The economic success of regions and cities is highly dependent on how products including raw materials and people flow through them. Hazardous Materials (HazMats, as they are also called) are included in the group of materials whose successful transportation is key for economical growth. With the evolution of societies, there is an increasing concern regarding the themes of safety and risk management. This means that these materials as well as the routes they flow through are increasingly becoming a topic of concern both for the population and the government entities. Even though the transport of fuel has a minority share of the amounts of goods transported by truck, the hazards that can come from a spill are especially severe due to the nature of these materials.

The objective of this research project is to contribute to increase road safety associated to the transportation of HazMats, whilst keeping that transportation economically viable. This work has expanded knowledge on a bi-level linear programming model, which, after being supplied with a road map containing information regarding road characteristics, will be able to find the best itineraries for the distribution of hazardous materials. These are chosen based on the risk of an accident and economic factors, represented by the distance travelled. Therefore, in the first level of the model the objective is risk minimization while the second level aims at maintaining the economic viability of the itineraries. This works builds upon and continues the research of Rodrigues (2014) with further challenges of the road network given that for this project the whole city of Lisbon was analyzed.

Risk modeling, which in this case is represented by the total number of people affected in the event of an accident, took into account not only data from people living in the surroundings of the network arcs (data taken from the BGRIs featured in the 2011 census), but also by introducing data related to hospitals and schools in the surrounding areas of the arcs. This allowed solving the model for four different “combinations” or population allocations to the network arcs.
Comparisons between different route selections were performed. The study of how the different combinations impacted not only the routes chosen but also the amount of computer resources such as computation time and number of iterations needed to solve the model with the different inputs was also carried out.

The solutions were in all cases optimal and most of them were obtained in short computation time, indicating the potential of this methodology for the industry to decide which routes should be chosen for fuel distribution in urban environments.

**Key words:** hazardous materials transportation, bi-level linear programming, road safety in urban areas, geographical information systems (GIS), Galp Energia.

### 2. Introduction

Societies nowadays are extremely dependent on HazMats, which fulfill a wide range of purposes, including the energy supply to cities, vehicles and also industries. (Erkut et al., 2007).

The study leading to this dissertation is centered on the transportation of fuels, which are also classified as HazMats (Verter & Kara, 2008).

Even though the number of accidents involving HazMats is relatively small, (approximately four times less than the probability of being struck by lightning (Transportation and Infrastructure Committee, 2011)), when accidents involving HazMats occur, the severity of damages (both personal, environmental and material) can be high (Erkut et al., 2007).

The objective of this project is to somehow contribute for an increase in the road safety in Lisbon, which is the case study but also worldwide, more precisely through the identification of routes for the transportation of HAZMATs that minimize the risk of accident without compromising the economical feasibility of this industry.

### 3. Optimization Model

In order to reach the objectives, namely the identification of the best safe routes for the transportation of HazMats based on the knowledge from Kara & Verter (2004), a bi-level linear programming model will be developed featuring two problems, which will be hierarchically solved. They are related and correspond to different entities, in which the choice that one makes is directly influenced by the choice of the other (Bianco et al., 2009). In this particular case, it is acceptable to consider that the regulator, whose objective is to minimize total risk, defined as total population exposure, is on the outer level. Then the transportation companies are on the inner level, and their objective is to minimize total cost. It is thus clear that the outer level chooses a network where risk is controlled and the inner level then chooses the minimum cost path from those available (Kara & Verter, 2004).

However, and in order to solve these kinds of problems using a commercial solver, a transformation must be applied to the initial model using the *Karush-Kuhn-Tucker* (KKT) conditions. These are used whenever there is the need to transform a bi-level into a linear programming model. A more detailed explanation of this transformation can be found at a later point in this chapter.

In the model by Kara & Verter (2004), the authors consider that the consequences of an accident involving HazMats occur until a given distance from the place where the accident happened. This
means that when a truck is driving through a given arc, only the people within that distance to the arc are exposed to risk. In this model, total risk is the measure used by the regulator and total time spent travelling is what the companies use for picking their best routes. However this can easily be adapted and other measurements can be used (Rodrigues, 2014).

