Valuation and Financing of

Early Stage Technological Companies:

Analysis with a Real Options Structural Model

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Abstract

This paper assesses the impact of a mixed capital structure model, considering both equity and debt, on the valuation of early stage technological companies (ESTCs).

Traditional valuation methods are not suitable for the analysis of ESTCs, not being able to correctly assess the impacts of the high uncertainty and opportunities associated with such ventures. Models based on real options analysis, have characteristics that make them particularly suited to deal with ESTCs, such as the incorporation of the volatility on the valuation to represent the uncertainty associated with the underlying assets, in this case the EBIT of the early stage companies. Modelling the EBIT growth as an arithmetic Brownian motion enables the possibility of negative cash flows. This is essential when dealing with early stage companies, which excludes most of the real options models, developed to study market traded assets, from being viable in assessing the value of ESTCs. The model is developed within a static capital structure framework, which restricts the issuance of debt to a single stage. The proposed model, built on these assumptions, is a flexible base that can be easily implemented and expanded.

A case study based on Fitbit Inc, was conducted, being, to the authors knowledge, the first application of a structural model to the valuation of an ESTC, in a reality based setting. The results of the model show that the valuation of the company is related with its capital structure and the case study provides a valuation for Fitbit's equity consistent with its market value.

Keywords: Early stage companies ; Real options valuation (ROA); Structural model; Arithmetic Brownian motion (ABM); Investment project analysis; Capital structure.

1. Introduction

The valuation of ESTCs requires specific models, suited do deal with their particularities. Real options based models allow increased flexibility, enabling the incorporation of real conditions such as options to pursue new projects, delay investments or expand existing ones, among many others. Having originated from the pricing of financial options, many of these models share underlying assumptions that make them unsuitable to the conditions faced by ESTCs. The field of study of structural models, which incorporate the optimization of the capital structure in the valuation process, offers increased possibilities to both management and investors, going beyond the limiting assumption of all equity financing to consider the use of debt in the fulfillment of the capital necessities of the company.

2. Structural Models

Structural models are based on the assumption that the dynamics of the project variable are described by a stochastic differential equation. From that assumption, it is possible to find the present values of different claims, such as debt, equity and taxes through the solution of a partial differential equation. This pricing framework is based on the work of Black and Scholes (1973) and Merton (1974). If differentiating conditions between debt and equity are incorporated, such as the tax advantage of debt, it is possible to derive the optimal coupon level which maximizes the firm value, thus defining the optimal capital structure. Most structural models, particularly early ones, share some core characteristics: the project variable is the value of the firm's assets, which is assumed to have dynamics described by a geometric Brownian motion (GBM) and they have a static setting, meaning that debt can be issued only once.

The first generation of structural models suffers from a particular inconsistency (Ammann and Genser, 2004): the state variable of the models is the unlevered firm value. This implies that both the levered and unlevered value of the firm exist simultaneously, which is not viable in an asset pricing context. Goldstein et al. (2001) proposed a model that uses an income measure that is unaffected by the capital structure decision, the EBIT. This type of model is more consistent with regular practices because debt coupons are financed with earnings, not by issuing new equity. The model divides the present value of the EBIT between several claims.

Ammann and Genser (2004) propose a model which features an alternative assumption for the dynamics of the project variable: instead of the commonly assumed GBM they opt for an arithmetic Brownian motion (ABM). Models that use ABM are all in a static setting, although the works of Ammann and Genser (2004) and Genser (2006) do allow for debt with finite maturities.

The use of geometric Brownian motion in all dynamic structural models, so far, is due to the fact that its mathematical properties allow a simplified progression from the static setup to the dynamic one, which is due to a homogeneity property inherent to this particular stochastic process. Since most start-ups do exhibit negative operating income, the assumption of GBM would be fairly limiting from a practical angle (Alexander et al., 2012).

3. Development of the Structural Model

The starting point of the model is the stochastic differential equation of the ABM process:

$$dx(t) = \mu dt + \sigma dW$$

Equation 1 – Arithmetic Brownian Motion

x – state variable; μ – drift (constant); t – time; σ – standard deviation (constant); W – Wiener process ; d – mathematical differential operator

Discounting the expected cash flows under the risk-neutral measure gives the pricing of the subjacent asset (Goldstein et al., 2001). In particular, the value of the claim to the entire EBIT flow is:

$$V_{expected}(x) = E^{Q}\left[\int_{t}^{+\infty} xe^{-rt}\,dt\right] = \frac{\mu}{r^2} + \frac{x}{r}$$

Equation 2 – Expected Present Value of the EBIT

E – expected value operator; r – risk free rate

The existence of an equivalent martingale measure, Q, guarantees that every contingent claim will receive fair expected return for the risk borne. With $\mu = \mu_m - \sigma\Phi$ being the risk neutral drift, μ_m the measured drift and Φ the risk premium.

