Stress Field Models for Structural Concrete – Application to the Strengthening of Existing Foundations

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Abstract
Foundations constitute the interface between the superstructure and the substructure, enabling the load transfer from one to another. Being regions of high structural discontinuity, it is important to have consistent and clear design methods. The study of these regions is extensively well-documented in what concerns new structures. However, the same is not applicable to existing structures. On that note, the purpose of this work is to systematize the most common situations of strengthening of reinforced concrete foundations, namely regarding their enlargement, the strengthening with micropiles and the strengthening with prestressed systems. To fulfil this objective, the proposed design models were obtained by applying stress field models to reinforced concrete, covering the stages of conception, modelling and design. The work carried out allowed to obtain a set of models that help understanding the topic under study. The following conclusions were made, based on the analysis of different models: the compressive strength of a strut crossing an interface between old and new concrete is reduced by at least 60%. This reduction can be even more significant when the surface's roughness is not appropriate; for solutions with enlargement of spread footing, it is generally necessary to extend the existing steel reinforcement in order to continue tensile stress fields; for solutions with micropiles, prestressed systems appear to be beneficial to the improvement of the bond strength conditions between the micropile and the foundation, allowing the reduction of the embedment length.

Keywords
Strengthening of existing foundations; Enlargement of shallow foundations; Micropiles; Discontinuity regions; Stress field models

1. Introduction
The building rehabilitation market has favorably evolved in the last few years. In this context, the need to reinforce existing structures' foundations becomes necessary. Taking into account that foundations constitute regions of high structural discontinuity (static and geometrical), there is the need to develop consistent models that allow evaluating the load transfer between the existing and the new structure. The aim of this work is to systematize the most common situations of strengthening of reinforced concrete foundations: enlargement of spread footing, micropiles and prestressed systems.

Enlargement of spread footing
The enlargement of spread footing is an interesting solution when the soil foundation presents appropriate capacity and when there is room available in plant. This solution aims at increasing the contact area between the footing and the soil foundation in order to increase the compressive capacity or redistribute soil stresses[1].

Micropiles
The strengthening with micropiles is one of the most common solutions in what concerns foundations’ strengthening. One of the most relevant aspects when using this solution is its connection to the existing structure. Two of the most commonly used solutions are the (1) application of micropiles through pre-drilled holes in the existing foundation and (2) placement of a micropile in widening areas (Figure 1)[2]. In the first case, the load transfer is performed through friction between the steel/grout and grout/concrete interfaces and strength of shear rings, in the case of textured micropiles[3]. In the second case, the load transfer from the structure to the micropile can be done through friction throughout the embedment length or through bending, if referring to the usage of special devices on top of the micropile[4].
2. The stress field models

The strut-and-tie and stress field models are currently recognized as a valuable tool for the design and detailing of discontinuity regions in structural concrete\(^6\),\(^5\),\(^6\), representing a complete and intuitive method that treats discontinuity and current regions with similar accuracy\(^7\). The method simulates the load paths, thus enabling to visualize the stress fields inside the region under study. It is considered that this subject is technically and normatively well-documented. For that reason, only additional work on load transfer from a strut across an interface will be presented in this study.

For a prismatic strut crossing an interface inclined at an angle \(\alpha_f\) (Figure 2), the design strength \(f_{cd,j}\) can be determined with expression 1:\(^8\):

\[
\sigma_{bd,j} = \nu_3 f_{cd}
\]

where \(f_{cd}\) is the design value of concrete compressive strength. Adopting the value proposed on Eurocode 2\(^9\) for the design value of the shear strength, the following expression is obtained for \(\nu_3\):

\[
\nu_3 = \frac{\nu f_{cd} \left( \rho f_{cd} (\mu \sin \alpha + \cos \alpha) \right) \left( \tan^2 \alpha_f + 1 \right)}{f_{cd}} \leq \frac{0.5 \nu}{\sin \alpha_f \cos \alpha_f}
\]

Graphs were obtained for interfaces with different roughness characteristics and different degree of reinforcement. The graphs were obtained for C25/30 concrete and an angle of 90\(^\circ\) was considered between shear reinforcement and shear plane \((\alpha = 90^\circ)\). For low values of \(\alpha_f\), the strut is practically perpendicular to the interface’s plane and thus \(\nu_3\) is unitary. As \(\alpha_f\) takes higher values, two distinct situations occur: (1) the strut’s tangential component increases, thus reducing \(\nu_3\); (2) the strut crosses the interface throughout a superior length \((l_2)\), thus increasing \(\nu_3\) and, consequently, the strength of the strut. Through the analysis of Figure 3 it is possible to conclude that, for surfaces with decreased roughness values, shear reinforcements play an important role and, with increasing of surface’s roughness, that relevance starts to fade. It is also possible to conclude that, for usual angles between struts and ties (from 30\(^\circ\) to 60\(^\circ\)), the fact that a strut crosses an interface implies a reduction of the design value of the compressive strength of the concrete of at least 60\% (see C25/30 in Figure 3). This reduction is associated with the limitation imposed by the second term of expression 2.
3. Design model proposals for different situation of foundation strengthening

