y), centered in Mars and the rotating (synodic - x, y) coordinate system, centered in Phobos.

In this reference frame, these orbits can be described as an elliptical reference trajectory, moving mainly forward and backward, in the direction of Phobos' orbital velocity. In the inertial reference frame, the S/C still describes an orbit about Mars, slightly perturbed by the presence of Phobos.

Throughout this work, the term *epicycle* will be adopted, to describe each ellipsoidal drifting trajectory of the S/C about Phobos, as seen in the synodic reference frame. A QSO refers to the entire dynamics of epicycles.

2.2. Stability

In order to analyze the stability of a certain orbit, the concept of stability must be defined.

Theoretically, an epicycle in a stable motion will continue to drift back and forth indefinitely. However, the naturally perturbed environment of the Mars-Phobos system will cause the spacecraft to eventually drift away [2].

The QSO explored in this work are observational orbits, and eventually we want to get to a high gate for a descending maneuver. From a mission's design point of view, it is enough to ensure that the epicycles are stable for sufficiently long period of time.

The stability definition used in this work is as follow:

- An orbit is considered to be sufficiently stable about Phobos if it orbits the moon for, at least, a period of 30 days. The S/C should neither collide against the surface of Phobos nor get more than 1000 km away from it.

Similar studies use identical conditions for defining stability.

2.3. State of the Art

Over recent years, fully numerical integration methods are used to simulate with high accuracy the environment in the Mars-Phobos system.

QSO are reviewed from a preliminary mission design point of view by Gil and Schwartz in 2010 [2]. Different phase spaces scan are completed, starting at a fixed position, in the y axis of the synodic reference frame. A non-coplanar orbit is presented, along with some important mission analysis issues. Results show that the velocity component toward Phobos, at the examined insertion point, needs to be more accurate, in the order of 1 $\mathrm{m\,s^{-1}}$. It also appears that it is harder to obtain small QSO than large ones, possibly due to the higher-order terms of Phobos' gravitational potential.

The conditions required for a stable QSO are inspected by Spiridonova et al, in 2017 [3]. At the same time, the feasibility of a global ground-track coverage is investigated. Systematic phase space explorations are performed, varying the out-of-plane velocity v_z, starting at the negative x axis of the synodic reference frame. The simulations start with Phobos at perimartem. Mars gravity field of order and degree 20, Solar radiation pressure and the point-mass gravity field of the moon are considered. Latitudes up to 30° N/S are possible, assuming that the control of the injection velocity errors is in the order of 0.2-0.3 $\mathrm{m\,s^{-1}}$. Orbits reaching latitudes higher than 45° are shown to be more rare and less stable.

3. Numerical determination of QSO

The formulation of the code used for this work is described, including a short runtime comparison. The outcome produced is then compared to the results in the available literature, in order to validate the code.

3.1. Code Formulation

A code is developed to simulate quasi-satellite orbits about Phobos. It replicates the space environment with accuracy to the 1 m range. Therefore, the equations of motion become highly complex and must be interpreted by numerical methods. Different numerical methods have been developed in order to numerically integrate ordinary equations of motion. The code developed in this work uses an 8th order Runge-Kutta (RK) integrator. Such methods are considered a standard technique for the numerical solution of ordinary differential equations.

The Range-Kutta integrator used for this type of work is an 8th order RK, with an embedded 7th order method for step size control. The RK integrator does not conserve energy, problem that becomes palpable over long times scales. The maximum number of days simulated in this work is 30, which is not considered a long time span. If the integration's step size is reduced, the RK should not suffer significant losses of energy. On the other hand, a very small step size increases the round-off errors and the computational effort.

Different step sizes are used to propagate the complete force model for the QSO $(x, y, z) = (0, -100, 0)$ (km), $(v_x, v_y, v_z) = (-20, 0, 0)$ $(\mathrm{m\,s^{-1}})$. The preferred step size is 10 s, as it proves to be a suitable compromise between the absolute error and the computational effort of the simulations. After 30 days, it has a position error in the 10^{-5} m order, when compared to 1 s time step. Errors associated with velocity are residual.

In order to estimate the error associated with the integrator, an orbit around Mars is simulated. The point-mass model is adopted, so a comparison with analytic results can be established. The absolute error is calculated, for different numbers of revolutions. Results are shown in Table 1.

Table 1: Position and velocity errors, computed for different numbers of revolutions. Orbital parameters considered for the integration: $a = 9377$ (km), $e = 0.0151$ and $i = 0°$.

time step: 10 s	Absolute Error	
	Position (km)	**Velocity** $(\mathrm{m\,s^{-1}})$
100 revs	0.1099e-03	2.518e-05
50 revs	3.727e-07	3.500e-07
10 revs	3.075e-07	3.700e-07

After 100 revolutions, which are completed in approximately 30 days, there is a position error associated with the integrator, and it is expected to be in the 1 m order of magnitude.