The differential equation that represents the problem takes the form:

$$\frac{dV}{dx}\mu + \frac{1}{2}\frac{d^2V}{dx^2}\sigma^2 - rV + mx + k = 0$$

Equation 3

This being a second order nonhomogeneous differential equation, a solution can be obtained:

$$V(x) = \frac{m(\mu_m - \sigma\Phi)}{r^2} + \frac{mx + k}{r} + A_1 e^{\beta_1 x} + A_2 e^{\beta_2 x}$$

Equation 4

$$\beta_1 = -\frac{\mu_m - \sigma\Phi}{\sigma^2} - \frac{\sqrt{(\mu_m - \sigma\Phi)^2 + 2r\sigma^2}}{\sigma^2} < 0$$

Equation 5 - β_1 coefficient

$$\beta_2 = -\frac{\mu_m - \sigma\Phi}{\sigma^2} + \frac{\sqrt{(\mu_m - \sigma\Phi)^2 + 2r\sigma^2}}{\sigma^2} > 0$$

Equation 6 - β_2 coefficient

The constant A_2 must me equal to 0, otherwise the value of the claim would increase exponentially with the cash flows (Ammann and Genser, 2004).

3.1. Value of unlevered ESTCs

The present value of the cash flows generated by an unlevered firm $V(x)$ must be divided among three claims: the equity claim, $E_u(x)$, the tax claim, $T(x)$, and the closure costs, $\mathrm{CC(x)}$. The abandonment threshold, x_a, can be derived by maximizing the equity value.

$$V(x) = \frac{\mu}{r^2} + \frac{x}{r} - \left(\frac{\mu}{r^2} + \frac{x_a}{r}\right) e^{\beta_1(x-x_a)}$$

Equation 7 – Present Value of the cash flows with abandonment condition

$$\mathrm{CC(x)} = \mathrm{C}e^{\beta_1(\mathrm{x}-\mathrm{x_a})}$$

Equation 8 – Closure Costs claim

$$T(x) = \tau\left(\frac{\mu}{r^2} + \frac{x}{r}\right) - \tau\left(\frac{\mu}{r^2} + \frac{x_a}{r} + C\right) e^{\beta_1(x-x_a)}$$

Equation 9 – Tax claim

$$E_u(x) = (1-\tau)\left(\frac{\mu}{r^2} + \frac{x}{r}\right) - (1-\tau)\left(\frac{\mu}{r^2} + \frac{x_a}{r} + C\right) e^{\beta_1(x-x_a)}$$

Equation 10 – Equity claim for the unlevered company

$$x_a = \frac{r - C\beta_1 r^2 - \mu\beta_1}{\beta_1 r}$$

Equation 11 – Abandonment threshold

3.2. Optimal capital structure for ESTCs

Levering the company creates a tax shield, TS(x), which represents the tax advantage of debt. The total value present in the EBIT created by the company remains constant with the changes in the capital structure. It is only redistributed among the claimants, with the claim due to the government via taxes being reduced. This is in accordance with the pie model of Modigliani and Miller (1958), extended to incorporate taxes and bankruptcy costs.

The optimal capital structure of a company is achieved when the impact of the tax advantage of debt is annulled by the increased cost of debt, which reflects the greater likelihood of financial distress of the indebted firm.

The model assumes that there is a default if the EBIT falls below a threshold x_b, at this level debt holders are entitled to the residual value of the company, after bankruptcy costs, $BC(x_b)$, and taxes are deducted. According to the procedure of Goldstein et al (2001) the costs of financial distress are represented as a fraction, α, of the pre distress asset base attributable to the equity owners, $BC(x_b) = \alpha E_u(x_b)$. The optimal coupon value, c, is obtained by maximizing by maximizing the firm value $VC(x)$ through Equation 18.

$$BC(x) = \alpha E_u(x_b) e^{\beta_1(x-x_b)}$$

Equation 12 – Bankruptcy Costs

$$TS(x,c) = \frac{\tau c}{r} - \frac{\tau c}{r} e^{\beta_1(x-x_b)}$$

Equation 13 – Tax shield

$$VC(x) = E_u(x) + TS(x) - BC(x)$$

Equation 14 – Company Value

$$E_l(x) = VC(x) - D(x)$$

Equation 15 – Equity claim for the levered company

$$D(x) = \frac{c}{r} + \left[(1-\alpha)E_u(x_b) - \frac{c}{r}\right]e^{\beta_1(x-x_b)}$$ *Equation 16 – Debt claim*

$$x_b = \frac{r - \mu\beta_1 + rc\beta_1}{r\beta_1}$$ *Equation 17 – Bankruptcy threshold*

$$\frac{\partial VC(x)}{\partial c} = 0$$ *Equation 18 - Condition for optimal capital structure*

There is no closed solution for this equation, hence a numerical solver is required for the problem.

