EVOLUTION OF A BEACH-DUNE PROFILE UNDER EROSIVE WAVES:
DATA ANALYSIS AND MODEL EVALUATION

João Lavrador Rocha

This study determines dune erosion parameters from the Tomasicchio et al. (2010) data, from physical model tests that were carried out in a flume at the Canal d'Investigació i Experimentació Maríntia (CIEM), of the Universitat Politècnica de Catalunya (UPC, Barcelona), with a sandy dune exposed to a combination of water levels and wave conditions. The dune erosion processes, including dune recession rates in terms of the retreat of the dune crest and the eroded volume, have been defined and correlated with the effect of significant wave height, wave peak period and total water level (including the storm surge effect). These laboratory data sets were also used to calibrate and verify the analytical models proposed by Kriebel and Dean (1993), Larson et al. (2004) and van Rijn (2009), in order to calculate and predict the time evolution of dune retreat and dune eroded-volume. Furthermore, it was evaluated the performance of the three numerical models using error parameters such as the Brier Skill Score (BSS). In conclusion, the van Rijn model (2009) showed the highest similarity between the modelled and the experimental data, followed by Kriebel and Dean (1993) and Larson et al. (2004) models.

Keywords: coastal erosion; dunes, large-scale experiment; morphodynamics models.

1. INTRODUCTION

Coastal dunes generally form a natural barrier along sandy coastlines protecting areas against high waves and water levels during severe storms. Disaster due to storm surge and waves in the era of global warming and accelerated sea level rise are major threats to coastal nations such as the U.S., Japan, the Netherlands and Portugal. Storm-driven surge, wind and waves can cause severe dune erosion with large-scale morphology changes, damages to infrastructures and loss of human lives (Tomasicchio et al., 2011).

The need for predicting the response of the dunes to coastal storms is crucial and is rising steadily as the population in coastal areas grows worldwide and loads on coastal systems increase due to rising sea levels (D’Alessandro et al., 2012). In the fields of civil engineering and coastal management the prediction of coastal erosion and coastal dune adaption determines a good design of the infra-structure options of the coastal hinterland.

Due to the importance of the ability to predict the impact of a storm on a dune in terms of recession distance and eroded volume, several experiments using large-scale physical models have been performed (Vellinga, 1986; Dette et al., 2002; van Gent et al., 2009; Tomasicchio et al., 2011). Alongside, numerical and analytical models have been developed to estimate the dune erosion and to predict the beach profile evolution (e.g., Kriebel & Dean, 1993; Larson et al. 2004; van Thiel de Vries et al., 2008; Roelvink et al., 2009; van Rijn, 2009; Oliveira, 2012a,b). The models still have some restrictions because dune erosion is a complicated process to describe. However analytical and numerical models offer advantages since the simplicity make them easy to apply, which is valuable in the initial stage of a project when approximate estimates are required.

The main objectives of this work are to: verify the ability of the analytical models proposed by Kriebel and Dean (1993), Larson et al. (2004) and van Rijn (2009) to predict the values of dune retreat and dune eroded volume for the Tomasicchio et al. (2010) data set; correlate the results of the dune retreat and eroded volume with the variation of the water depth, the wave height, and the wave period. The following sections describe the set-up of the physical model, the results of the dune erosion tests, the analytical models and their predictions, and a discussion of the model performance and the physical phenomena.

