Unbalance between phases and Joule's losses in low voltage electric power distribution networks

Part II – Optimal strategies for reducing the imbalance

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Abstract **– Power quality, system efficiency and reliability are key aspects of distribution systems planning and operation. The imbalances in the node voltages and branch currents affect both power quality and efficiency. Imbalances cause increased energy losses and increased risk of overloads. Thus, the quality and energy prices are affected. Phase swapping can economically and effectively balance the feeder currents to improve power quality and reduce power system operation costs. In this thesis, methods are proposed for to solve the phase swapping optimization problem in order to minimize losses and swapping effort. A greedy algorithm and a genetic algorithm are developed to find a Pareto optimal set of solutions leaving up to the network operator the selection of the most convenient tradeoff solution.**

Key Words: Distributions systems, unbalance feeders, optimization, genetic algorithm, Pareto optimal set

I. INTRODUCTION

The power distribution system has an extremely important role in energy supply, as it provides consumers concentrated in cities, suburbs and remote areas. As such, the operation of the distribution system seeks to ensure service to all customers with quality and reliability. In power distribution systems, unbalanced loading is common phenomena. As a result, feeders are also unbalanced. The loading is unbalanced because it consists mostly of single-phase loads to feed, which are not evenly distributed through the phases. On the other hand, increased consumption and growth of decentralized production linked to low voltage system can emphasize imbalances.

The main problem of imbalanced feeders is the increase of energy losses. Besides this problem, the imbalanced feeders affects the quality of energy. For example, the unbalanced load causes the imbalanced voltages even when voltages in the source is balanced. Feeder imbalance describes a situation in which the voltages of a three-phase voltage source are not identical in magnitude, or the phase differences between them are not 120 electrical degrees, or both. It affects motors and other devices that depend upon a well-balanced three-phase voltage source. Unbalanced systems also increase the risk of overloading on a phase line or ground line. Overloading can cause overheating of cables and thus damaging them. In addition, an unbalanced feeder has a large ground current, which may trip the protective devices. Furthermore, in unbalanced feeders, the improvement of the utilization factor is limited by the capacity of the ground line or one phase line. This limitation can lead to investment in new lines to increase its capacity, representing huge investments costs. In certain situations, these investments can be avoided by phase balancing to increase the utilization factor [1, 2, 3].

Thus, it is necessary to optimize the distribution systems to achieve a phase balancing. A balanced system has a lower peak load, lower voltage drop and lower energy losses, resulting in increased reliability, power quality and lower price. There are two ways for balancing of three phases electrical systems. One is feeder's reconfiguration at system level; and the other one is phase swapping at the feeder level. The first one has as purpose to balance loads among feeders, therefore is not an effective technique to settle the unbalance problem. Phase swapping is an effective way to balancing a feeder in terms of its phases, which consists of change the connection of the loads or lateral branches among the phases of the line.

This thesis aims to develop methods for finding a minimum number of phase swapping to be held to minimize losses and imbalances of current between phases. The methods to be developed must provide a set of optimal solutions in order to leave the decision to the distribution system operator. The application of these methods will focus on the unbalanced networks detected by work of Pedro Gonçalves, under Part I of this dissertation. These methods should be as quick as possible.

II. NETWORK FEATURES

In power distribution system, the feeders are radial. The optimization is performed to the peak power of load because at this point the losses are greater and the risk of overload is greater too. The loads are modeled by current magnitudes [4], thus relieved from calculating voltage drop and increased the speed of the methods. The power of each load varies according to its installed capacity and its utilization factor. It is still used a simultaneity factor since the loads are not all the power peak at the same time [5].

III. Optimization Problem

A. Phase swapping

Phase swapping can be classified as nodal phase swapping and lateral phase swapping. Lateral phase swapping is to switch the laterals to the primary trunk. If lateral phase swapping is applied, all the nodes on this lateral will not be allowed for nodal phase swapping. Therefore, the lateral can be treated as a fictitious node on the primary trunk. It must ensure that the sequence of phases is the same for this don't affect the operation of three phase equipment. Thus, optimization may not be satisfactory. Nodal phase swapping is the load swapping at a node. Only single-phase loads are considered. Thus, these loads can be swapped for a certain phase independently of the other loads at the same node.

Phase balancing has several significant benefits, such as improving power quality and utilization factor of existing facilities, and reducing energy losses and price. However, each phase swapping operation is associated with certain costs on lineman expenses, maintenance expenses, and considered outage duration. The number of phase swapping need to be compromised between benefits and costs. Thus, the optimization problem is to perform the minimization of power losses and minimizing the number of exchanges. The problem is a combinatorial optimization problem with two objectives (multi-objectives). However, objectives under consideration conflict with each other.