3.3. Option to Invest in an ESTC

Until this point, the present analysis concerns the evaluation of a company, and the optimization of its capital structure, given the EBIT value. Underlying this procedure is the option to invest in the firm. The investment, which typically represents the acquisition of an equity stake, has a cost IC. The cost of the investment dictates that it is only logical to invest if EBIT reaches a level x_i. The cash flows associated with the investment option follow a ABM process, with adjusted drift $\mu_I = \mu - \delta$, where δ represents the opportunity costs associated with the wait to invest.

$$dx = (\mu - \delta)dt + \sigma dW$$ *Equation 19*

A non-trivial solution can be found:

$$I(x) = A_3 e^{\beta_3 x} + A_4 e^{\beta_4 x}$$ *Equation 20*

$$\beta_3 = -\frac{\mu - \delta}{\sigma^2} - \frac{\sqrt{(\mu - \delta)^2 + 2r\sigma^2}}{\sigma^2} < 0$$ *Equation 21*

$$\beta_4 = -\frac{\mu - \delta}{\sigma^2} + \frac{\sqrt{(\mu - \delta)^2 + 2r\sigma^2}}{\sigma^2} > 0$$ *Equation 22*

It is necessary to set $A_3 = 0$, otherwise the option could increase its value with increasingly negative EBIT values. What is left is an exponential function. To constrain the possible values of the option, $I(x)$, a value-matching condition must be set:

$$I(x) = [VC(x_i) - (1-\tau)IC]e^{\beta_4(x-x_i)}$$ *Equation 23*

The boundary condition, so that x_i can be optimized, is the smooth-pasting condition:

$$\frac{\partial I(x)}{\partial x} = \frac{\partial VC(x)}{\partial x}; x = x_i$$ *Equation 24 – Smooth-pasting condition (2)*

An analytical solution could not be found, therefore a numerical solution is required. It is relevant to note that solutions are only valid with $x < x_i$, that is when the current EBIT value is still inferior to the optimal investment value, otherwise the exponential factor in Equation 23 would increase the value of the option above the value of the acquired assets net of the investment cost, which loses practical value. When the this validity condition is violated, the option to invest, $I(x)$, should be valued considering $x_i = x$, in which case nothing can be said about the optimal conditions for the investment. When this happens additional information can be obtained by introducing an extra parameter, λ, representing the minimum stake in the company that the investor should accept in return for the investment cost, IC.

$$I(x) = \lambda VC(x) - (1-\tau)IC$$

Equation 25

$$\lambda = \frac{I(x) + (1-\tau)IC}{VC(x)}$$

Equation 26 – Equity stake

4. Case Study

This study is based on Fitbit Inc, a company that went through its initial public offering (IPO) in June 2015. Fitbit is a highly technological company that operates in the “wearables” market. The case study uses data obtained from the period immediately before the IPO.

4.1. Model Parameters

The risk neutral drift, $\mu = \mu_m - \sigma\Phi$, requires an estimate of a measured drift, μ_m, a standard deviation, σ, and a risk premium, Φ. The first two components were obtained from a simulation following ABM, with initial drift and volatility measured from Fitbits statements (Fitbit, 2015). The risk premium was estimated using this model with an arbitrated $\mu = \mu_m$ and remaining parameters equal to those used in the case study, to obtain a valuation of the same magnitude of that of Fitbit. To estimate the risk premium the arbitrated drift was changed to obtain an increase in the valuation of 39.5% which is, according to Fester et al (2013) the return that venture capital investors demand, on average. The necessary change in drift was assumed to represent $\sigma\Phi$, enabling the computation of Φ, since the standard deviation was already known. The tax rate, τ, is the corporate tax rate in the USA (Deloitte, 2015). The initial EBIT, x, is the EBIT of the company for the year 2014 (Fitbit, 2015). The risk free rate, r, was estimated to be equal to the US 30 year Treasury Bond yield, the value corresponds to the average of the daily yield values (US Department of the Treasury, 2015) of the period from 11-06-15 to 17-06-15, the week before the Fitbits IPO. Closure costs, C, were arbitrated to be equal to the general and administrative costs of the company for 2014 (Fitbit, 2015). The bankruptcy stake, α, was defined according to the research of Branch (2002). The opportunity cost, δ, was arbitrated to be equal to the risk neutral drift. The equity stake, λ, is set to 100%, end the investment costs, IC, were assumed to be last reference valuation for the IPO (Fitbit, 2015).