2. SET-UP OF THE PHYSICAL MODEL AND WAVE CONDITIONS

The physical model tests have been performed at the CIEM wave flume of LIM/UPC in Barcelona and are described in detail in Tomasicchio et al. (2010, 2011). The CIEM wave flume is 100 m long, 3 m wide and 5 m deep. The initial cross-shore profile (Figure 1) was inspired by a down-scaled beach profile north of Figueira da Foz, along the Atlantic coast of Portugal. The length scale was approximately 1:4.8. The horizontal coordinate, x, has been taken to be positive onshore with x = 0 at the wave paddle. The crest of the dune was 2 m long, and the seaward and landward slopes were 1:2.30. The sand in the flume had a median diameter, d50, equal to 0.246 mm with measured fall velocity, w, of 34 mm/s.
A wedge-type wave paddle has been used to generate irregular waves, based on a Jonsaw spectrum with peak enhancement factor, $\gamma$, equal to 3.3. Table 1 presents from A to L the ten tests of Tomasicchio et al. (2011), where $h_0$ symbolises the water depth, $H_{m0}$ the significant wave height at the wave-maker, $T_p$ the peak period, and $\xi$ the corresponding wave steepness. To simulate the water level surge that occurs in storm conditions, two different values of water depth, $h_0$, have been considered in the flume: $h_0 = 2.35$ m (tests A to F) and $h_0 = 2.50$ m (tests G to L). The experimental set-up resembles the scenario of a beach-dune system with no berm foreshore and a high water level. The duration of a single test was divided into different wave attacks each composed by $n$ waves with $n = 250, 250, 250, 500, 1000$ and $1000$, respectively. After each wave series, the test was interrupted to perform profile measurements along the longitudinal flume axis by means of a mechanical profiler. The interested reader is referred to Tomasicchio et al. (2011) for a detailed description of the physical model set-up.

### 3. LABORATORY DATA ANALYSIS

Some erosion parameters were defined in order to inspect the data from Tomasicchio et al. (2010, 2011) experiments. During a storm, sand is eroded from the dunes to build up a new foreshore. Storm impact on a dune system is frequently described by the lineal dune retreat and the consequent eroded volume. The eroded volume per alongshore unit length, $V_E$, is here defined as the dune volume loss restricted vertically between the onshore dune face and the landward back-face limit, $x=75.31$ m, according to the dune boundaries represented in Figure 1, and horizontally above the mean water level (see Figure 2a). The dune erosion volume can be also the integrated between the pre-storm, $z(x)|_{t_0}$, and the post-storm, $z(x)_{t_f}$ profiles, above the mean water level, as defined by equation (1). Equation (2) represents the relative eroded volume, where $V_{i}$ is the initial dune volume at the beginning of each test, above the mean water level, and $V_{Ei}$ is the eroded volume at instant $i$.

$$V_E = \int_{x_0}^{x_1} (z(x)_{t_1} - z(x)|_{t_0}) dx$$  \hspace{1cm} \text{(1)}$$

$$V_i = \frac{V_{Ei}}{V_0} \times 100$$  \hspace{1cm} \text{(2)}$$

$$R(t) = \Delta x = x_i - x_0$$  \hspace{1cm} \text{(3)}$$

<table>
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<th>Test</th>
<th>$H_{m0}$ (m)</th>
<th>$T_p$ (s)</th>
<th>$\xi$</th>
<th>Number of waves</th>
<th>Wave regimes</th>
<th>Duration (s)</th>
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Figure 1: Sketch of the dune initial cross-shore profile geometry

Table 1- Tests programme with wave conditions at wave paddle

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**Table 1. Tests programme with wave conditions at wave paddle**
According to equation (3), the dune retreat (Figure 2b) is the horizontal distance between the \( x_0 \) position of the dune crest before the wave attack and the \( x_i \) position of the dune crest at the end of the wave attack. In order to calculate the dune retreat variation in percentage values (equation 4), the horizontal distance, \( \Delta x \), was divided by the initial length of the dune, \( l_0 = 2 \) m (see Figure 1).

Figures 3a and 3b show the cumulative time evolution of the non-dimensional dune crest retreat (see equation (3)). It appears evident that retreat presents a different behaviour in the two intervals, \( n<500 \) and \( n>500 \), respectively; the larger values of the retreat rate have been observed during the first wave attack, with \( n=250 \), corresponding to the highest slope on the graphics. The reason for this behaviour is because the first wave series breaks directly onto the steep dune-face. When the dune erosion phenomenon is in progress, as a consequence of scarping at the toe of the dune, part of the eroded sand moves seaward and settles, forming a bar/step that dissipates the energy of the breaking waves (D’Alessandro et al., 2012). For tests E and F, after 2250 waves, the dune was totally eroded with dune retreats above 99% and 93%, respectively.

**Figure 2** – (a) Erosion volume, \( V_E \), boundaries. (b) Dune retreat, \( R \).**
the mean water level. All tests show, after a certain period, a steadiness in the retreat and eroded volume rates visible in the linear trends past the 500 waves, this stabilization meaning that the beach-dune profile is approaching an equilibrium state. It was observed that the eroded volume function has the same behaviour as the dune-crest retreat curve. It was found that the time evolution of the eroded volume has a logarithmic behaviour that follows the equation \( V_E = a \ln(n) + b \), for \( n > 0 \), where \( n \) is the number of waves and \( a, b \) are data-fitting constants. Minimum, average, and maximum values of \( a \) and \( b \) are -0.390, -0.240, -0.126, and 2.182, 3.023, 3.904, respectively.