Force Model

The force model applied in this work includes all major perturbation forces, Mars gravity field (4th degree and order spherical harmonic model), Phobos gravity field (3rd degree and order spherical harmonic model), Sun as 3rd body perturbation and the Solar radiation pressure.

The relative position of the planets and the Sun is taken from an ephemerides file from SPICE. Numerical simulations with ephemerides data grants the inclusion of all relevant perturbations and, thus, gives confidence in the obtained results for practical applications.

In order to avoid unexpected behavior of the S/C with respect to Phobos, the motion of the moon is also determined by numerical integration methods, along with the motion of the spacecraft.

Code Runtime Analysis/ Comparison

Every function relevant to the simulation is written in C code, including the integration itself, using Python only as an user's interface, to call these functions.

Initially, numerical simulations were conducted in pure Python and afterwards in Cython (tool that allows a Python module to be implemented like C code). Using Cython, it is also possible to call pure C functions from Python, allowing for Python code to bypass its own (slow) runtime and run it in other context, which is much faster.

A standard QSO, defined by $(x, y, z) = (0, -100, 0)$ (km) and $(v_x, v_y, v_z) = (-20, 0, 0)$ ($\mathrm{m\,s^{-1}}$), is numerically propagated for 10 days, in each environment. The runtime of the simulations is noted and compiled in Table 2.

Table 2: Runtime for a 10 days simulation

	Python	**Cython**	**C**
time (s)	877.78	279.13	20.55
time (normalised)	1.0	0.32	0.023

A simulation run on C code wrapped in Python represents a speed-up of 43 times, when compared to the pure Python routine. As expected, compiled languages are much faster than interpreted ones.

3.2. Code Validation

The code is validated by comparing its outcome to some of the results presented in literature. In each validation process, the force model is adapted accordingly to the perturbations included in the reference. Overall, the results show that the code developed is consistent with the literature. Nonetheless, it should be further tested, in order to strengthen its validity.

4. QSO Solutions and Stability

The phase space is investigated for 2D and 3D orbits in order to find to find sufficiently stable QSO. Such orbits are presented, and additional mission analysis issues are inspected.

A method is developed to test the robustness to injection errors on the position, velocity and both of them simultaneously. For each orbit investigated, a stability region is identified. This region defines the boundaries for a sufficiently stable orbit insertion.

4.1. Sufficiently Stable QSO

Sufficiently stable QSO are defined as orbits that are stable, under the stability criteria discussed in 2.2, for a period of 30 days.

A point in the y axis seems to be the preferable for a QSO insertion, as it appears to be more forgiving to insertion errors [2]. For this reason, throughout this chapter, every simulation starts at a point in the y axis of the synodic reference frame. For convenience, the simulations were initiated with Phobos at periapsis of its orbit around Mars. The initial epoch used is defined as the first periapsis' passage since 14 November, 2025 [1]. A phase space exploration is completed, exclusively for planar orbits, i.e. orbits with no out-of-plane component in the initial condition. A stable QSO is identified. Based on such orbit, another phase space exploration is completed for the out-of-plane components, $z - v_z$. A 3D orbit is then identified, and issues related with mission analysis issues are investigated (Sun position, eclipse conditions).

Phase Space Exploration

In order to find sufficiently stable, planar QSOs, a search is conducted in the phase space of the system. For such exploration, the initial position y and the initial velocity v_x vary, while all other initial position and velocity components are zero. The results are depicted in Figure 3.

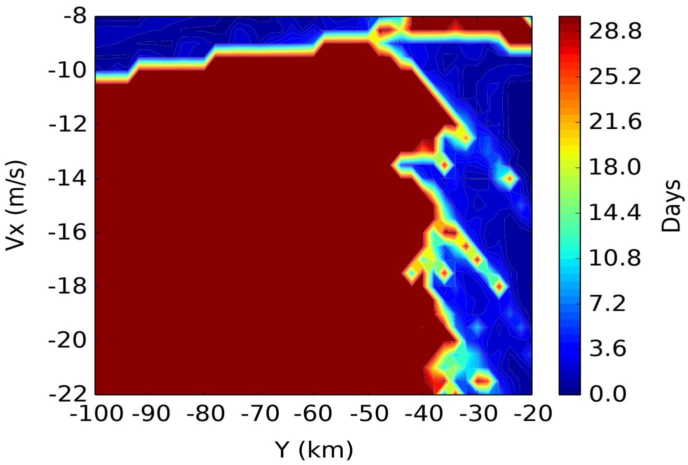

Figure 3: Phase Space Exploration of 2D QSOs. The color gradient represents the number of days the orbit is stable, warmer colors represent a longer period of stability.

Two different regions are noted, separated by injection radius $y_0 \approx -35$ (km). In the first region, characterized by $y < -35$ (km), a wide region of stability is observable. Only for values $v_x > -10$ ($\mathrm{m\,s^{-1}}$) instability can be perceptible, in this region. In the region closer to Phobos, a much smaller stability area appears for $v_x > -9$ ($\mathrm{m\,s^{-1}}$).