Table 1 – Complete input parameters

Input Parameters[1]									
$\boldsymbol{\mu}$	σ	r	x	C	τ	α	δ	IC	λ
9.40	56.7	3.09%	158	33.556	0.35	0.44	9.40	3680	100%

[1] The units in which the parameters are expressed are: μ, δ – millions of US dollars per year; σ, x, C, IC – Millions of US dollars. The remaining parameters are dimensionless.

4.2. Model Results

Table 2 - Model Results – Unlevered Company

Unlevered Company				
Values in million US dollars				
$\boldsymbol{V}$	T	E_u	CC	x_a
14991	5246	9744	0.28	-427.29

The value of the unlevered equity, E_u, can be compared to the market value of the equity of Fitbit Inc. In the three months following the IPO of the company, that value ranged between US$6180M and US$10700M (Yahoo, 2015). The valuation resulting from the model, $E_u = US\$9744M$, is coherent with the market value.

Table 3 - Model Results – Optimally Leveraged Company

Optimally Leveraged Company							
Values in million US dollars							
$\boldsymbol{VC}$	T	TS	D	E_l	BC	c	x_b
12813	1823	3423	10233	2580	355	358.81	-67.45

In the conditions of this test it is not possible to reach conclusions concerning the optimal conditions for the investment, according to what was explained in section 4.2.4. These conditions are, in any case, favorable to the investor, the option to invest is valued at $I = \mathrm{US\$10421M}$. The threshold stake in the company, above which the investment is viable is $\lambda = 18.67\%$.

4.3. Sensitivity Analysis and Model Robustness

Each of the parameters that is analyzed in the sensitivity analysis is inserted into the model with three different values: a reduction of 50%, the original value and an increase of 50%, with all remaining parameters unchanged.

Table 4 - Sensitivity Analysis to x (percentage values).

EBIT								
$\boldsymbol{x}$	VC	T	D	E_l	BC	c	x_b	DER[2]
-50%	-17.7%	-12.1%	-19.0%	-12.6%	-10.4%	-16.8%	89.5%	-7.3%
158	12813	1823	10233	2580	355	358.81	-67.45	3.97
+50%	18.0%	10.9%	19.7%	11.6%	8.7%	17.4%	-92.4%	7.1%

Table 5 - Sensitivity Analysis to μ_m (percentage values)

Drift									
$\boldsymbol{\mu_m}$	μ	VC	T	D	E_l	BC	c	x_b	DER
-50%	-100.6%	-59.6%	-43.0%	-63.6%	-43.8%	-40.6%	-49.5%	-31.8%	-35.3%
18.92	9.40	12813	1823	10233	2580	355	358.81	-67.45	3.97
+50%	100.6%	76.2%	8.3%	91.8%	14.1%	-10.1%	76.5%	-21.4%	68.0%

[2] Debt-to-Equity ratio – this financial ratio will be used in the following sections as a measure of the leverage of the company. It is computed according to: $DER = \frac{D}{E}$.

Table 6 - Sensitivity Analysis to σ (percentage values)

Volatility								
$\boldsymbol{\sigma}$	VC	T	D	E_l	BC	c	x_b	DER
-50%	7.1%	-41.1%	18.3%	-37.3%	-53.2%	7.9%	-166.8%	88.4%
56.70	12813	1823	10233	2580	355	358.81	-67.45	3.97
+50%	-1.0%	18.8%	-5.6%	17.1%	23.7%	4.1%	127.0%	-19.4%

Table 7 - Sensitivity Analysis to r (percentage values)

Risk free rate								
$\boldsymbol{r}$	VC	T	D	E_l	BC	c	x_b	DER
-50%	236.6%	187.6%	247.7%	192.3%	172.1%	67.3%	115.3%	18.9%
3.09	12813	1823	10233	2580	355	358.81	-67.45	3.97
+50%	-48.4%	-45.5%	-49.0%	-45.8%	-44.8%	-21.8%	-51.5%	-6.0%

Table 8 - Sensitivity Analysis to τ (percentage values)