**Mathematic Formulation before KKT**

**Objective Function**

\[
\min \sum_{p \in P} \sum_{(i,j) \in A} \sum_{c \in C} n_i^p \rho_{ij}^p m(c) X_{ij}^c
\]  

(1)

**Constraints**

\[Y_{ij}^m \in \{0, 1\} \ \forall i, j \in A, m \in M\]  

(2)

And \(X_{ij}^c\) solves:

\[
\min \sum_{c \in C} \sum_{(i,j) \in A} n_i^c l_{ij} X_{ij}^c
\]  

(3)

**Constraints**

\[
\sum_{(k,l) \in A} X_{ik}^c - \sum_{(k,i) \in A} X_{ki}^c = \begin{cases} 
+1, & i = o(c) \\
-1, & i = d(c) \\
0, & O.W.
\end{cases} \ \forall i \in N, c \in C
\]

(4)

\(o(c)\) – origin node for freight \(c\)

\(d(c)\) – destination node for freight \(c\)

\[X_{ij}^c \leq Y_{ij}^m(c) \ \forall (i,j) \in A, c \in C\]

(5)

\(m(c)\) – HazMats class transported in freight \(c\)

\[X_{ij}^c \in \{0, 1\} \ \forall (i,j) \in A, c \in C\]

(6)

The model is constituted by two levels: the outer problem, defined by the regulator and the inner problem, defined by the transporting companies.

The outer problem decision regards which arc should be made available for the transportation of HazMats and is represented by objective function (1) and constraints (2), where the value of \(Y_{ij}^m\) is 1 if arc \((i,j)\) is available for transport of HazMat \(m\) and 0 if not. The inner problem is centered on the problem of choosing which routes will actually be used for transport and is represented by objective function (3) and constraints (4) through (6). The binary decision variables for the outer problem \((Y_{ij}^m)\) constitute parameters for the inner problem, therefore the inner problem is essentially the minimum cost flow problem, which minimizes the distance (or time) traveled.

The constraints regarding the flux through the different arcs are implemented through constraint (4). The first term in the left hand side regards the exit of an arc while the second term regards the entry. This means that the sum (right hand side of constraint 4) for node \(i\) can be 1 if there is an exit but no entry (starting point of a freight), -1 if there is an entry but no exit (destination of a freight) and 0 for every other arc where the freight enters and exits.
Constraint (5) guarantees that the transporting companies can only choose the arcs available by the regulator. For that purpose, it indicates that for the choice of arcs used in each route, \((X_{ij}^c = 1)\), only arcs that were previously defined as available for the transport of that HazMats class by the regulatory entity \((Y_{ij}^m = 1)\) can be chosen.

Constraint (6) indicates that the decision variable \(X_{ij}^c\) is binary and therefore either an arc is used for a freight or it is not and that the pool of usable arcs correspond to set \(A\).

### Mathematical Formulations after KKT

**Objective Function**

\[
\min \sum_{p \in P} \sum_{(i,j) \in A} \sum_{c \in C} n^c p_{ij}^p m(c) X_{ij}^c
\]

**Constraints**

\[
\sum_{(lk) \in A} X_{lk}^c - \sum_{(kl) \in A} X_{kl}^c = \begin{cases} +1 & i = o(c) \\ -1 & i = d(c) \\ 0 & \text{O.W.} \end{cases} \quad \forall i \in N, c \in C
\]

\(X_{ij}^c \leq Y_{ij}^m(c) \quad \forall (i,j) \in A, c \in C\)

\[
n^c l_{ij} - \omega_i^c + \omega_j^c - v_{ij}^c + \lambda_i^c = 0 \quad \forall c \in C, (i,j) \in A
\]

\[
v_{ij}^c \leq R \left(1 - X_{ij}^c\right) \quad \forall c \in C, (i,j) \in A
\]

\[
\lambda_i^c \leq R \left[1 - \left(Y_{ij}^m(c) - X_{ij}^c\right)\right] \quad \forall c \in C, (i,j) \in A
\]

\[
v_{ij}^c \geq 0, \lambda_i^c \geq 0 \quad \forall c \in C, (i,j) \in A
\]

\[
\omega_i^c \in \mathbb{R} \quad \forall c \in C, (i,j) \in A
\]

**Binary constraints**

\(X_{ij}^c \in \{0,1\} \quad \forall (i,j) \in A, c \in C\)

\(Y_{ij}^m \in \{0,1\} \quad \forall (i,j) \in A, m \in M\)

The model composed of expressions (1) through (10) is of the mixed integer linear programming (MILP) type and is focused on the generic problem of identifying the best routes for the transport of HazMats, by minimizing risk while maintaining economic feasibility.
4. An illustrative example of model application

This model was created in order to familiarize the reader with the procedures carried out by the solver through the understanding of a smaller dimension representative problem.

![Network diagram](image)

**Figure 1- Network chosen for the practice model**

The network of Figure 1 (where the value associated to each arc is the travel time) is then introduced in GAMS and the set of equations used is the one described in section above.

Three freights were considered with:

- \( n_c \) – Number of trucks used in freight \( c \), for each \( O-D \) pair:
  - 1 for freight \( O1-D1 \)
  - 3 for freight \( O2-D2 \)
  - 2 for freight \( O3-D3 \)

The model indicated the routes that should be chosen in order to minimize risk.

This data can be transferred into an excel file or it can also be presented in GAMS.