Figure 4 shows that 40% to 60% of the initial dune volume have been eroded at the end of the tests. The tests G, H, I, L have the highest values of \( V_E \) essentially because this group belongs to the highest \( h_0 \) level.

### 3.1. Effect of storm surge level

The pairs of tests A-G, B-H and E-I varied the water depth from 2.35m to 2.50m, respectively, and kept constant \( H_s, o \) and \( T_p \) in order to simulate solely the storm surge effect. D’Alessandro et al. (2012), who also studied the B-H sequence, concluded that the results indicate that the storm surge influences the dune retreat and the eroded volume, in particular, high values of \( h_0 \) increase the dune retreat and eroded volume, \( R \) and \( V_E \).

Figure 5 (a) (b) and (c) show the comparison between the values of the dune retreat, the absolute volume and the relative volume for tests B and H, respectively, and confirms D’Alessandro et al. (2012) findings. It’s evident that dune crest retreat \( R \) and the eroded volumes, \( V_E \) and \( V_r \), increase for larger values of \( h_0 \) (case H). Furthermore, it’s noticed that different regimes (see Sallenger et al., 2003) of the dune-wave interaction were present, namely, collision regime for the test B and overwash conditions for test H. As expected, the wave regime is more intense for high water levels. The pairs A-G and E-I have the same behaviour as B-H set.

![Figure 4](image1.png)

**Figure 4** - Time evolution of the relative eroded volume to tests A to L.

### 3.2. Effect of wave peak period

The sets of tests D-E-F and B-C vary the peak period, \( T_p \), from 2.5, 3.0 and 3.5 s, respectively, keeping \( H_s, o \) and \( h_0 \) constants. According with van Gent et al. (2008), it was found that the retreat dune face is largest for the test with the longest wave period and the smallest for the shortest wave period. Also they
found that dune erosion volumes increase for a larger wave period. The results of Tomasicchio et al. are in concordance with van Gent, as figure 6 shows that test E ($T_p=3.0s$) and test F ($T_p=3.5s$) have larger values of dune retreat and eroded volumes than test D ($T_p=2.5s$). Furthermore, it is noticed that different regimes of the dune-wave interaction were present: collision regime for test D, minor overwash conditions for test E, and major overwash conditions for the test F. The wave regime is more intense during high $T_p$. The set B-C has the same behaviour as the D-E-F set.

3.3. Effect of significant wave height

The sets of tests A-B-D and C-E and G-H vary the significant wave height, $H_{o,s}$, from 0.25, 0.30 and 0.35 m, respectively, keeping the $T_p$ and $h_0$ constant. As figure 7 shows the dune crest retreat $R$ and the absolute and relative eroded volumes, $V_E$ and $V_r$, increase for larger values of $H_{o,s}$. For the two largest wave heights, however, results are almost similar, suggesting that, upon a certain threshold, an increase of the incident wave height will not cause an increase in the dune retreat and eroded volume. The sets C-E and G-H has the same behaviour as A-B-D set.

4. ANALYTICAL MODELS

4.1. Kriebel and Dean model

The basis for the convolution method (Kriebel and Dean, 1983) is the observation that beach response to steady-state forcing conditions is approximately exponential in time. For laboratory conditions, where an initially plane beach is subjected to a fixed water level and steady wave action, the response of any depth contour as a function of time, $R(t)$, according to this model may be approximated by the form:

$$R(t) = R_{\infty}(1 - e^{-t/T_s})$$

where $R_{\infty}$ is the maximum response of the contour (advance or retreat) that occurs after the system reaches equilibrium, and $T_s$ is the characteristic time scale of the exponential response.