B. Multi-objective optimization

Consider a decision vector $\mathbf{x} = \{x_1, x_2, \dots, x_n\}$ of dimension n, the solution space $\mathbf{X}$. The formulation of the problem of multi-objective optimization is defined as the minimization of a set of K objective functions,

$$\min_{x} \boldsymbol{F}(\mathbf{x}) = \{f_1(\mathbf{x}), \dots, f_K(\mathbf{x})\} \quad (1)$$

The solution space X is generally restricted by a series of constraints,

$$\begin{aligned} g_i(\mathbf{x}) &\leq 0 & i &= 1, \dots, m \\ h_j(\mathbf{x}) &= 0 & j &= 1, \dots, p \\ l_z \leq \mathbf{x} &\leq u_z & z &= 1, \dots, q \end{aligned} \quad (2)$$

There are two general approaches to multiple-objective optimization. One is to combine the individual objective functions into a single composite function. Determination of a single objective is possible with methods such as weighted sum method,

$$\min_{x} \boldsymbol{F}(\mathbf{x}) = \alpha f_1(\mathbf{x}) + \beta f_2(\mathbf{x}) + \dots + \gamma f_k(\mathbf{x}) \quad (3)$$

In practice, it can be very difficult to precisely and accurately select these weights, even for someone very familiar with the problem domain. Unfortunately, small perturbations in the weights can lead to very different solutions. On the other hand, there are non-commensurable objectives which do not convert to the same extent as other objectives, making it impossible to assign a weight [6].

The second general approach is to determine an entire Pareto optimal solution set or a representative subset. A Pareto optimal solutions is a set of solutions that are non-dominated with respect to each other [7]. A feasible solution $\mathbf{x}$ is said to dominate another feasible solution $\mathbf{y}$ ($\mathbf{x} \succ \mathbf{y}$), if and only if,

$$\begin{cases} f_i(\mathbf{x}) \leq f_i(\mathbf{y}), & \forall i \in \{1, \dots, k\} \\ f_j(\mathbf{x}) < f_j(\mathbf{y}), & \exists j \in \{1, \dots, k\} \end{cases} \quad (4)$$

A Pareto optimal solution cannot be improved with respect to any objective without worsening at least one other objective. The set of all feasible non-dominated solutions in $\mathbf{X}$ is referred to as the Pareto optimal set. The set of all possible Pareto optimal solutions constitutes a Pareto frontier in the objective space. Fig. 1 shows an example. In this way, a number of solutions can be found which provide the decision maker with insight into the characteristics of the problem before a final solution is chosen.

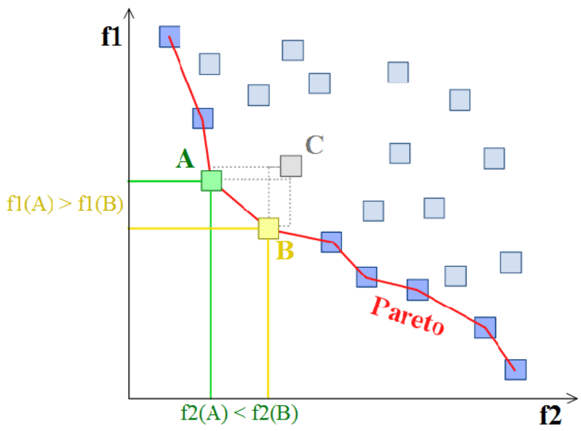

Fig. 1. Example of Pareto front and these optimal solutions

C. Mathematical formulation

Considering the problem to solve in this thesis, the minimization of power losses (P) and minimizing the number of phase swapping (M) to be made, the problem is defined by a set of N single phase loads, or single phase consumers, $C = \{C_1, C_2, \dots, C_N\}$. Each load is defined by its installed capacity, utilization factor, phase, and the network node that is connected. However, only the phase of each load is a decision variable due to its restrictions. Thus, the decision vector is defined according to:

$$\mathbf{x} = [C_j(f_i)] = \begin{bmatrix} C_1(f_i) \\ \vdots \\ C_N(f_i) \end{bmatrix}, \quad \begin{matrix} i = \{a, b, c\} \\ j = \{1, \dots, N\} \end{matrix} \quad (5)$$

Where i is the phase of the load C_j.

The objective corresponds to minimizing power losses and the number of phase swapping:

$$\min_{\mathbf{x}} \{P(\mathbf{x}), M(\mathbf{x})\} \quad (6)$$

Where:

$$P(\mathbf{x}) = \sum_{i=1}^{T} r_{i,a} I_{i,a}^2 + r_{i,b} I_{i,b}^2 + r_{i,c} I_{i,c}^2 + r_{i,0} I_{i,0}^2 \quad (7)$$

$$M(\mathbf{x}) = \frac{1}{2}\sum_{w=1}^{N}\left|\alpha_{a,w}^{j} - \theta_{a,w}^{j}\right| + \left|\alpha_{b,w}^{j} - \theta_{b,w}^{j}\right| + \left|\alpha_{c,w}^{j} - \theta_{c,w}^{j}\right| \quad (8)$$