A stable 2D orbit is then selected from Figure 3, defined by the following initial conditions, $(x, y, z) = (0, -50, 0)$ (km) and $(v_x, v_y, v_z) = (-12, 0, 0)$ ($\mathrm{m\,s^{-1}}$). To inspect the stability of different 3D orbits, and based on the latter nominal QSO, the phase space is explored for the out-of-plane components, z and v_z. The exploration is

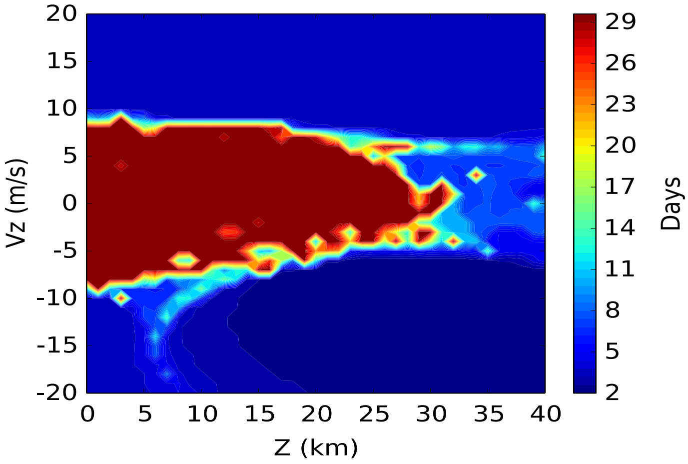

Figure 4: Phase Space Exploration of the nominal orbit selected.

depicted in Figure 4.

A stable orbit is randomly selected from Figure 4. The orbit is described by the initial conditions: $(x, y, z) = (0, -50, 15)$ (km) and $(v_x, v_y, v_z) = (-12, 0, 7)$ $(\mathrm{m\,s^{-1}})$. Figure 5 shows the latitude and longitude wrt Phobos, reached by this trajectory.

This particular orbit is capable of consistently reaching latitudes as high as 40°, while being relativity close to Phobos (≈ 40 (km)). A pattern is noted: when the spacecraft is at its farthest point from Phobos, it tends to be at relatively low latitudes, typically inside the $[-10°, 10°]$ interval. Only then, it will approach Phobos, in a upward/ downward motion, covering higher/ lower latitudes.

In a 3D orbit, it is obvious that higher latitudes are achievable. Comparing to 2D orbits, the difference in latitudes is notable, and it indicates that these types of orbits are ideal for high latitudes observations.

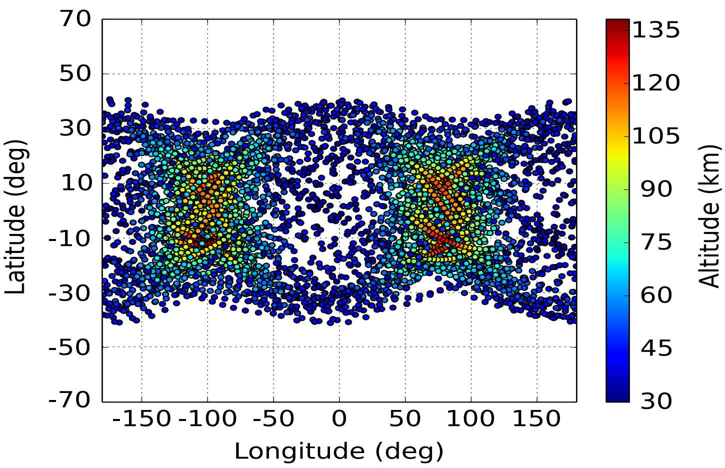

Figure 5: Latitude, longitude and altitude with regards to Phobos, reached by the spacecraft. Cooler colors represent lower altitudes.

Eclipse conditions and Sun elevation angle are evaluated over 30 days. This orbit is much less propitious to suffer from eclipses caused by Phobos, when compared to other 2D QSOs studied. This fact is explained by the rate of change in the z direction, which removes the S/C from the moon's shadow cone, along with Phobos relatively small dimensions. In the 30 days simulated, the spacecraft is in eclipse conditions for only 3.6 days. The closer an orbit is from Phobos, the faster it is. This causes the frequency of the Sun elevation angle to increase as the distance to the moon decreases, while preserving a sinusoidal shape.