It is assumed that for a time-varying water level, the maximum potential response may be determined for the peak water level while the erosion-forcing function is proportional to $R_{\infty}$ times a unit-amplitude function of time, $f(t)$. Thus, a linear differential equation governing the profile response to variations in water level is assumed to have the form:
\[
\frac{dR(t)}{dt} = \frac{1}{T_s} [R_\infty f(t) - R(t)]
\]

The general solution can be obtained by the convolution integral method and has the form of:

\[
R(t) = \alpha R_\infty \int_0^t f(\tau)e^{-\alpha(t-\tau)}d\tau
\]

where \(\alpha = 1/T_s\) and \(\tau\) is the time-lag. As a result, equation (6) suggests two important and general characteristics of beach-profile response: that the beach profile response will lag behind the erosion forcing; and that it will be damped relative to the maximum erosion potential of the system. Furthermore, it is evident that a beach has a certain “memory” so that the response at any one time is dependent on the forcing conditions that have occurred over some preceding period of time. In addition to the recession or retreat of the berm, the convolution method may be used to approximate the time-dependent volume of sand eroded from the beach face. In this case, it is assumed that all elevation contours erode at the same relative rate so that the dimensionless erosion, \(R(t)/R_\infty\), is the same everywhere (Figure 8). The relative volume eroded is then the same as the relative berm retreat such that:

\[
\frac{V(t)}{V_\infty} = \frac{R(t)}{R_\infty}
\]

where \(V(t)\) is the time-dependent volume eroded above some reference datum while \(V_\infty\) is the equilibrium eroded volume above that same datum.

In the Tomasicchio et al. (2011) cases presented above, the value of storm surge is null (S=0), which determines the impossibility to use directly the Kriebel and Dean equilibrium beach-profile formula (7). On the other hand, it was observed that the time evolution of the eroded volume follows a logarithmic function that enables the calculation of \(V_\infty\) and \(R_\infty\) using equation (8). The time scale of profile response, \(T_s\) is given by the empirical expression in the form (Kriebel and Dean, 1983):

\[
T_s = C_1 \frac{H_b^{3/2}}{g^{1/2}} \left(1 + \frac{h_b}{B} + \frac{m x_b}{B h_b}\right)^{-1}
\]

where \(C_1\) is an empirical coefficient, \(H_b\) is the breaking wave height, \(h_b\) is the water depth in the breaking zone, \(x_b\) is the surf zone width, \(B\) is the elevation above mean sea level, \(m\) is the linear beach-face slope and \(A\) is a parameter that governs the overall steepness of the profile:

\[
A = 2.25 \left(\frac{\alpha^2}{g}\right)^{1/3}
\]

**4.2. Larson et al. model**

A basic assumption in estimating dune erosion from ‘wave impact theory’ is that there is a linear relationship between the impact (force \(F\) on the dune due to change in the momentum flux of the bores impacting the dune) and the weight (\(\Delta W\)) of the sediment volume eroded from the dune. This may be written (Fisher et al., 1986; Overton et al., 1987; Nishi and Kraus, 1996; see Fig. 9 for a definition sketch) as

\[
\Delta W = C_E F
\]

where \(C_E\) is an empirical coefficient. The weight \(\Delta W\) can also be expressed as a function of the eroded volume \(\Delta V\):

\[
\Delta W = \Delta V \rho_s (1 - p) g
\]
where \( \rho_s \) is the density of the sediment, \( p \) is the porosity, and \( g \) is the acceleration of gravity. For a number of bores impacting the dune during a time period \( \Delta t \), the total swash force can be written as:

\[
F = \frac{1}{2} \rho u_0^4 \frac{\Delta t}{T}
\]

(13)

where \( u_0 \) is the speed of the bore and is given by Cross (1967) and Miller (1968) as \( u_0 = C_u \sqrt{g h_0} \), \( h_0 \) is the height of the bore, \( T \) is the period at which waves hit the dune taken to be equal to the incident wave period, \( C_u \) is an empirical coefficient.