In this chapter, design model proposals are presented for the most usual strengthening situation of reinforced concrete shallow foundations: (1) enlargement of spread footing; (2) use of micropiles; (3) use of prestressed reinforcement. 20 models were developed but, in this section, only the most relevant are to be analysed.

Centered loading

Model 1 – Enlargement of spread footing

Model 1 (Figure 4) illustrates a solution of strengthening with enlargement of spread footing. Through the analysis of that model, it is possible to conclude that the existing reinforcement in the spread footing must be extended in order to continue the tensile stress fields. In cases where existing reinforcement is not sufficient, two alternatives are proposed: (1) increasing the force in the tie through a solution with prestressed reinforcement (see Model 2); (2) reduce the force on the ties. This reduction of forces on the ties can be done by increasing the height of the footing. In designing the struts and nodes it must be taken into account that struts cross the interface of concrete with different ages. Regarding interface 2 (Figure 4), the stresses in the transversal section of the strut must be lower than the design value presented in expression 1 ($\sigma_{\text{rd},j} = \nu \sigma_{fcd}$). For node verification, node 1 is subdivided in two nodal regions (node 1.1 and node 1.2) and it is considered the verification of a second nodal region (node 2). For node 1 (CCC), stress in the nodal sub-region 1.2 must be evaluated, taking into account that the node is submitted to a multiaxial state of compression. When verifying nodes 1.2 and 2, the stress in the diagonal face.

Figure 3 – Variation of the factor $\nu_3$ with the angle $\alpha_f$ for interfaces with different roughness and different reinforcement ratios: a) $c=0.025; \mu=0.5$; b) $c=0.2; \mu=0.6$; c) $0.4; \mu=0.7$; d) $c=0.5; \mu=0.9$
must be evaluated and lower than the value presented in expression 1. It is possible to enhance the load transfer conditions across the interface by improving the angle between the strut and the interface (see node 1 in Figure 4).

![Elevation and Plan Bottom View](image)

**Figure 4 - Model 1: Enlargement of spread footing**

**Model 2 – Enlargement of spread footing with prestressed reinforcement**

In any prestressed system (Figure 5), it is necessary to define the prestressed cables’ position. Considering that it is mandatory that the position of the stresses on the soil and the forces acting on the ties coincide (in plant) for an uniform stress distribution on the soil, it is possible to conclude that, in most cases, cables ought to be placed in the inside of the existing footing, being crucial to proceed its drilling. If considering the placement of prestressed cables in the new overlay of concrete, the closer the cables are to the existing footing’s face, the higher the percentage of load that can be balanced on the soil.

![Elevation and Plan Bottom View](image)

**Figure 5 - Model 2: Enlargement of spread footing with prestressed reinforcement**

**Model 3 – 4 Micropiles**

When obtaining Model 3, three aspects should be taken into account: (1) an uniform stress distribution is considered along the embedment length of the micropile; (2) the embedment length of the micropile is determined by the tie’s position, once the resulting vertical forces transferred across the embedment length of the micropile must be balanced and coincide with
the position of tension forces on the tie; (3) distance $d_2$ is determined by the embedment length of longitudinal reinforcement bars placed at the inferior face of the footing.

The force on the tie mainly results from the contribution of the reinforcement grid located in the inferior face of the footing. However, if the reinforcement placed in the lateral faces of the footing is appropriate detailed, it is possible to take them into account. In that case, a new position of the tie can be determined through the weighted average of the contribution of each tension force (see detail in Figure 6). This solution can be complemented with the utilisation of prestressed steel. In that case, these solutions presented the following advantages in relation to ordinary reinforcement: (1) increasing the confinement of the embedment region of the micropile allowing to obtain higher bond strength values and, consequently, reduce the embedment length; (2) introducing compressions in the nodal regions, thus enhancing the multiaxial state of stress; (3) transferring soil loads to the micropiles if the induced prestressed force is higher that necessary to balance the load the micropiles are balancing; (4) allowing the positioning of the cable in order to obtain the most convenient embedment length for the micropile.