For O-D1 arc14 and arc45 were chosen, meaning that these are the arcs that minimize \( Z \) for this route, when compared to other alternative routes. This is pretty straightforward and makes sense in the point that a truck departing from node 1 will make its way through arco14, arriving at node4, where he will encounter arco45 leading it directly to its final destination. For O-D2 and O-D3 the routes chosen were arc25 – arc54 and arc36 – arc64 – arc47 respectively.
5. Case Study Description and Results

This dissertation will focus on identical methodology as Rodrigues (2014) but with a different road network for the Lisbon case study and so a larger problem size. In fact, Rodrigues (2014) used a network for a specific neighborhood of Lisbon (Olivais parish). This author used a network that had much detail, with 654 nodes and 6 O-D pairs in a small area. For the present work, the entire network for Lisbon will be analyzed but in this case, only the higher hierarchy routes will be considered. Even though only a part of the routes is featured, this network consists of 2685 arcs and 2285 nodes, which represents a significant increase compared to the networks that have been used previously in the reviewed literature. The problem will also have 26 O-D pairs, which also represents a considerable increase in complexity of the problem to be solved, compared to the 6 O-D pairs in Rodrigues (2014).

In Figure 2 it is possible to see the chosen network that featured 2685 arcs and 2285 nodes. The locations of the destinations were given by GALP under the form of coordinates. They were transformed into data that could be read on ArcGIS, and after that, there was the need to find out which arcs were the closest to each of these locations. That was done using the nearest function available in the software. Figure 3 shows in blue the destinations operating in 2013 (26 in total) and in black those that operated in 2012 but were shut down during 2013 (6 in total).
Different solutions were found for different population. The combinations are as follows:

**Table 1 - Weighted population combinations tested**

<table>
<thead>
<tr>
<th>SOLUTION</th>
<th>POPULATION COMBINATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1 x arc population + 0 x hosp_school</td>
</tr>
<tr>
<td>B</td>
<td>0 x arc population + 1 x hosp_school</td>
</tr>
<tr>
<td>C</td>
<td>1 x arc population + 1 x hosp_school</td>
</tr>
<tr>
<td>C'</td>
<td>0.5 x arc population + 0.5 hosp_school</td>
</tr>
<tr>
<td>D</td>
<td>0.5 x arc population + 0.5 x hosp_school (800 m)</td>
</tr>
</tbody>
</table>

The comparison of results in terms of objective function values cannot be done directly because the amount of people that was accounted varies greatly among the different solutions. This is among other factors, one of the causes for the variation in people affected for each solution. These values (population) increased significantly with the introduction of the hosp_school population.

With C' having twice the population from C, given the same network and parameters, in order for both the models to be optimal, the objective function from C' would also have to be double that of the one corresponding to C. This in fact occurred, reassuring that the model was coherent.

In figures 4 to 7 it is possible to see different routes chosen for "frete23" depending on which population combination was tested.
Figure 4 - Route for combination A

Figure 5 - Route for combination B

Figure 6 - Route for combination C

Figure 7 - Route for combination D
Here is one example of an output table (table 2) for one of the combinations tested:

<table>
<thead>
<tr>
<th>FREIGHT</th>
<th>POPULATION EXPOSURE</th>
<th>TIME (MIN)</th>
<th>NO. OF ARCS USED</th>
<th>NO. OF TRUCKS</th>
</tr>
</thead>
<tbody>
<tr>
<td>frete1</td>
<td>3979</td>
<td>11.8</td>
<td>61</td>
<td>45</td>
</tr>
<tr>
<td>frete2</td>
<td>25380</td>
<td>20.0</td>
<td>124</td>
<td>119</td>
</tr>
<tr>
<td>frete3</td>
<td>1299</td>
<td>5.9</td>
<td>38</td>
<td>241</td>
</tr>
<tr>
<td>frete4</td>
<td>2865</td>
<td>6.7</td>
<td>44</td>
<td>195</td>
</tr>
<tr>
<td>frete5</td>
<td>14459</td>
<td>11.4</td>
<td>67</td>
<td>112</td>
</tr>
<tr>
<td>frete6</td>
<td>14259</td>
<td>11.6</td>
<td>68</td>
<td>66</td>
</tr>
<tr>
<td>frete7</td>
<td>23687</td>
<td>19.7</td>
<td>122</td>
<td>41</td>
</tr>
<tr>
<td>frete8</td>
<td>25248</td>
<td>20.0</td>
<td>123</td>
<td>21</td>
</tr>
<tr>
<td>frete10</td>
<td>18114</td>
<td>27.8</td>
<td>141</td>
<td>58</td>
</tr>
<tr>
<td>frete11</td>
<td>9045</td>
<td>12.7</td>
<td>62</td>
<td>69</td>
</tr>
<tr>
<td>frete12</td>
<td>8741</td>
<td>12.2</td>
<td>64</td>
<td>53</td>
</tr>
<tr>
<td>frete13</td>
<td>21963</td>
<td>18.2</td>
<td>95</td>
<td>53</td>
</tr>
<tr>
<td>frete14</td>
<td>11603</td>
<td>12.7</td>
<td>65</td>
<td>36</td>
</tr>
<tr>
<td>frete15</td>
<td>6192</td>
<td>10.1</td>
<td>61</td>
<td>124</td>
</tr>
<tr>
<td>frete16</td>
<td>6530</td>
<td>11.0</td>
<td>55</td>
<td>97</td>
</tr>
<tr>
<td>frete17</td>
<td>11419</td>
<td>10.5</td>
<td>51</td>
<td>257</td>
</tr>
<tr>
<td>frete18</td>
<td>16778</td>
<td>25.4</td>
<td>136</td>
<td>132</td>
</tr>
<tr>
<td>frete19</td>
<td>6815</td>
<td>17.7</td>
<td>70</td>
<td>145</td>
</tr>
<tr>
<td>frete20</td>
<td>21945</td>
<td>19.0</td>
<td>117</td>
<td>69</td>
</tr>
<tr>
<td>frete21</td>
<td>12124</td>
<td>12.7</td>
<td>66</td>
<td>85</td>
</tr>
<tr>
<td>frete22</td>
<td>11603</td>
<td>12.7</td>
<td>65</td>
<td>12</td>
</tr>
<tr>
<td>frete23</td>
<td>25019</td>
<td>19.4</td>
<td>97</td>
<td>271</td>
</tr>
<tr>
<td>frete24</td>
<td>4417</td>
<td>9.6</td>
<td>59</td>
<td>180</td>
</tr>
<tr>
<td>frete25</td>
<td>14215</td>
<td>8.6</td>
<td>38</td>
<td>174</td>
</tr>
<tr>
<td>frete26</td>
<td>17374</td>
<td>26.2</td>
<td>138</td>
<td>87</td>
</tr>
<tr>
<td>frete27</td>
<td>7198</td>
<td>10.1</td>
<td>50</td>
<td>173</td>
</tr>
<tr>
<td>Sum</td>
<td>341974</td>
<td>383.8</td>
<td>2077</td>
<td>2915</td>
</tr>
<tr>
<td>Average</td>
<td>13153</td>
<td>14.8</td>
<td>79.9</td>
<td>112.1</td>
</tr>
</tbody>
</table>
6. Conclusions

The subject of HazMat transportation will continue to see its importance grow in the future and eventually new virtually riskless ways of distributing fuel and other HazMats will be invented. Until then it is up to the responsible entities to mitigate the risks and further develop the knowledge in this area.

There are requirements in order to use this methodology for planning HazMat routes. Firstly, a map, preferably in a Geographical Information System platform, and with enough road network information to enable the user to distribute population, either in the surroundings or in fixed points in the proximity of the arcs. Subsequently, the data extracted from this GIS network must be compiled into a mathematical modeling software such as GAMS and the MILP model solved so that the safe routes can be computed and extracted. Provided that further work, such as interpretation of the results, is developed, and that the network becomes a more accurate representation of roads and population distribution and that the calibration of the model is done correctly, this methodology is judged appropriate for decision making in the industry.

The results reached were satisfactory, having identified different best routes (the ones with the least population exposure while still being economically viable) for the different population combinations. This means that with an optimized method to represent real population exposure along the arcs it is possible to compare population exposure for the different routes and have an informed computerized choice of which routes to take when distributing or transporting HazMats. Furthermore, optimal solutions where attained from the MILP model in CPU times that ranged from 30 s to 1h20min, which shows the capabilities of current MILP solvers associated to modeling systems such as GAMS.

7. Recommendations for continuing this research

Improvements in this area of investigation should occur mainly in the following topics.

Firstly the roads network, which needs to be developed in terms of connectivity (all the arcs should have an entry and exit point). This was the key main challenge of this work, as it is hard to find a network with more detail than the one used in this dissertation, but which would still maintain accuracy regarding the arcs connectivity.

Further studies on how to model the population associated to the arcs of the road network is probably the biggest challenge to be developed here and this is a relevant aspect for future studies to focus on. Resident population, the one that is on the location during the day, bystanders or a combination of all of these and if so, with which proportion?

Future models should also include traffic measures for accuracy reasons. Although it is difficult to quantify traffic flows along different arcs, if this could be attained that would lead to the most accurate description of what happens in reality. This way the routes would be influenced in two ways. Firstly the actual speed at which trucks can go through them and secondly the model would also take into consideration how many people in adjacent travelling vehicles would be affected in case of an accident.

The importance of this theme will continue to grow in an increasingly energy dependent world where maximum efficiency and lower costs are needed.

1 www.gams.com
8. References