From equation (11), (12) and (13) after some rearranging yields the rate of dune erosion during a storm, \( q_D \):

\[
q_D = \frac{\Delta V}{\Delta t} = -\frac{1}{2} C_E \frac{\rho}{\rho_s} \frac{u_0^4}{g^2 T (1-p)} C_u \frac{u_0^4}{g^2 T}
\]

(14)

where \( C_s \) can be written as:

\[
C_s = \frac{1}{2} C_E \frac{\rho}{\rho_s} \frac{1}{g^2 T (1-p)}
\]

(15)

In order to continue the derivation and arrive at an analytical model, some simplifying assumptions are introduced. First of all, the bore speed in front of the dune face \( (u_0) \) is needed, and it is estimated as:

\[
u_0^2 = u_s^2 - 2g z_0
\]

(16)

where \( u_s \) is the velocity of the bore as it starts its travel up the foreshore, and \( z_0 \) is the elevation difference between the dune foot and the beginning of the swash. If at the limit of the run-up, \( z_0=R \), the velocity \( u_s=0 \), then:

\[
u_s^2 = 2gR
\]

(17)

By substituting equation (17) and (16) into (14), and consider that the variation in \( z_0 \) with time is ignored, results:

\[
\frac{dV}{dt} = -4 C_s \frac{(R-z_0)^2}{T}
\]

(18)

For the case when \( R \) is constant and \( z_0=0 \), the following solution may be written as:

\[
\Delta V_E = 4 C_s R^2 \frac{t}{T}
\]

(19)

Figure 9 - Definition sketch for modeling dune erosion due to the impact of runup waves.

The run-up extension, \( R \), has been calibrated based on the observed values of \( \Delta V \):

\[
R = \tan(\beta) \sqrt{H_0 L_0}
\]

(20)

In the present work, the least square method was applied in order to reach a value of \( \beta = 12.79^\circ \), which minimizes the difference between the experimental and the estimated values of the eroded volume. This value is slightly greater than that used by Larson et al. (2004), \( \beta=9^\circ \), most likely due to the fact that
the run-up in the experiments of Tomasicchio et al. (2011) occurred over the steep slope of the dune face. The \( C_s \) value was determined using the empiric equation
\[
C_s = A e^{-\frac{H_{rms,0}}{d_{50}}}
\]  
where A and b are empirical coefficients and \( H_{rms,0} \) is the deepwater root-mean-square wave height.

4.3. Van Rijn model (2009)

The results of the sensitivity study based on the CROSMOR-model runs were used by van Rijn (2009) to develop a simplified dune erosion rule (DUNERULE-model) and calculate dune erosion volumes per longitudinal unit through the expression:
\[
A_{d,t=5} = A_{d,ref} \left( \frac{d_{50,ref}}{d_{50}} \right)^{\alpha_1} \left( \frac{S}{S_{ref}} \right)^{\alpha_2} \left( \frac{H_{rms,0}}{H_{rms,0,ref}} \right)^{\alpha_3} \left( \frac{T_p}{T_{p,ref}} \right)^{\alpha_4} \left( \frac{\tan \beta}{\tan \beta_{ref}} \right)^{\alpha_5} (1 + \frac{\theta_o}{100})^{\alpha_6}.
\]  
with:
- \( A_{d,t=5} \) = dune erosion area above storm surge level after 5 hours (m\(^3\)/m),
- \( A_{d,ref} \) = dune erosion area above S storm surge level after 5 hours in Reference Case= 170 (m\(^3\)/m),
- \( S \) = storm surge level above mean sea level (m),
- \( S_{ref} \) = storm surge level above mean sea level in Reference Case= 5 (m),
- \( H_{rms,0} \) = offshore significant wave height (m),
- \( H_{rms,0,ref} \) = offshore significant wave height in Reference Case= 7.6 (m),
- \( T_p \) = peak wave period (s),
- \( T_{p,ref} \) = peak wave period (s) in Reference Case= 12 (s),
- \( d_{50} \) = median bed material diameter (m),
- \( d_{50,ref} \) = median bed material diameter in Reference Case= 0.000225 (m),
- \( \tan \beta \) = coastal slope gradient defined as the slope between the -3 m depth contour (below mean sea level) and the dune toe (+3 m),
- \( \tan \beta_{ref} \) = coastal slope gradient defined as the slope between the -3 m depth contour and the dune toe (+3 m) for the Reference Case= 0.0222 (1 to 45),
- \( \theta_o \) = offshore wave incidence angle to coast normal (degrees),
- \( \alpha_1 \) = exponent=1.3,
- \( \alpha_2 \) = exponent=1.3 for \( S<S_{ref} \) and \( \alpha_2=0.5 \) for \( S>S_{ref} \),
- \( \alpha_3=\alpha_4=\alpha_5=0.5 \) (exponents),
- \( \alpha_6 \) = exponent=0.3.