Tax Rate								
$\boldsymbol{\tau}$	VC	T	D	E_l	BC	c	x_b	DER
-50%	6.9%	-43.4%	-7.8%	64.9%	-25.1%	-11.8%	62.8%	-44.1%
35%	12813	1823	10233	2580	355	358.81	-67.45	3.97
+50%	-6.1%	44.0%	1.7%	-37.2%	-6.2%	5.8%	-30.8%	61.7%

Table 9 – Evolution of the debt to equity ratio with growing EBIT

Debt to Equity Ratio											
EBIT	158	177	196	215	234	253	272	290	309	328	189
DER	3.97	3.57	3.24	2.97	2.73	2.52	2.19	2.19	2.05	1.92	1.81

Table 10 - Sensitivity Analysis to α (percentage values)

Bankruptcy Stake								
$\boldsymbol{\alpha}$	VC	T	D	E_l	BC	c	x_b	DER
-50%	1.6%	-3.8%	5.4%	-13.6%	-36.3%	5.6%	-30.0%	21.9%
44%	12813	1823	10233	2580	355	358.81	-67.45	3.97
+50%	-1.3%	4.3%	-4.4%	11.3%	22.8%	-4.5%	24.1%	-14.1%

Table 11 - Sensitivity Analysis to IC (percentage values)

Investment Costs				
$\boldsymbol{IC}$	VC	I	x_i	λ
-50%	0.0%	221.1%[2]	-88.4%[3]	-75.0%
14720	12813	3245[3]	104.66[4]	74.67%
+50%	0.0%	-220.3%	156.0%	-

[3] These values for the investment option, $I(x)$, correspond to the totality of the equity.

[4] As these values for x_i are lower than the EBIT level, x, they are not valid solutions.

Table 12 – Comparative relevance of the input parameters in the results of the model

	VC	T	D	E_l	BC	c	x_b	DER
Descending order of impact	r	r	r	r	r	μ_m	σ	σ
	μ_m	τ	μ_m	τ	σ	r	r	τ
	x	σ	x	μ_m	α	x	x	μ_m
	τ	μ_m	σ	σ	μ_m	τ	τ	α
	σ	x	τ	α	τ	σ	α	r
	α	α	α	x	x	α	μ_m	x

The risk free rate, r, is the parameter that has the greater impact on the valuation of all the claims. From Table 12 it is clear that it is the most relevant parameter for all results except those related with the leverage and the bankruptcy threshold of the company. An increase of 50% in the risk free rate, which historically can be verified in periods much shorter than that of this analysis induces a decrease in the valuation of the company of 48.6%. A similar decrease would have an even greater effect, it is important to refer that the yield rate used as reference is at historically low values (US Department of the Treasury, 2015). The bankruptcy threshold and the debt-to-equity ratio are most sensitive to changes in the volatility, measured by the standard deviation, there is some uncertainty in the measurement of this parameter as it is obtained from a simulation based on the available data from Fitbit, a short supply of information, considering the short life of the company. The same applies to the drift, μ_m, which is the second most influent parameter for the valuation of the company. Although there is no uncertainty in the measurement of the tax rate, τ, it can be changed by governments during the duration of the investment. This consideration is relevant and should be considered when evaluating the exposure do debt, and the consequent leverage; an increase of 50% in the tax rate reduces the value of the company in 6.1%, but increases the debt coupon, c, in 5.8% resulting in an increase of 61.7% in the DER. The analysis to the investment cost, IC, confirms the expectations about the dynamics of the investment option, $I(x)$: when the investment cost is raised, resulting in a negative present value for the option to invest, the investment threshold, x_i, increases above the EBIT level, x, signaling the investor to wait until the EBIT of the company has reached that level to enter in the venture.

5. Conclusions

This work had for its foremost objective to contribute to the literature related with the study of the valuation of ESTCs. By employing a structural model with the assumption that the EBIT of the company can be modelled with an ABM, it was possible to do a valuation of the company and study its optimal capital structure.

The sensitivity analysis shows that the valuation is highly affected by changes in the risk free rate; there is uncertainty in the long term estimates of this parameter that must be considered. The volatility parameter also has considerable impact in the results, particularly in the optimal leverage proposed by the model. Providing estimates for this parameter is one of the biggest challenges in the application of real options models to ESTCs, due to the lack of available data on which to base the simulations.

The application of the model to a reality based setting is a relevant contribution to the available literature, since it is, to the extension of the author knowledge, the first case study for a structural model based on a real ESTC. The results indicate that there is a relation between the value of the company and its capital structure.

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