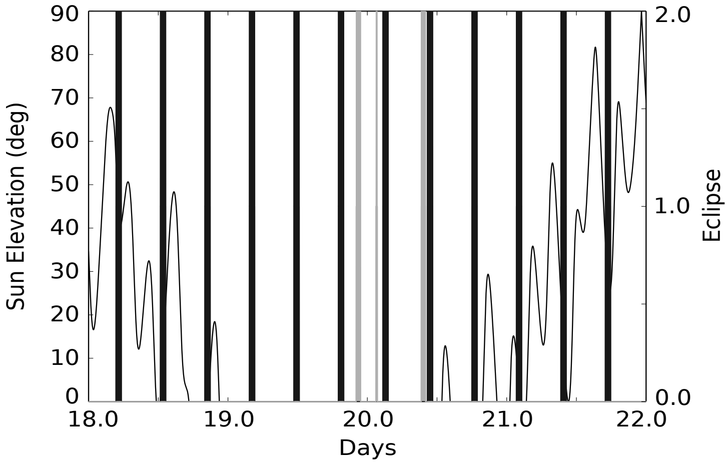

Figure 6: Eclipse conditions and Sun elevation, day 18-22. Detail of a complete 30-day simulation.

Figure 6 presents a 4 days detail of the simulation. It shows a combination of eclipse conditions and Sun elevation. While only a small eclipse is caused by Phobos around day 20 (represented by light gray columns), Mars' caused eclipses appear to be a periodic event (represented by dark columns), with a period rounding the 6 hours. Similar investigations to different QSOs show similar eclipse conditions, caused by Mars. These results are plausible, because even though the position of the spacecraft with relation to Phobos differs, the relative position regarding Mars is approximately the same, causing the eclipse conditions generated by this planet to be rather similar. Note that all the QSOs propagations start at the same epoch.

Overall it is noticeable the relative ease to establish a stable orbit farther away from Phobos. Closer QSO have a much smaller stability region, as seen from Figure 3. This outcome corroborates the results presented in [2].

Different 3D orbits from Figure 4 are explored. Yet, none of them is capable of reaching latitudes higher than 45°. This results are confirmed by [3]. According to ESA [1], higher latitudes are aspired

(50° - 60° N/S). Besides, it is also desired to get closer to the moon's surface for better observations.

These evidences, along with the favorable Sun elevation conditions for closer orbits, seem to suggest the investigation of a nominal 3D QSO, closer to the moon. Hence, a phase space exploration is performed, based on the nominal QSO $(x, y, z) = (0, -30, 0)$ (km), $(v_x, v_y, v_z) = (-8.5, 0, 0)$ $(\mathrm{m\,s^{-1}})$. Such exploration is limited to the out-of-plane components z and v_z, while all other components remain unaltered, from the nominal orbit. However, the exploration proves that the stability region is much smaller, making it much more difficult to obtain a 3D, stable QSO, the closer we get from Phobos.

An alternative strategy to reach higher latitudes at low altitudes is presented in section 5.

4.2. Sensitivity and Stability of QSOs

In this section a method is implemented for studying the sensitivity to injection errors (non-nominal position and velocity), and to assess the orbit stability afterwards. The methodology used is divided in two steps.

In the first step, each component of the orbit's initial conditions (x, y, z) and (v_x, v_y, v_z), is individually adjusted and the respective stability is inspected.

In the second step, a not-ideal orbital injection maneuver is simulated, so simultaneous injection errors must be taken into account. These small errors are modulated by probability distributions, in sets of 200 simulations. For this injection simulation, the position and velocity errors are individually evaluated, to gain some valuable insight about the sensitivity to perturbations on each of them. Then, both position and velocity errors are assessed at the same time, using the information collected before to better estimate the deviations allowed by the system. Based on the latter investigation, a stability region is defined as a set of constraints (boundaries), that ensures sufficiently stable QSOs.

Due to the apparent facility to obtain stable orbits further away from Phobos, as highlighted in Figure 3, an orbit closer to the moon is chosen as QSO A, defined by these initial conditions, $(x, y, z) = (0, -25, 0)$ (km), $(v_x, v_y, v_z) = (-9, 0, 0)$ $(\mathrm{m\,s^{-1}})$. This nominal orbit is stable for, at least, 30 days.

Sensitivity to Perturbation on each component

Figure 7 represents the sensitivity analysis on the y. It is observable that the nominal trajectory, though stable, is close to an area of instability, which is defined by $y < -26$ (km). Also, it is not possible to obtain a stable orbit using an injection radius $y >= -15$ (km), as these orbits eventually collapse with the surface of Phobos.

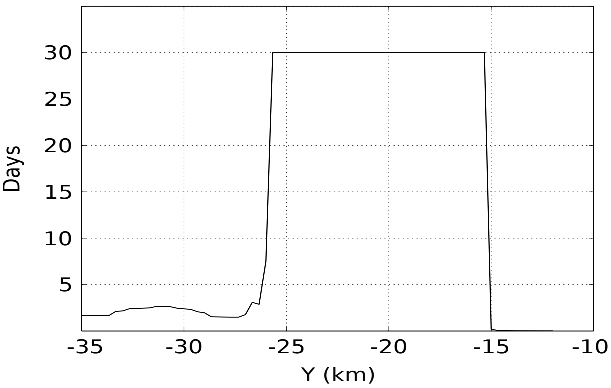

Figure 7: Sensitivity to perturbation on y component.