The time development over \( t \) hours can be estimated from:
\[
A_{d,t=t} = A_{d,t=5} \left( \frac{t}{t_{ref}} \right)^{\alpha_7}
\]  
\( t \) = time in hours (\( t_{ref}=5 \) hours),
- \( \alpha_7 \) = exponent=0.5 for \( t<t_{ref} \) and 0.2 for \( t>t_{ref} \).

Essentially, the proposed method produces dune erosion values with respect to a defined “Reference Case”, studied by Vellinga (1986) and Delft Hydraulics (2004, 2006a,b, 2007), that represents a storm with a constant storm surge level, wave height and a duration of 5 hours. Equation (22) yields zero erosion for \( S=0 \) (no storm surge set-up). Tomasicchio et al. tests have no value for storm surge set-up, \( S \), during each specific test. To reach an \( S \) value valid for the model calibration, the least square method was applied, and it was obtained \( S=0.45 \)m for group tests A to F (\( h_0=2.35 \)m) and \( S=0.43 \) m for group tests G to H (\( h_0=2.50 \)m). It is noticed that the values for storm surge are consistent with the hydrodynamic conditions and the dune-beach geometry.

5. RESULTS AND DISCUSSION

5.1. Eroded volumes

Figure 10 presents, for each test of Tomasicchio et al. (2010), the time evolution of the eroded volume, observed in the laboratory and calculated by the different analytic models: Kriebel and Dean (1993), Larson et al. (2004) and van Rijn (2009).
It is concluded that the evolution of the dune eroded-volume predicted by the van Rijn method follows, in much the same way, the behaviour of the observations. For tests A, B, C, D and G this model overestimates the experimental values, i.e., at the end of each experiment the values are higher by about 9%, 4%, 15%, 3% and 9%, respectively. In tests E, F and H this model features an underestimation, by about 24%, 8% and 10% lower than the experimental observation. For the van Rijn model, for example, for tests E and H the differences of the eroded volume between the estimation and the observed, for n=250 waves, are below 1%, which is an excellent prediction after calibration of the model. So it could be concluded that the model van Rijn has a better agreement in the initial phase of the test than later. One reason for this issue is due to the variation of the face dune slope and the formation of underwater bars at the foreshore, that the model does not include.

The Kriebel and Dean model gives always underestimates the dune eroded-volume for all tests of Tomasicchio et al. (2010). It produces values for all tests between 6% and 33% lower than the experimentally observed values. The model of Larson et al. (2004) produces the dune eroded-volumes values with a linear trend over time, so that deviates from the behaviour observed experimentally. This method yields over estimates of the eroded volume in tests A, B, F and G, by 20%, 20%, 5% and 5%, respectively. For Tests C, D, E and H the model origins underestimates with estimated values of the dune eroded volume about 35%, 6%, 13% and 10% lower than those observed in the laboratory.

5.2. Performance indicator: Brier Skill Score (BSS)

The Brier Skill Score (BSS) gives an objective assessment on the capability of a model to provide accurate predictions (Eq. 24). It was described by van Rijn et al. (2003) and Roelvink et al. (2009) and compares the set of N dune eroded-volume data observed experimentally, $V_{\text{exp}}$, and the estimations of the dune eroded-volume for each model, $V_{m_i}$, using equation (24):

$$BSS = 1 - \frac{\text{Var}(V_{m_i} - V_{\text{exp}})}{\text{Var}(V_{\text{exp}})} \quad (24)$$

Interpretation of values of BSS were provided by Van Rijn et al. (2003), and indicated bad model performance for values BSS<0.3, poor for values BSS>0.3, reasonable/fair for 0.3-0.6, good for 0.6-0.8, and excellent over 0.8.
Table 1. BBS results for Tomasicchio et al. tests, from A to H, for the three different empirical models.