![Figure 6 – Model 3: 4 Micropiles](image)

**Model 4 – 4 Micropiles with enlargement of spread footing**

If increasing the width of the footing, it is possible to distinguish between two different solutions of load transfer from the footing to the micropile: with or without plate on the top of the micropile (Figure 7 shows the solution with plate). In designing these solutions, the following aspects must be taken into account: (1) node and struts' verification must be performed as presented in Model 1, ensuring that the fact struts are crossing concrete of different ages\(^1\) is being considered; (2) in the solution with a single plate on top of the micropile, considering the adherence’s contribution to the micropile along its embedment length is debatable, since sliding of the micropile is necessary in order to activate this mechanism, which is not compatible with the fact there is a plate on top of it; (3) the solution with one plate allows to reduce the embedment length when compared to the solution without it. For this last solution, the design of the plate must be done.

\(^1\) The design is conservative when the strut is not crossed on its totality by the interface.
Eccentric Loading

Model 5 – Enlargement of spread footing

In this model, the detailing of the column tensioned reinforcement are anchored (details in Figure 8) condition the position of the horizontal tie, where it may be necessary to choose a solution with prestressed steel so that the horizontal reinforcement forces match the friction forces transferred across the length $l_b$. In most of the situations, it is necessary to enlarge the column and, on that case, extend the tensioned reinforcement of the column.

With the aim of reducing the embedment length, the following specifications are proposed: (1) use of small steel bar diameters; (2) use of grouts with characteristics that allow increasing the bond strength value; (3) use of prestressed reinforcement in order to cause transversal compressions in the embedment region. Furthermore, it is also possible to anchor the reinforcement in the inferior face of the footing, which should however be avoided since this solution implies a considerable excavation volume.

Modelo 6 – 4 Micropiles

Model 6 (Figure 9) illustrates the strengthening with 4 micropiles of a footing submitted to a loading with high eccentricity. When obtaining this model, the embedment length of the compressed micropile must be consistent with the embedment length of the column’s tensioned reinforcement and the height of node 1 must be compatible with the embedment length of the tensioned micropile. In most cases, the footing’s superior face steel reinforcement are not enough to balance loads, where there are two different solutions to this
problem: (1) increasing the height of the footing and placing steel reinforcement on the new layer of concrete; (2) applying prestressed reinforcements. These two alternatives can equally come with the fact that steel reinforcement placed at the inferior face of the existing footing is not sufficient to balance loads.

4. Practical examples

In the present section, the design of two strengthening solutions are performed based on the models developed in section 3. The different solutions concern the strengthening of an existing foundation that presents the characteristics described in Table 1. For the presented examples, the design procedures are performed according to Eurocode 2\cite{9}.

| Table 1 - Characteristics of the initial footing |
| --- | --- |
| **Geometry** | **Materials** |
| A=B [m] | 2.50 | C25/30 ($f_{cd}=16.7\text{MPa}$) |
| H [m] | 0.75 | A500 ($f_{yd}=435.8\text{MPa}$) |
| $A_{s,\text{inf}}$ | (10.05 cm$^2$/m - $\Phi16/0.20$) |

A – footing’s smaller dimension in plant; B – footing’s bigger dimension in plant; H – footing’s height; $f_{cd}$ – design value of the compressive strength of concrete; $f_{yd}$ – design yield strength of reinforcement; $A_{s,\text{inf}}$ – steel cross sectional area

Example 1 – Enlargement of spread footing (centered loading)

Example 1 illustrates the strengthening of a footing with the geometric and loading characteristics presented in the Table 2.
Table 2 – Geometric and loading characteristics of the existing footing

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_{Ed}$ [kN]</td>
<td>5000</td>
</tr>
<tr>
<td>$A=B$ [mm]</td>
<td>3000</td>
</tr>
<tr>
<td>$H$ [mm]</td>
<td>1300</td>
</tr>
<tr>
<td>$\sigma_{Ed,s}$ [MPa]</td>
<td>0,56</td>
</tr>
<tr>
<td>$\sigma_{Rd,s}$ [MPa]</td>
<td>0,70</td>
</tr>
</tbody>
</table>

$N_{Ed}$ – design axial loading; $A$ – footing’s smaller dimension in plant; $B$ – footing’s bigger dimension in plant; $\sigma_{Ed,s}$ – soil’s design loading; $\sigma_{Rd,s}$ – soil’s design strength.

The value of forces in the model and respective geometry are illustrated in Figure 10. In what concerns the design procedures, the design of strut $C_2$ and nodes 1 and 2 is presented in Tables 2-3.