Orbits with different values for the y position are represented in Figure 8. All other components remain unaltered from the nominal QSO. The extreme proximity between the orbit defined by $y = -16$ (km) and the surface of the moon is noticeable. This QSO is defined as one of the closest to Phobos surface that is stable for 30 days.

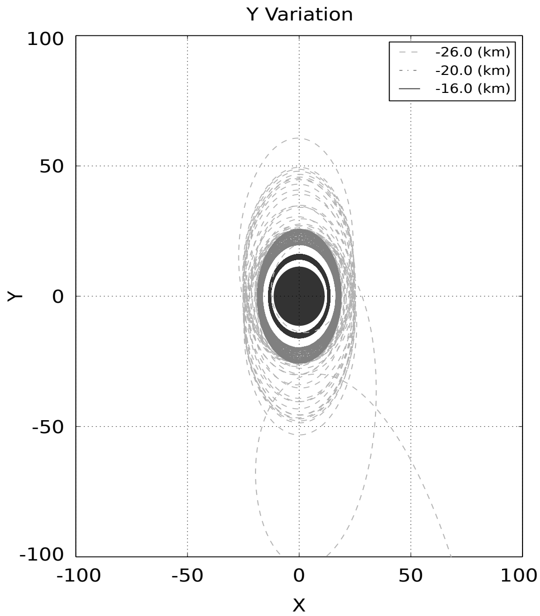

Figure 8: Different orbits representation, with different y components.

Even though other components have different tolerances, their respective sensitivity analysis is not presented, due to their similar "step-like" shape. Tolerances for each individual component are noted and presented in Table 3.

It is already apparent that an orbit closer to Phobos demands tight control over the initial injection components, especially on the velocities as seen in Table 3.

Table 3: Stability Intervals for each component of QSO A.

	QSO A	Stabilty Interval
Position (km)	x	$[-1.3; 1.2]$
	y	$[-26; -16]$
	z	$[-2.0; 1.8]$
Velocity ($\mathrm{m\,s^{-1}}$)	v_x	$[-9; -7.6]$
	v_y	$[-0.7; 0.6]$
	v_z	$[-0.7; 0.8]$

Real Injection Simulation
The second step is to simulate an error of an injection maneuver, which can occur simultaneously in the 6 component analyzed. Three test campaigns are carried out to determine the robustness to initial position dispersion, to velocity dispersion, and to both of them simultaneously.

In order to simulate this errors efficiently, each of the components is modeled by a Gaussian distribution $N(\mu, \sigma^2)$, characterized by a mean μ and a standard deviation σ. According to the distribution definition, 95.44% of the samples of observations are within the interval $[\mu - 2\sigma, \mu + 2\sigma]$. 200 simulations are completed, for each investigation conducted.

On a first phase, the position error is studied. Two investigations are conducted. Table 4 presents the number of stable orbits for, at least, 30 days, from the set of 200 simulations completed.

Table 4: Position dispersion investigation.

Nr	Pos (km)	μ	σ	30 days
1	x	0	0.2	148
	y	-25	1	
	z	0	1	
2	x	0	0.2	198
	y	-20	1	
	z	0	1	

The difference between the distributions used in investigation number 1 and 2 lies on the mean value for the y component. This adjustment is made based on the tolerance presented in Figure 7, because the μ used for the y distribution in investigation number 1 is very close to an instability region. 198 stable orbits are obtained for investigation number 2, which indicates a much more robust alternative position for the y parameter.

On a second phase, the velocity error is assessed. Table 5 summarizes the results from the two investigations carried out.

Table 5: Velocity dispersion investigation.

Nr	Vel ($\mathrm{m\,s^{-1}}$)	μ	σ	30 days
3	v_x	-9	0.5	109
	v_y	0	0.2	
	v_z	0	0.5	
4	v_x	-8.5	0.5	156
	v_y	0	0.2	
	v_z	0	0.5	

Once again, it is noticeable that the only difference between the distributions used for investigation number 3 and 4 is the μ used to modulate the v_x component.

As suggested by the v_x stability interval, presented in Table 3, the mean value for the v_x distribution is adapted for a non-nominal value. This decision results in a bigger number of stable orbits, with 156.

The position and velocity errors are inspected in the third phase. Two investigations are conducted, and the results are presented in Table 6.

Table 6: Position and velocity dispersion investigation.

Nr	Component	μ	σ	30 days
5	x	0	0.5	126
	y	-25	0.25	
	z	0	0.5	
	v_x	-9	0.1	
	v_y	0	0.2	
	v_z	0	0.2	
6	x	0	0.5	154
	y	-20	0.25	
	z	0	0.5	
	v_x	-8.5	0.1	
	v_y	0	0.2	
	v_z	0	0.2	

Based on investigation number six, a 6-dimensions stability region is identified. Within such region, 87 orbits are recognized, and all of them are stable for 30 days.