<table>
<thead>
<tr>
<th>Model</th>
<th>Kriebel &amp; Dean</th>
<th>Larson</th>
<th>Van Rijn</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test A</td>
<td>0.688</td>
<td>0.556</td>
<td>0.989</td>
</tr>
<tr>
<td>Test B</td>
<td>0.819</td>
<td>0.731</td>
<td>0.980</td>
</tr>
<tr>
<td>Test C</td>
<td>0.783</td>
<td>0.723</td>
<td>0.962</td>
</tr>
<tr>
<td>Test D</td>
<td>0.791</td>
<td>0.639</td>
<td>0.987</td>
</tr>
<tr>
<td>Test E</td>
<td>0.851</td>
<td>0.709</td>
<td>0.933</td>
</tr>
<tr>
<td>Test F</td>
<td>0.794</td>
<td>0.663</td>
<td>0.991</td>
</tr>
<tr>
<td>Test G</td>
<td>0.806</td>
<td>0.654</td>
<td>0.980</td>
</tr>
<tr>
<td>Test H</td>
<td>0.766</td>
<td>0.590</td>
<td>0.985</td>
</tr>
<tr>
<td><strong>Average</strong></td>
<td><strong>0.787</strong></td>
<td><strong>0.658</strong></td>
<td><strong>0.976</strong></td>
</tr>
</tbody>
</table>

It is presented in Table 12 the results of the BSS for the three empirical models, applied to all tests of Tomasicchio et al. (2010), from A to H. For all tests, the van Rijn model always achieves a rating greater than 0.93, which means that the predictions are considered excellent. Generally, the time evolution of the eroded-volume is best predicted by the van Rijn model, followed by the Kriebel and Dean model and, finally, the Larson et al. model, which produces less good results, but that can be still classified as “good” predictions. However, it can be said that the three models are a simple and effective tool for the evaluation and prediction of dune eroded volume.

6. CONCLUSIONS

Concerning the experimental observations, it is noted that, for all the tests of Tomasicchio et al. (2010), there is a maximum of the dune crest retreat at the beginning of each test, i.e., during the first range of incident waves (with a duration corresponding to n=250 waves), meaning that the dune erosion rate is more intense in the first attack of waves that break directly on to the dune face. Around 2250 waves the dune retreat is almost independent of the incident wave conditions and will be approaching to constant values, around 10% of the original width of the dune. At that stage, the beach-dune profile achieves a more stable configuration and closer in the balance with the incident energy. This reduction on dune erosion rates are mainly due to the natural barrier that dissipates the energy of the hydraulic action, and underwater bar, that forms with the deposit of the sediments that erode from the dune face. It is therefore evident that dune retreat presents a different behaviour in the two intervals: n<500 and n>500, respectively. Alongside, the volume of the dune will decrease over the successive waves of attacks and is revealed that this temporal decay has a logarithmic behaviour.

In this study, it were also compared the hydrodynamic conditions of the tests (the water depth, h₀, the wave peak period T_p, and the significant wave height, Hₛₐ) with the dune erosion results. It was observed that:

- an increase of the water depth, to simulate the storm surge effect, increased the eroded volume.
- a gradual increase in the wave peak period increased the dune-volume erosion rate and dune retreat values. It were also observed more intense collision regimes with the increase of the peak period.
- a gradual increase of the significant wave height always resulted in an increase of the eroded volume and decrease of the dune crest. Furthermore, it were observed more intense collision regimes with overwash conditions for larger values of the significant wave height.

In the second part of this work, the laboratory data sets were used to calibrate and verify the analytical models proposed by Kriebel and Dean (1993), Larson et al. (2004) and van Rijn (2009), in order to calculate and predict the values of the dune retreat and the dune eroded volume at specific time intervals. The models’ performances were computed using the Brier Skill Score (BSS). It was concluded that:

- the Kriebel and Dean model presents a good performance. However, it underestimates the values of the dune erosion volume, also in accordance with Oliveira (2013).
- the model of Larson et al. (2004) produces dune eroded-volumes values with a linear behaviour over time, so that deviates from the behaviour observed experimentally. The calibration of the model by the β value for the dune face slope did not improve the correlation between the numerical and the experimental profile. This model shows the least performance of all, according to BBS evaluation.
- the analytical model proposed by van Rijn shows to predict the dune eroded-volume evolution with excellent accuracy. The van Rijn results follow, in much the same way, the behaviour of the experimental observations.
Therefore, it is recommended the application of any of the above models to preliminary estimates in engineering problems, being preferable the use of van Rijn method.

7. ACKNOWLEDGMENTS

I would like to express my gratitude to my supervisors, Prof. Dr. Antonio Trigo Texiera and Dr. Francisco Sancho, for the patient guidance, encouragement, advice and fruitful discussion and comments, they have provided.

8. REFERENCES