Subject to:

$$I_{i,\varphi} = \sum_{k} I_{k,\varphi} + \sum_{w} \theta_{\varphi,w}^{j} d_{w}^{j} \quad (9)$$

$$\theta_{a,w}^{j} + \theta_{b,w}^{j} + \theta_{c,w}^{j} = 1 \quad (10)$$

$$\theta_{\varphi,w}^{j} \in \{0,1\} \quad (11)$$

In the formulation, the equation (7) is the power losses of solution **x**; the equation (8) determines the number of phase swapping of solution **x**, where $\alpha_{a,w}^{j}$ represents the original phase of w-th load and $\theta_{a,w}^{j}$ represents the phase of w-th load of solution **x**; equation (9) represents the current in branch i by application of Kirchoff's currents law; equation (10) ensures that the load w is linked to only one phase; and equation (11) is defined by the decision variable **x** and determines if the load w, in node j, is linked to phase φ.

D. Optimization techniques

General search and optimization techniques are classified into three categories: enumerative, deterministic, and stochastic. Although an enumerative search is deterministic a distinction is made here as it employs no heuristics. Techniques are developed for each category. However, the enumerative method is not much interest. Thus only deterministic and stochastic techniques are presented.

1) Greedy algorithm

The deterministic technique used is the greedy algorithm. This sort of myopic behavior is easy and convenient, making it an attractive algorithmic strategy. Greedy algorithms build up a solution piece by piece, always choosing the next piece that offers the most obvious and immediate benefit.

Since this is a combinatorial problem, application of this greedy algorithm reduces significantly the number of combinations to compute to find acceptable solutions for each phase swapping introduced. This reduction is due to the fact that each step of the algorithm, the load that contributes most to the reduction of power losses is chosen to be performed a phase swapping. In the next stage, the previous loads are considered and chosen another one load from among the possible combinations. Thus, the algorithm proceeds at pace in the number of phase swapping. The maximum number of combinations to be calculated by the greedy algorithm is given (12). This is an upper limit since it corresponds to the worst case, i.e., to a situation in which all loads are connected to same phase.

$$n = 2N + \sum_{i=2}^{M} 2.\left(N - (i-1)\right).2^{(i-1)} \quad (12)$$

2) Genetic Algorithm

Genetic algorithms differ from conventional techniques of demand, since they use an initial set of random solutions called population. Thus, working with the entire population and not a single point which provides for simultaneous searching across the solution space. That is, the genetic algorithm based on a stochastic technique.

The theoretical background of genetic algorithms were developed by Holland, in the beginning of the 70s, with the idea of imitating the evolutionary process which takes place within biological organisms in nature. It may be understood as a process of "intelligent" probabilistic search, which could be applied in a series of combinatory optimization problems. In the first place, it is known that evolution takes place by means of chromosomes, which store the genetic code that defines individual characteristics. Through a process of natural selection, individuals that are better fitted to the environment are able to reproduce with more frequency, transmitting their genetic traits to their descendants. Reproduction is the key point, in which evolution takes place. The recombination of the genetic code of the ancestors generates new chromosomes, which eventually undergo a process called mutation. With this process, descendants may present traits that are different from their ancestors, and eventually these traits will allow the individual to possess greater ability to adapt to the environment [8].

The genetic algorithm can be represented by pseudo code:

t=0
Generating an initial population P(t);
Assessing the fitness of the individuals in this population P(t)
Repeat
    t = t + 1;
    Select the fittest from P(t-1) to build P(t)
    Cross P(t)
    Mutate some solution from P(t)
    Evaluate P(t)
***Until** stopping criterion*

Within this perspective, the relevant aspects to be discussed are: representation of the chromosome, fitness evaluation and processes of natural selection, crossover and mutation.

Chromosome representation and population initialization

Every possible solution within a search space, or population, is represented as a sequence of elements where each element is called a gene, and each of these sequences formed by genes are the chromosomes. Thus, each solution is coded by a different sequence of genes.

In this thesis, each chromosome is represented by a listing and encoding is performed so that each gene represents the phase of each load. The possible values that can take these phases corresponds to 1, 2 or 3 that are connected in phase a, b or c, Fig. 2 shows an example.

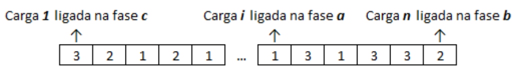

Fig. 2. Chromosome representation

To use the genetic algorithm is necessary to create a set of solutions, called population, from which will develop the entire evolutionary process. Typically, the population is generated randomly. In this case, the optimal solution is not unique and can be obtained by various combinations. Thus, the population is randomly generated but around the initial configuration of the phases to minimize the number of phase swapping.

The optimum size of a population depends on the type of problem to solve and the complexity of it. However, according to [9] the optimal size is between N and $2N$, where N is the number of genes in the chromosome.

Selection

The selection is based in a binary tournament. The idea is to select (at random) two solutions from the population and choose the solution with better fitness for the next generation (new population to be crossed). This process is repeated until the number of selected solutions matches the size of the population.