Table 7: Stability region, identified for an injection maneuver.

Position (km)	Velocity ($\mathrm{m\,s^{-1}}$)
$-0.25 \leq x \leq 0.25$	$-9 \leq v_x \leq -8$
$-20.5 \leq y \leq -19.5$	$-0.3 \leq v_y \leq 0.3$
$-1 \leq z \leq 1$	$-2 \leq v_z \leq 2$

Any injection maneuver whose components (position and velocity) are contained inside such region, should guarantee a sufficiently stable QSO.

The QSOs investigated suggest that the stability region is smaller, for an orbit closer to Phobos. This evidence advises that it is preferable to perform the injection maneuver farther away from Phobos. Results also suggest that for an injection maneuver in the y axis, the components x and v_y are the most sensible to injection errors.

This methodology is generic and can be applied to any QSO.

5. Phobos' Fly-by

Different methodologies to approach Phobos' surface are explored. For convenience of description, the term *fly-by* is used to describe such approaches. The objective is to have the spacecraft go over Phobos' surface at low altitudes, 5-10 km, while reaching high latitudes, 50° to 60° N/S.

Planar fly-by and out-of-plane fly-by are separately presented. For each fly-by inspected, the respective return to a QSO is assessed. The feasibility of a Phobos fly-by is also discussed, and the results are compared with predictions from literature.

Every simulation conducted starts at the same epoch. The diverse maneuvers presented are also initiated from an identical position, obtained after one revolution of the nominal QS0 defined by $(x, y, z) = (0, -50, 0)$ (km) and $(v_x, v_y, v_z) = (-10, 0, 0)$ $(\mathrm{m\,s^{-1}})$.

5.1. Planar Fly-by

Planar fly-by are defined as close approaches to the surface, defined on the $x-y$ plane, the orbital plane of the Mars-Phobos system.

Preserving the initial position, the phase space is explored for the velocity components v_x and v_y, constrained to $[-10, -7.5]$ and $[-1, 1]$ $(\mathrm{m\,s^{-1}})$ respectively.

Trajectories that pass within $[5, 10]$ (km) of altitude (in the closest point to the moon), are selected and highlighted in Figure 9. The phase angle is measured in the synodic reference frame, counting from the positive x axis in a counter-clockwise direction.

Two different regions are notable. The first region is characterized by phase angles between -50° and 50°, and the second region is limited to angles between -125° and -150°. Even though there is a discontinuity in the phase angle distribution, the entire range of altitudes seems to be accessible, in both regions.

Three distinct fly-by are computed, and different altitudes are achieved, while maintaining similar phase angles. The collection of maneuvers is represented in Figure 10.

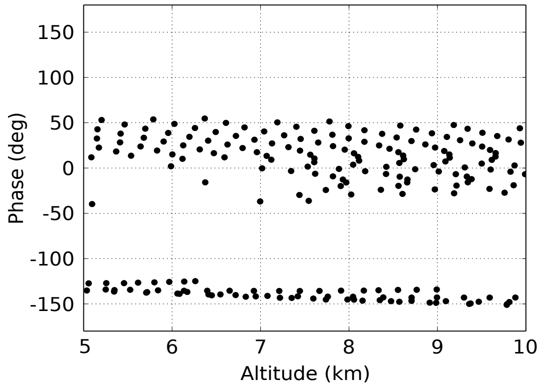

Figure 9: Collection of points selected, for a potential fly-by maneuver.

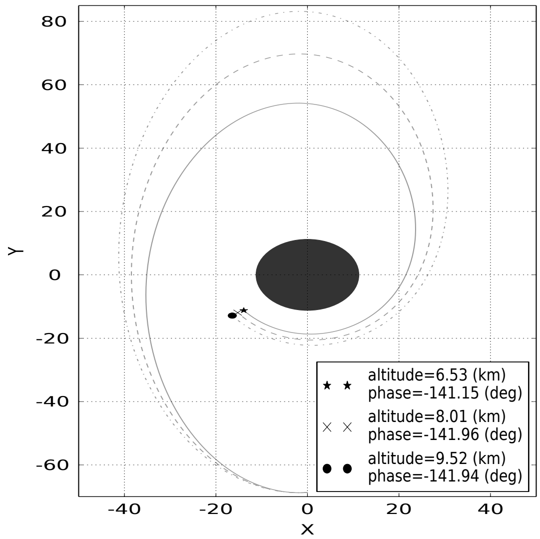

Figure 10: Representation of the fly-bys, in the synodic reference frame.

5.2. Return to QSO

After the fly-by is executed, the S/C will either drift away from Phobos or collide with the moon's surface. A trajectory correction maneuver (TCM) is required, to prevent either situation.