Table 3 – Check of the strut $C_2$

<table>
<thead>
<tr>
<th>Geometry and stresses</th>
<th>Interface 2 ($c=0,5$; $\mu=0,9$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_2$ [mm]</td>
<td>117</td>
</tr>
<tr>
<td>$b_2$ [mm]</td>
<td>1555</td>
</tr>
<tr>
<td>$F_{Ed,c}$ [kN]</td>
<td>571,1</td>
</tr>
<tr>
<td>$\sigma_{Ed,c}$ [MPa]</td>
<td>3,15</td>
</tr>
<tr>
<td>$\sigma_{Rd,j}$ [MPa]</td>
<td>9,02</td>
</tr>
</tbody>
</table>

$a_2$ – strut’s width; $b_2$ – strut’s depth; $F_{Ed,c}$ – strut’s design compression force; $\sigma_{Ed,c}$ – strut’s design compression stress; $\alpha_i$ – angle between the strut’s transversal section and the interface; $\rho$ – degree of reinforcement crossing the interface; $\nu_3$ – see expression 2; $\sigma_{Rd,j}$ – interface’s design strength.

Figure 10 - Stress field models of example 1
Example 2 – Strengthening with 4 micropiles (eccentric loading)

In this example, the soil presents a bearing capacity of 0.7 MPa and the footing is submitted to an axial loading of 2200 kN and a bending moment of 2200 kN in one of its directions. These loading demands cause a stress in the soil of 1.76 MPa. Taking into account that it is not possible to increase the height of the footing due to unavailable space, the solution was to strengthen the foundation with 4 micropiles. The stress field model for this solution is presented in Figure 11. When obtaining the model, the following was taken into account: (1) use of a prestressed solution (exterior prestress) in the inferior region of the footing in order to enhance the anchoring conditions of the column’s tensioned reinforcements and the micropile’s embedment conditions; (2) the column’s tensioned reinforcement bars are embedded in the footing; (3) the micropile presents a textured tube and a toothed hole’s surface. The design of nodes 1-4 is presented in Table 4. To determine the embedment length of the column’s tensioned reinforcement, only steel bars placed on the column’s over-width were considered which were subject to embedment with a grout with a bond strength value of 15 MPa.

Table 4 - Check of the nodes 1 and 2 stresses

<table>
<thead>
<tr>
<th>Geometry [mm]</th>
<th>Stresses [MPa]</th>
<th>Interface 1</th>
<th>(c=0.5; μ=0.9)</th>
</tr>
</thead>
<tbody>
<tr>
<td>a₁</td>
<td>50</td>
<td>σ_{Ed,1}</td>
<td>13.89</td>
</tr>
<tr>
<td>a₂</td>
<td>59</td>
<td>σ_{Ed,2}</td>
<td>16.06</td>
</tr>
<tr>
<td>a₃</td>
<td>33</td>
<td>σ_{Ed,3}</td>
<td>19.53</td>
</tr>
<tr>
<td>θ [°]</td>
<td>46.85</td>
<td>σ_{Rd,max}</td>
<td>37.02</td>
</tr>
</tbody>
</table>

Node 1

| a₁ | 50 | σ_{Ed,1}    | 13.89 | α₁ [°] | 46.85 |
| a₂ | 59 | σ_{Ed,2}    | 16.06 |        |       |
| a₃ | 33 | σ_{Ed,3}    | 19.53 |        |       |
| θ  | 46.85| σ_{Rd,max}  | 37.02 |        |       |

Node 2

| a₁ | 155 | σ_{Ed,1}    | 16.7  |       |       |
| a₂ | 236 | σ_{Ed,2}    | 12.7  |       |       |
| a₃ | 200 | σ_{Ed,3}    | 7.7   |       |       |
| θ  | 59  | σ_{Rd,max}  | 31.6  |       |       |

Example 2 - Strengthening with 4 micropiles (eccentric loading)

In this example, the soil presents a bearing capacity of 0.7 MPa and the footing is submitted to an axial loading of 2200 kN and a bending moment of 2200 kN in one of its directions. These loading demands cause a stress in the soil of 1.76 MPa. Taking into account that it is not possible to increase the height of the footing due to unavailable space, the solution was to strengthen the foundation with 4 micropiles. The stress field model for this solution is presented in Figure 11. When obtaining the model, the following was taken into account: (1) use of a prestressed solution (exterior prestress) in the inferior region of the footing in order to enhance the anchoring conditions of the column’s tensioned reinforcements and the micropile’s embedment conditions; (2) the column’s tensioned reinforcement bars are embedded in the footing; (3) the micropile presents a textured tube and a toothed hole’s surface. The design of nodes 1-4 is presented in Table 4. To determine the embedment length of the column’s tensioned reinforcement, only steel bars placed on the column’s over-width were considered which were subject to embedment with a grout with a bond strength value of 15 MPa.