Crossover

The crossover operator is responsible for the exchange of genetic material between chromosomes. The recombination takes place between chromosomes chosen at random and occurs with a given probability. In this work the single point crossover is considered, which consists in choosing a random sectioning point (locus) in the structure of the ancestral chromosomes, and combining the left section of an ancestor with the right section of another ancestor. This operation allows two new descendants to be generated for each pair of ancestors selected. In Fig. 3 an example of application of this operator is presented.

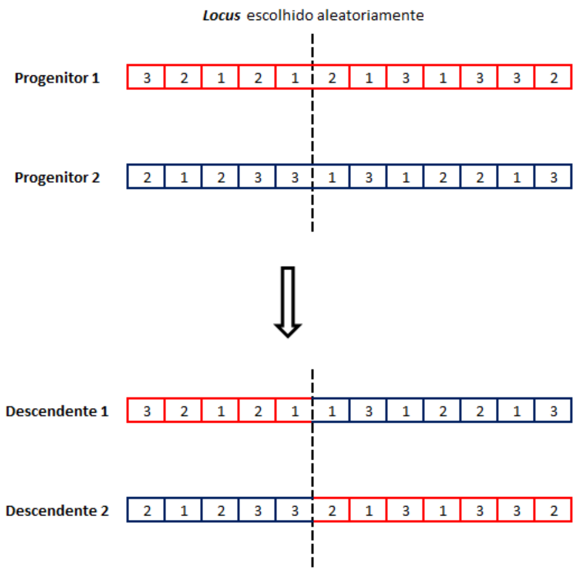

Fig. 3. Application of the single point crossover

Mutation

The mutation allows the introduction and maintenance of genetic diversity in the population. This operator is applied after the crossover and occurs with a given probability of mutation, typically less than the probability of crossover. The mutation consists of changing the value of a gene chosen randomly from a chromosome. In this thesis is used a ternary coding, the gene mutated assumes a randomly value within possible values, as depicted in Fig. 4.

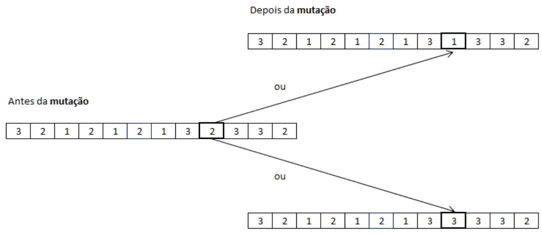

Fig. 4. Example of mutation

Fitness Function and Objective Function

The objective function, or evaluation function, provides a measure of performance with respect to a particular set of parameters. The fitness function transforms that measure of performance into an allocation of reproductive opportunities. In this thesis, there are two objectives and therefore two objective functions [10].

The merit function can be considered mono-objective considering only one objective function or combining the objective functions through a weighted sum as in equation (3).

The merit function can also be considered multi-objective to determine the Pareto optimal set. In these cases the merit function assigns the merits of each solution based on its dominance. One of these cases, in the ε-constraint technique [6, 11] is developed a selection scheme based on a tournament where the winning solution has to dominate another solution with a tolerance ε, such that:

$$\begin{cases} f_j(\mathbf{x}) < f_j(\mathbf{y}), & \exists j \in \{1, \ldots, k\} \\ f_i(\mathbf{x}) \leq f_i(\mathbf{y}) + \varepsilon, & \forall i \in \{1, \ldots, k\} \end{cases} \tag{13}$$

IV. Results and Comparison of Methods

The presented algorithms are now applied and compared. The main objective lies in obtaining the Pareto optimal set. The networks used have all imbalances in the feeder in order of 50% between phases with higher and lower amplitudes. The networks were obtained using D-Plan 2 software [12]. First, both algorithms are applied to an urban network with 53 nodes and 100 single-phase loads. After only the best method is applied to a large network and methods are presented to assist in choosing a solution.

A. *Greedy algorithm*

The results obtained by the greedy algorithm are present in the Fig. 5 and Table I. In Fig. 5, presents all the solutions determined in the course of the algorithm. Are still represented the solutions of the Pareto optimal set. The value of these solutions are shown in Table I.