Figure 11 show a fly-by maneuver, followed by a re-insertion in a stable QSO. Point 1 represents the initial position, where the first impulse is applied. Point 2 represents the position in which the QSO re-insertion maneuver is exercised. This point is selected as a crossing of the y axis (identified before as preferable for QSO insertion), after the surface approach.

Table 8 shows the cost ($\triangle V$) of each maneuver, as well as the QSO in which the spacecraft is inserted afterwards. Note that the orbits insertion are based on similar positions, at a similar epoch. This is an heuristic process, so less expensive trajectories might be found.

After the re-insertion maneuver, every trajectory

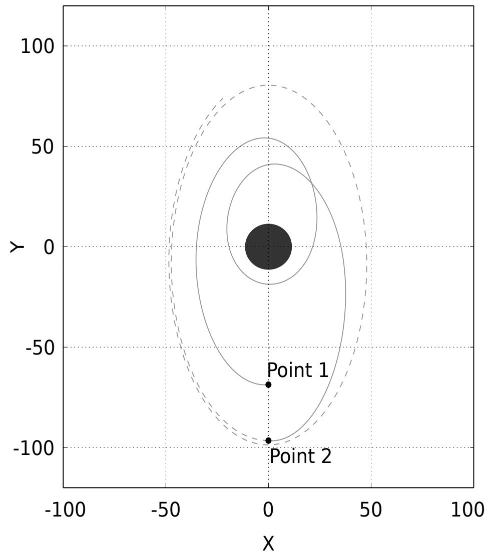

Figure 11: Representation of a fly-by maneuver and respective QSO re-insertion.

is stable at least for 5 days. This period is enough to correct the trajectory, if necessary.

Table 8: Required $\triangle V$ for impulses in Point 1 and 2. Re-insertion QSO is also noted, for each Fb.

$\triangle V$ ($\mathrm{m\,s^{-1}}$)	Fb 1	Fb 2	Fb 3
Figure Line Style	10 —	10 - - -	10 - . -
Point 1 Point 2	1.94 4.93	1.46 4.91	1.21 5.90
Total	6.87	6.37	7.11
QSO			
(x, y, z) (km) (v_x, v_y, v_z) ($\mathrm{m\,s^{-1}}$)	(0,-94,0) (-12,0,0)	(0,-118,0) (-12.30,0,0)	(0,-130,0) (-14,0,0)

The phase angle reached by the spacecraft, represented in Figure 9, is influenced by the maneuver's initial position. The epoch also influences the range of phase angles reached. This indicates that it might be possible to reach any point in the Mars-Phobos orbital plane through a coplanar fly-by.

The spacecraft behavior should be inspected if no impulse is exercised at Point 2. Obviously, a spacecraft drifting away from Phobos is preferable to a collision with the moon's surface, as eventually it can return to Phobos close proximity (the S/C is still orbiting Mars), whereas a crash is irreversible. An operational delay of approximately 8 hours should also be considered [1].

Table 9 summarizes the outcome of the 3 maneuvers simulated, 8 hours and 24 hours after crossing Point 2, if no correction maneuver is applied.

Table 9: Spacecraft behavior, after 8 and 24 hours, if no TCM is applied at Point 2.

	Fb 1	Fb 2	Fb 3
Figure Line Style	10 —	10 - - -	10 - . -
8 hours 24 hours	Crash -	- Drift Away	- Drift Away

All these issues must be considered when designing a fly-by mission to Phobos.

6. Out-of-Plane Fly-by

Out-of-plane fly-by can be defined as any approach to the moon's surface, not constrained to the $x - y$ plane of the synodic reference frame.

The objective here is to simulate a high latitude, low altitude fly-by, departing from a planar QSO. The initial position for the fly-by maneuver remains unaltered.

The first approach is to directly apply an impulse in the z component of the initial velocity. It is noted that only at the expanse of high delta-V it is possible to reach 50 °. A new strategies should be implemented for an out-of-plane fly-by.

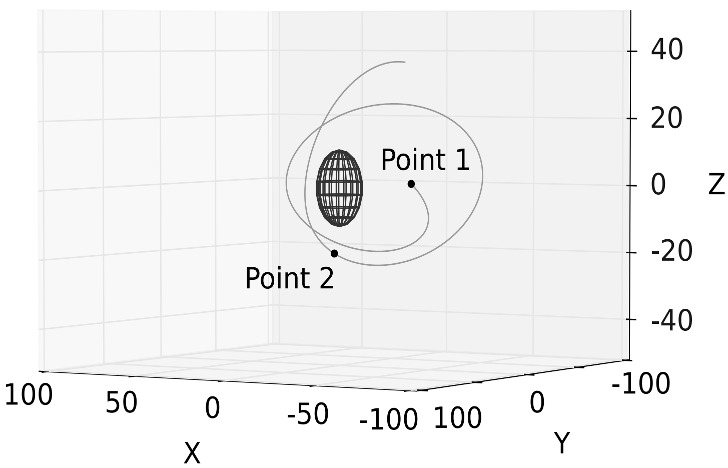

Figure 12: Representation of the out-of-plane fly-by, trajectory A, in the synodic reference frame.