Table 5 - Check of nodes 1-4 stresses

<table>
<thead>
<tr>
<th>Geometry [mm]</th>
<th>Stresses [MPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nodes 1D 1E 2 3D 3E 4</td>
<td>Stresses [MPa]</td>
</tr>
<tr>
<td>a₁</td>
<td>16.7</td>
</tr>
<tr>
<td>a₂</td>
<td>12.7</td>
</tr>
<tr>
<td>a₃</td>
<td>7.7</td>
</tr>
<tr>
<td>θ</td>
<td>31.6</td>
</tr>
</tbody>
</table>

a₁ – node’s width; a₂ – node’s diagonal length; a₃ – node’s depth; θ – angle between the strut and the tie; σ_{Ed,1} – face a₁ design stress; σ_{Ed,2} – face a₂ design stress; σ_{Ed,3} – face a₃ design stress; σ_{Rd,max} – node’s design strength
5. Conclusions and future works

This work enabled the development and systematization of a set of design models for the most current situations in strengthening of reinforced concrete foundations, namely regarding the enlargement, strengthening with micropiles and the use of prestressed systems. As a starting point, the mechanisms of load transfers specific of enlargement and strengthening of foundations with micropiles were studied. In what concerns the load transfer between concrete of different ages, it was seen that a strut that crosses an interface with an angle of 30° to 60° presents a significant reduction of the compressive design strength of at least 60%. In cases where the interface’s surface does not present the appropriate roughness or any reinforcement, the reduction can be quite superior. As for the models developed, the following conclusions were made:

**Centered Loading**

For the strengthening solution with enlargement of spread footing, in most of the cases, in order to balance vertical loads for the entire area of the footing, it is necessary to extend existing steel reinforcement. In this solution, to take into account the fact that struts cross the interface...
between concrete cast at different ages, the design strength of the strut is reduced by the factor $v_3 \left( \sigma_{rd,j} = v_3 f_{cd} \right)$. In this case, special attention must be taken to how the interface’s surface is treated since its roughness is one of the main aspects that affect $v_3$. For solutions with strengthening of micropiles, one of the most relevant factors in determining the embedment length of the micropile is the positioning of inferior reinforcement layer of the existing footing. On that note, in order to increase the embedment length of the micropile, it is possible to take into account the footing’s lateral reinforcement (for reduced to moderate values of tension forces). Another alternative is to introduce prestressed steel, a solution that presents advantages in the optimization of friction conditions of the embedded connection of the micropile, thus allowing to reduce the embedment length.

**Eccentric Loading**

For solutions with eccentric loadings, one of the most relevant aspects is the anchorage of the column’s tensioned steel reinforcement. The length across which the steel bars are disposed and the detailing of the anchorage region have a significant influence on both the model’s geometry and the node’s stresses check.

Aiming at refining the proposed model, it is considered relevant to deepen, in future works, the following aspects: (1) performing experimental testing that allows to gauge the distribution of stresses along the embedment length of the micropile. The goal of these tests is to determine the position of the bond forces, since these represent a quite relevant parameter to obtain the model; (2) performing experimental testing to micropiles with plates on top that allow to determine the contribution of the bond forces across the embedment length of the micropile; (3) analysis of the models considering their tridimensional geometry. This last aspect must be taken into account in the definition of the nodal regions, in the evaluation of design strength of the nodes and in determining the embedment lengths of tensioned steel reinforcement.

**6. Notation**

- $A$ – footing’s smaller dimension in plant
- $A_0$ – steel cross sectional area
- $a_1$ – node’s width
- $a_2$ – node’s diagonal length
- $a_3$ – node’s depth
- $B$ – footing’s bigger dimension in plant
- $c$ – cohesion
- $H$ – footing’s height
- $f_{cd}$ – design value of the compressive strength of concrete
- $f_{yd}$ – design yield strength of reinforcement
- $N_{Ed}$ – design axial loading
- $\alpha$ – angle between the shear reinforcement and the shear plane
- $\alpha_r$ – angle between the strut’s transversal section and the interface
- $\rho$ – degree of reinforcement that cross the interface
- $v_3$ – reduction factor of the strength of the strut
- $f$ – design value of the tensile strength of concrete
- $\tau$ – friction
- $\sigma$ – steel reinforcement embedment length
- $\sigma_1$ – face $a_1$ design stress
- $\sigma_2$ – face $a_2$ design stress
- $\sigma_3$ – face $a_3$ design stress
- $\rho_{rd,max}$ – node’s design strength

**7. References**