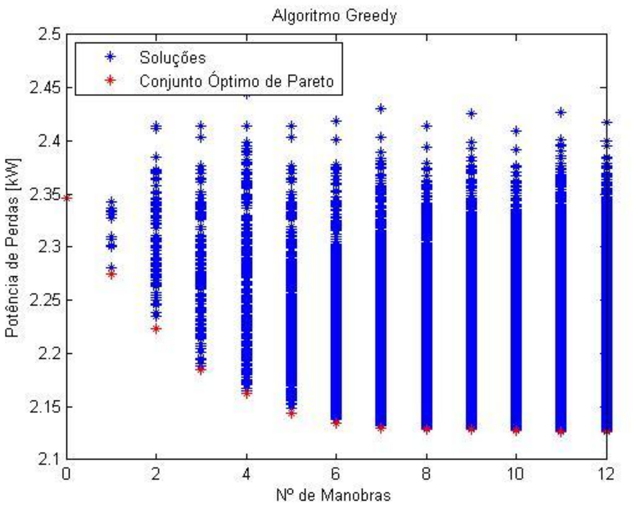

Fig. 5 - Representation of the solutions determined by the greedy algorithm

TABLE I
SOLUTIONS OF THE PARETO OPTIMAL SET

Runtime [s]	12997,82 (3h 36m 38s)		
Nº of Solutions	736929		
Peak Power [kW]	74,5050		
Swapped phases	**Power Losses**		
	W	*p.u.*	$\Delta P = \frac{P_i - P_0}{P_0} * 100$ [%]
0	2346,01	0,0315	0
1	2274,99	0,0305	-3,03
2	2222,98	0,0298	-5,24
3	2185,20	0,0293	-6,85
4	2162,51	0,0290	-7,82
5	2143,72	0,0288	-8,62
6	2134,10	0,0286	-9,03
7	2129,79	0,0286	-9,22
8	2129,14	0,0286	-9,24
9	2128,26	0,0286	-9,28
10	2127,73	0,0286	-9,30
11	2126,79	0,0285	-9,34
12	2126,65	0,0285	-9,35

The results are very good, however the Pareto optimal set is not completely defined. To be fully determined it was needed a bigger number of swapped phases, but the number of solutions increases exponentially with the number of swapped phases. In this case, solutions for up to 12 swapped phases, 736929 solutions were determined. This number of solutions are very high which makes the algorithm too slow. Thus, it is unfeasible to make more load swapping since the algorithm is already too slow.

B. Genetic algorithm

Genetic algorithm is used with two different merit functions. In the first case the merit function only minimizes power losses, i.e., find the solution that corresponds to the global minimum power losses without put any objection to the number of swapped phases. In the second case the merit function is multi-purpose and attributes the merit of each solution depending on their losses and the number of swapped phases.

1) Mono-objective genetic algorithm

For the case of single-objective genetic algorithm, the results are presented in Fig. 6 and Table III. It is expected that in the last generation the entire population has converged to the solution with the lowest losses. Thus, in order to determine the Pareto optimal set, the best solutions for each swapped phase are stored, without any interference in the AG.

The genetic parameters used in the process are presented in Table II.

TABLE II
PARAMETERS USED IN GENETIC ALGORITHM

Population dimension	150
Number of Generations	100
Probability of Crossover	0,7
Probability of Mutation	0,1

The number of solutions determined by the genetic algorithm is given by the population size times the number of generations. Thus, 15000 solutions are determined.

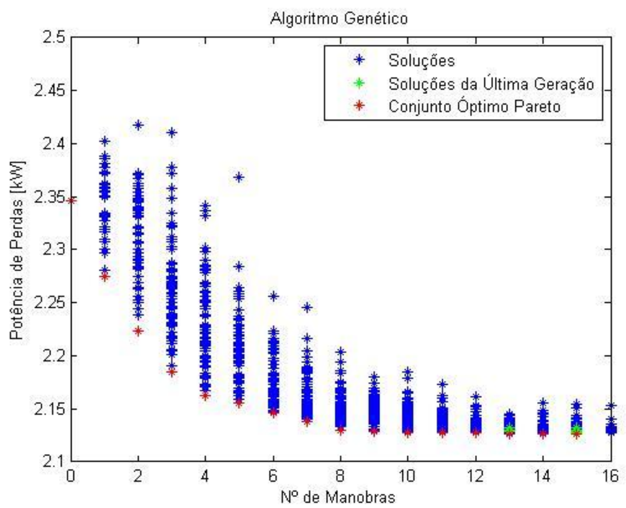

Fig. 6. Representation of the solutions determined by the mono-objective genetic algorithm

TABLE III
SOLUTIONS OF THE PARETO OPTIMAL SET

Runtime [s]		13,62		
Peak Power [kW]		74,5050		
Swapped phases	**Generation**	**Power Losses**		
		W	*p.u.*	$\Delta P = \frac{P_i - P_0}{P_0} * 100$ [%]
0	0	2346,01	0,0315	0
1	1	2274,99	0,0305	-3,03
2	4	2222,98	0,0298	-5,24
3	7	2185,20	0,0293	-6,85
4	11	2162,51	0,0290	-7,82
5	10	2155,16	0,0289	-8,14
6	15	2146,58	0,0288	-8,50
7	31	2137,71	0,0287	-8,88
8	28	2129,79	0,0286	-9,22
9	28	2129,15	0,0286	-9,24
10	32	2127,90	0,0286	-9,30
11	31	2127,57	0,0286	-9,31
12	42	2127,22	0,0286	-9,33
13	47	2126,79	0,0285	-9,34
14	60	2126,65	0,0285	-9,35
15	68	2126,60	0,0285	-9,35

The solutions of the Pareto optimal set are determined as the genetic algorithm evolves to minimize losses, as shown in column Generation of Table III. As expected, the surface formed by these solutions, the Pareto surface, is similar to an exponential function, $e^{-\alpha t}$, where the first swapped phases produce a significant reduction of losses and other swapped phases cause an negligible reduction.