Previous results for coplanar fly-bys show that the initial velocity in the y component is much more determinant for the trajectory contour. Therefore, the phase space is explored exclusively for the v_y and v_z components, constrained to $[-2, -2]$ and $[-5, 5]$ ($\mathrm{m\,s^{-1}}$) respectively. A trajectory reaching 63°S after 9 hours, at 9.82 (km) of altitude is selected, hereafter called trajectory A. Such trajectory, defined by $(v_x, v_y, v_z) = (-9.57, 1.60, -4.50)$

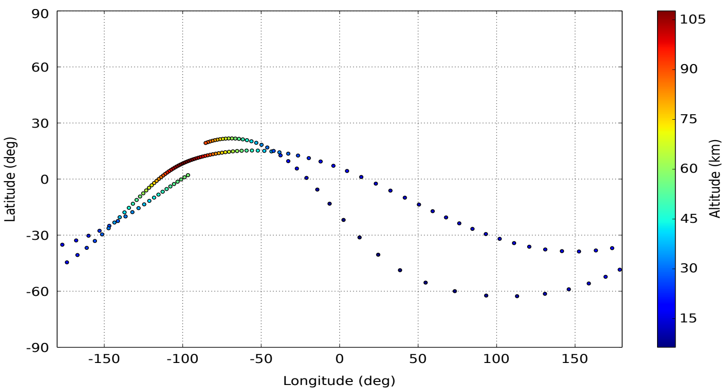

Figure 13: Latitude, longitude and altitude with respect to Phobos, in trajectory A.

($\mathrm{m\,s^{-1}}$), is presented in Figure 12. Figure 13 represents the Phobos' latitude and longitude reached by such trajectory.

6.1. Return to QSO

Considering that the departure orbit is a 2D QSO, the orbit should also be re-inserted into a planar QSO.

Trajectory A eventually drifts away from Phobos, if no TCM is applied. Therefore, two additional maneuvers are needed to correct the trajectory: phasing and reacquisition [1].

The phasing maneuver can be seen as the impulse that counteracts the velocity in z, which originates a planar trajectory (depending on the position where the impulse is applied). The reacquisition corresponds to the final QSO insertion maneuver. Both maneuvers can be applied either simultaneously or separately. The cost of each maneuver is noted and assembled in Table 10, for trajectory A.

Table 10: Cost of each maneuvers, for trajectory A.

$\triangle V$ (m/s)	Trajectory A	
Phasing (P)	8.58	11.50
Reacquisition (R)	6.94	
Total	15.52	11.50

Results from the phase space exploration confirm that the deorbiting maneuvre can always be kept below 5 ($\mathrm{m\,s^{-1}}$) approximately, according to [1].

Trajectory A is a relatively faster fly-by (compared with other fly-by maneuvers not presented). This option is preferable, as it minimizes the propagation's error and uncertainties of Phobos gravitational field. It is, however, more expensive.

The total $\triangle V$ for phasing and reacquisition maneuvers can be estimated on the order of magnitude of 10 ($\mathrm{m\,s^{-1}}$) [1]. Table 10 shows results consistent with such estimation.

7. Conclusions

In this work we are are able to obtain several QSOs around Phobos, that can be used as observation orbits.

The phase space is partially explored, as the primary search is limited to the an injection position in the y axis, and an injection velocity v_x. All other parameters are considered to be zero. As a consequence of that, investigations conducted in this work are limited to insertion on the y axis region. The explorations suggests that orbits closer to Phobos are more difficult to obtain, possibly due to the irregular gravity field of the moon. Sufficiently stable orbits are determined. Additional mission design issues are interpreted, such as eclipse conditions and Sun elevation angle. Eclipse conditions can not be avoided, due to the relative close proximity between the probe and Mars/Phobos. A methodology is developed to examine the sensitivity to injection errors, and the respective stability. Stability regions can be assessed as a result of such methodology. Results suggest that, for an injection maneuver in the y axis, the initial components x and v_y should be carefully controlled, injection speed should be controlled to within a fraction of a meter per second and the injection position should be controlled within the order of 250 meters. It is confirmed that 3D QSO are able to consistently reach latitudes as high as 40° N/S. Therefore, the requirement of high latitude exploration can be met using fly-by maneuvers. The feasiblity of a Phobos' fly-by to high latitudes is also successfully demonstrated. The cost for such maneuvers is in accordance with ESA estimations.

In conclusion, we were able to simulate the close proximity operations, as prescribed by the Phobos Sample Return CDF Study Report from ESA.

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[1] ESA. CDF Study Report - Phobos Sample Return - Phobos Moon of Mars Sample Return Mission. 145(June):1–254, 2014.

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