On the Table IV are compared the results with the greedy algorithm and the mono-objective genetic algorithm.

TABLE IV
COMPARISON OF RESULTS BETWEEN THE GREEDY ALGORITHM AND GENETIC ALGORITHM

Swapped phases	**Power Losses [W]**		**Error [%]**
	Greedy Algorithm	**Genetic Algorithm**	
1	2274,99	2274,99	0
2	2222,98	2222,98	0
3	2185,20	2185,20	0
4	2162,51	2162,51	0
5	2143,72	2155,16	0,534
6	2134,10	2146,58	0,585
7	2129,79	2137,71	0,372
8	2129,14	2129,79	0,031
9	2128,26	2129,15	0,042
10	2127,73	2127,90	0,008
11	2126,79	2127,57	0,037
12	2126,65	2127,22	0,027
13	-	2126,79	-
14	-	2126,65	-
15	-	2126,60	-

The error of the genetic algorithm over the greedy algorithm is less than 1%. This error is negligible, since comparing these values with respect to peak power, i.e., values per unit, the results are the same. The solution with 15 swapped phases of all Pareto optimal set by the genetic algorithm has a power losses smaller than the last solution found by the greedy algorithm. However we cannot say that this solution corresponds to the global minimum power losses. The genetic algorithm does not guarantee that the final population converges to the global minimum due to its randomness.

2) Multi-objective genetic algorithm

In multi-objective genetic algorithm, it is expected that the solutions of last generation are dispersed along the Pareto surface. The selection is made in a tournament where the non dominated solutions win with ε-tolerance. That is, the solution x dominates solution y (x ≻ y) if and only if:

$$\begin{cases} M(\mathbf{x}) < M(\mathbf{y}) \\ P(\mathbf{x}) \leq (1+\varepsilon) * P(\mathbf{y}) \end{cases} \tag{14}$$

The ε-tolerance value was obtained experimentally. It was found that there is a range of ε that is between 1% and 4% of the power losses of the solution to compare.

The results obtained with this method are present in Fig. 7 and Table V, using ε equal to 0.025 and the parameters of table 2.

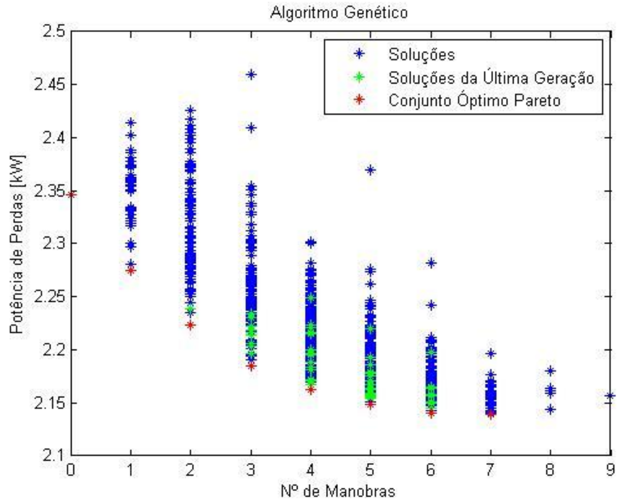

Fig. 7. Representation of the solutions determined by the multi-objective genetic algorithm

As expected, the solutions of the latest generation of multi-objective genetic algorithm are scattered in the area of the Pareto surface. However, this operation does not determine the entire surface as in the previous situation. The solutions along the 100 generations only focus on the area where the variation of power losses is higher. This is positive, since the solutions with practical significance are in this part of the Pareto surface, and thus it is possible to find the best solutions in this part of the Pareto surface in greater detail. These solutions are slightly better than the solutions of the case mono-objective genetic algorithm, however the surface of the Pareto set may be insufficient.

TABLE V
SOLUTIONS OF THE PARETO OPTIMAL SET

Runtime [s]		13,57		
Peak Power [kW]		74,5050		
Swapped phases	**Generation**	**Power Losses**		
		W	*p.u.*	$\Delta P = \frac{P_i - P_0}{P_0} * 100$ [%]
0	0	2346,01	0,0315	0
1	1	2274,99	0,0305	-3,03
2	6	2222,98	0,0298	-5,24
3	8	2185,20	0,0293	-6,85
4	26	2162,51	0,0290	-7,82
5	27	2143,72	0,0288	-8,62
6	76	2139,79	0,0287	-8,79
7	77	2138,91	0,0287	-8,83

Fig. 8 is represented the evolution of the average of power losses and the average of number of swapped phases. Observe that this situation of operation of genetic algorithm is oscillatory and never converges to a solution to the previous case. This can be explained by the existence of complex conjugate poles. There is also, as expected, that the number of operations varies inversely with power losses.

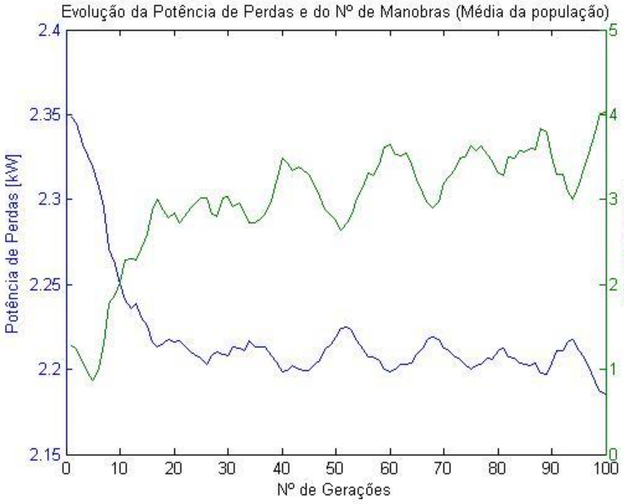

Fig. 8. Representation of evolution of the average of power losses and the average of number of swapped phases

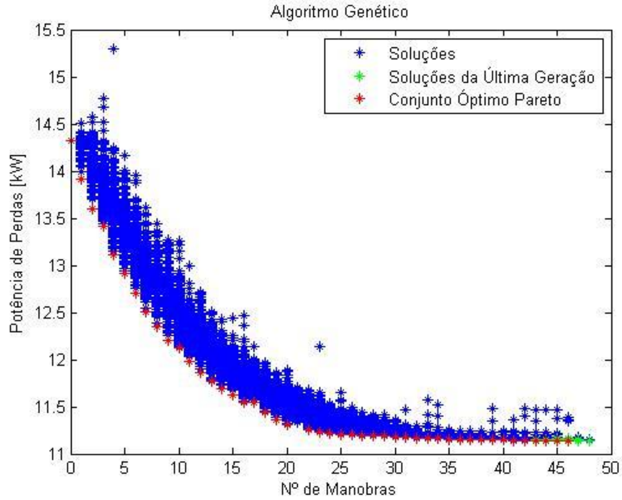

Fig. 9. Representation of the solutions determined by the mono-objective genetic algorithm

C. Comparison of used methods

The solutions of the Pareto optimal set determined by the greedy algorithm are better than solutions of genetic algorithm. However, the runtime of the greedy algorithm is very large and only it is possible to apply in networks with lesser loads. The solutions of the Pareto optimal set of genetic algorithm (single-objective and multi-purpose) in both cases are slightly worse than the solutions of the greedy algorithm. However, the error is negligible (lesser than 1%) and runtime is extremely fast.

The difference between the mono-objective and multi-objective genetic algorithm relates to the Pareto surface. Mono-objective genetic algorithm determines entire Pareto surface while the multi-objective genetic algorithm determines only the area with the greatest variation in losses. In some cases this surface may be insufficient.

D. Application to large rural network

Typically, the networks with higher losses and higher imbalances are rural networks, because of their size and dispersion of the loads. Thus, the optimization is presented to a rural network with 18 nodes and 176 single-phase loads with mono-objective genetic algorithm.

The population size is adjusted to ensure it is within the limits given earlier. The number of generations is also increased to ensure the convergence of the algorithm. The parameters used are in the table 6.

TABLE VI
PARAMETERS USED IN GENETIC ALGORITHM

Population dimension	300
Number of Generations	120
Probability of Crossover	0,7
Probability of Mutation	0,1

Initial losses of this network (0.1337 per unit) are much larger than the previous network (0.0315 per unit) due to greater extension of the network. The Pareto surface obtained for this network is identical to that obtained in previous network. The difference relates to the scale factor. Thus, we conclude that the number of operations is dependent only on the number of loads and power losses was mainly dependent on the type of network.

E. Helper methods for choosing a solution

The choice of a solution of Pareto optimal set depends of the decision of operator's network. To facilitate this choice, two helper methods are presented, one based on normalization of the power losses and the number of swapped phases and the other based on the level of improvement introduced by the number of swapped phases.

1) Normalization and H-square norm

The normalization should be based on costs associated with phase swapping and the reduction of losses. However these values are not available, and normalization is done based on the variation of losses and the maximum number of swapped phases in accordance with (15).

$$P_{i,Norm} = \frac{P_i - P_{min}}{P_{max} - P_{min}}$$
$$M_{i,Norm} = \frac{M_i}{M_{max}} \tag{15}$$

Performed normalization proceeds to the verification of the solution that minimizes the H-square norm.

$$min\|\mathbf{Y}\|_2$$
$$\|y_i\|_2 = \left(\left|P_{i,Norm}\right|^2 + \left|M_{i,Norm}\right|^2\right)^{\frac{1}{2}} \tag{16}$$

Where **Y** represents solutions of Pareto optimal set and $y_i = (P_{i,Norm}, M_{i,Norm})$.

The results are shown in Fig. 10.

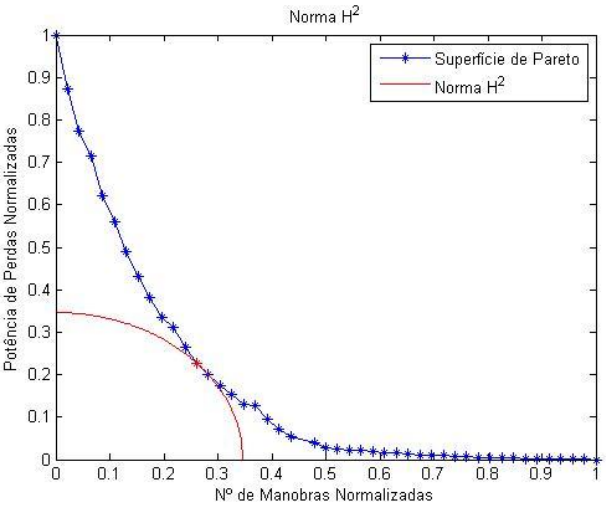

Fig.10. Representation of the solutions of Pareto optimal set and the minimum solution under H-square norm

The best solution is a solution of the Pareto optimal set with 12 swapped phases.

2) *Level improvement introduced by each swapped phase*

The introduction of each swapped phase has a different weight in reducing power losses. Thus, it is possible to have the solutions according to their impact in reducing power losses. Is considered that the solution with lower power losses, and consequently more swapped phases, corresponds to the global minimum and corresponds to optimizing the network to 100% in terms of power losses. In contrast, the initial configuration of the network, with any swapped phase and the highest power losses corresponds to 0% optimization.

The accumulated level of improvement in the number of swapped phases m, until a maximum of M, is given by:

$$F(m) = \frac{P_m - P_0}{P_M - P_0} * 100\% \quad (17)$$

And the level of improvement associated with each swapped phase can be obtained from:

$$f(m) = F(m) - F(m-1) \quad (18)$$

Applying this method to the case of the previous network, we obtain the following results presented in Table 7.

TABLE VII
REPRESENTATION OF THE LEVEL IMPROVEMENT INTRODUCED BY EACH SWAPPED PHASE IN REDUCING POWER LOSSES

Nº de Manobras	Potência de Perdas [W]	$F(m)$ [%]	$f(m)$ [%]
0	14320	0	0
1	13916	12,70	12,70
2	13603	22,54	9,84
3	13415	28,44	5,90
4	13118	37,78	9,34
5	12916	44,12	6,34
6	12698	50,96	6,84
7	12513	56,78	5,82
8	12347	61,98	5,20
9	12205	66,45	4,47
10	12126	68,92	2,47
11	11984	73,40	4,48
12	11861	77,25	3,85
13	11775	79,95	2,70
14	11691	82,61	2,66
15	11623	84,75	2,14
16	11547	87,12	2,37
17	11540	87,35	0,24
18	11436	90,60	3,25
19	11366	92,82	2,22
20	11306	94,69	1,87
22	11263	96,05	1,36
23	11225	97,24	1,19
24	11216	97,51	0,27
25	11206	97,82	0,31
26	11204	97,90	0,08
27	11194	98,20	0,29
28	11190	98,35	0,15
29	11186	98,47	0,12
30	11178	98,70	0,23
31	11172	98,89	0,19
32	11167	99,05	0,16
33	11165	99,11	0,06
34	11162	99,22	0,11
35	11156	99,40	0,18
36	11153	99,51	0,11
37	11151	99,55	0,04
38	11149	99,63	0,08
39	11146	99,71	0,08
40	11145	99,76	0,05
41	11143	99,83	0,07
42	11142	99,85	0,02
43	11140	99,92	0,07
44	11139	99,95	0,03
45	11137	99,99	0,04
46	11137	100,00	0,01

Note that in only 6 of 46 swapped phases is made more than 50% reduction of maximum power losses.

V. CONCLUSIONS

In this thesis, optimization methods have been proposed to reduce energy losses and improve quality of service in strongly unbalanced low-voltage networks. The optimization is based on the phase swapping, i.e., single-phase load switching between phases, in order to balance the most important branches currents.

The objective of the optimization methods developed has been set to minimize the number of swapped phases and power losses. These objectives are conflicting, which makes the simultaneous minimization a difficult task. Therefore, methods have been developed to determine a Pareto optimal set, leaving up to the network operator the selection of the best tradeoff between losses reduction and number of phase swapping operations.

A greedy algorithm and a genetic algorithm have been developed to find the Pareto set. The greedy algorithm has shown to perform better but with very high runtimes for real sized networks. The genetic algorithm has shown to perform slightly worse than the greedy algorithm (solutions' errors below 1%) but with very small runtimes when compared to the greedy.

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